The Chain Rule

Problem 1 (jS, Exercises 14.5.53)**.** *If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that*

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}.$$

Problem 2 (jS, Exercises 14.5.54)**.** *Suppose that $z = f(x, y)$, where $x = g(s, t)$ and $y = h(s, t)$. Show that*

$$\frac{\partial^2 z}{\partial t^2} = \frac{\partial^2 z}{\partial x^2}\left(\frac{\partial x}{\partial t}\right)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\frac{\partial x}{\partial t}\frac{\partial y}{\partial t} + \frac{\partial^2 z}{\partial y^2}\left(\frac{\partial y}{\partial t}\right)^2 + \frac{\partial z}{\partial x}\frac{\partial^2 x}{\partial t^2} + \frac{\partial z}{\partial y}\frac{\partial^2 y}{\partial t^2}.$$

Problem 3 (UC Berkeley midterm)**.** *Assume that the two equations $f(x, y, z) = 0$ and $g(x, y, z) = 0$ together implicitly define y and z as functions of x. Find formulas for $y' = dy/dx$ and $z' = dz/dx$ in terms of the partial derivatives of f and g. [Hint: Use chain rule with respect to x to differentiate both equations $f(x, y(x), z(x)) = 0$ and $g(x, y(x), z(x)) = 0$ and then solve the 2×2 remaining system.]*

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.