

## PROBLEM SET 7: CHAIN RULE AND IMPLICIT DIFFERENTIATION

Note: Most of the problems were taken from the textbook [1].

**Problem 1.** Differentiate the next functions.

a)  $y = \sin(\tan 2x)$ ;

b)  $y = \frac{\cos \pi x}{\sin \pi x + \cos \pi x}$ ;

c)  $f(x) = \sqrt{x + \sqrt{x + \sqrt{x}}}$ .

**Problem 2.** Find the points on the graph of the function  $f(x) = 2 \sin x + \sin^2 x$  at which the tangent line is horizontal.

**Problem 3.** If  $F(x) = f(3f(4f(x)))$ , where  $f(0) = 0$  and  $f'(0) = 2$ , find  $F'(0)$ .

**Problem 4.** If  $f(xf(xf(x)))$ , where  $f(1) = 2$ ,  $f(2) = 3$ ,  $f'(1) = 4$ ,  $f'(2) = 5$ , and  $f'(3) = 6$ , find  $F'(1)$ .

**Problem 5.** Find  $dy/dx$  by implicit differentiation.

a)  $\tan(x/y) = x + y$ ;

b)  $\sqrt{xy} = 1 + x^2y$ ;

c)  $\tan(x - y) = \frac{y}{1+x^2}$ .

**Problem 6.** If  $xy + y^3 = 1$ , find the value of  $y''$  at the point where  $x = 0$ .

**Problem 7.** Find an equation of the tangent line of the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point  $(x_0, y_0)$ .

**Problem 8.** Find  $y'$  if  $x^y = y^x$ .

**Problem 9.** Find a formula for  $f^{(n)}(x)$  if  $f(x) = \ln(x - 1)$ .

### REFERENCES

- [1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.