Vector Functions

Problem 1 (Stewart, Exercise 13.1.50). *Two particle travel along the space curves*

$$r_1(t) = \langle t, t^2, t^3 \rangle \qquad t_2(t) = \langle 1+2t, 1+6t, 1+14t \rangle.$$

Do the particles collide? Do their paths intersect?

Problem 2 (Stewart, Exercise 13.2.41). *Find* $r(t)$ *if* $r'(t) = \langle 2t, 3t^2, \sqrt{t} \rangle$ *and* $r(1) = \langle 1, 1, 0 \rangle$.

Problem 3 (Stewart, Exercise 13.2.53). *Show that if* r *is a vector function such that* r'' *exists, then*

$$\frac{d}{dt}[r(t) \times r'(t)] = r(t) \times r''(t).$$

Problem 4 (Stewart, Exercise 13.2.55). *If* $r(t) \neq 0$*, show that*

$$\frac{d}{dt}|r(t)| = \frac{1}{|r(t)|} r(t) \cdot r'(t).$$

[Hint: $|r(t)|^2 = r(t) \cdot r(t)$*.]*

Problem 5 (Stewart, Exercise 13.2.55). *If a curve has the property that the position vector* $r(t)$ *is always perpendicular to the tangent vector* $r'(t)$*, show that the curve lies on a sphere with center the origin. [Hint: Show that* $|r(t)|$ *and* $r(t) \cdot a$ *are constant.]*

Problem 6 (Berkeley Midterm 1). *The position vector* $r(t)$ *of a particle moving in three dimensions satisfies* $r' = r \times a$*, where* a *is a fixed vector. Show that the particle moves withing a circle.*

Problem 7 (Stewart, Exercise 13.3.24). *Find the curvature of* $r(t) = \langle t^2, \ln t, t \ln t \rangle$ *at the point* $(1, 0, 0)$.

Problem 8 (Stewart, Exercise 13.3.48). *Find the vectors* T*,* N*, and* B *of the curve* $r(t) = \langle \cos t, \sin t, \ln(\cos t)$ *at* $(1, 0, 0)$.

Problem 9 (Stewart, Exercise 13.3.51). *Find equations of the osculating circles of the ellipse* $9x^2 + 4y^2 = 36$ *at the points* $(2, 0)$ *and* $(0, 3)$.

Problem 10 (Stewart, Exercise 13.3.55). *Find equations of the normal and osculating planes of the curve of intersection of the parabolic cylinders* $x = y^2$ *and* $z = x^2$ *at the point* $(1, 1, 1)$.

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.