

PROBLEM SET 5: NOTION AND FORMAL DEFINITION OF DERIVATIVES

Note: Most of the problems were taken from the textbook [1].

Problem 1. Find an equation of the tangent line to the curve at the given point.

a) $y = 4x - 3x^2$, $(2, -4)$;

b) $y = \sqrt{x}$, $(1, 1)$;

c) $y = \frac{2x+1}{x+2}$, $(1, 1)$.

Problem 2. Sketch the graph of a function f where the domain is $(-2, 2)$, $f'(0) = -2$, $\lim_{x \rightarrow 2^-} f(x) = \infty$, f is continuous at all points in its domain except ± 1 , and f is odd.

Problem 3. If $f(x) = 3x^2 - x^3$, find $f'(1)$ and use to find an equation of the tangent line to the curve $y = 3x^2 - x^3$ at the point $(1, 2)$.

Problem 4. Find $f'(a)$ by using the definition of derivative.

a) $f(x) = 2x^3 + x$;

b) $f(x) = x^{-2}$;

c) $f(x) = \frac{4}{\sqrt{1-x}}$.

Problem 5. Find the derivative of the function $f(x)$ using the definition of derivative.

a) $f(x) = 4 + 8x - 5x^2$;

b) $f(x) = 1/\sqrt{x}$;

c) $f(x) = x^4$;

d) $f(x) = \frac{1-2x}{3+x}$.

Problem 6. Show that the function $f(x) = |x - 6|$ is not differentiable at 6. Find a formula for f' and sketch its graph.

Problem 7. Sketch the graph of the function $f(x) = x|x|$, and find the values of x where f is differentiable. Find a formula for f' .

REFERENCES

- [1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.