General Changes of Coordinates

Problem 1 (Stewart, Example 15.9.(23,25,27)). *Evaluate the following integrals.*

(1) $\iint_R \frac{x-2y}{3x-y}\,dA$, *where* R *is the parallelogram enclosed by the lines* $x-2y=0$, $x-2y=4$, $3x-y=1$, *and* $3x-y=8$.

(2) $\iint_R \cos\left(\frac{y-x}{y+x}\right)dA$, *where* R *is the trapezoidal region with vertices* $(1,0)$, $(2,0)$, $(0,2)$, *and* $0,1$.

(3) $\iint_R e^{x+y}\,dA$, *where* R *is given by the inequality* $|x|+|y|\leq 1$.

Problem 2 (Stewart, Example 15.9.3). *Evaluate* $\int\int_R e^{(x+y)/(x-y)}d\,A$, *where* R *is the trapezoidal region with vertices* $(1,0)$, $(2,0)$, $(0,-2)$, *and* $(0,-1)$. *[Hint: Define* T *by* $u=x+y$ *and* $v=x-y$ *and find that* T^{-1} *is defined by* $x=1/2(u+v)$ *and* $y=1/2(u-v)$ *and, therefore,* $\partial(x,y)/\partial(u,v)=-1/2$*][Answer:* $3/4(e-e^{-1})$*]*

Problem 3 (Stewart, Exercise 5.9.28). *Let* f *be continuous on* $[0,1]$ *and let* R *be the triangular region with vertices* $(0,0)$, $(1,0)$, *and* $(0,1)$. *Show that*

$$\iint_R f(x+y)\,dA=\int_0^1 uf(u)\,du.$$

Problem 4 (Stewart, Exercise 5.56). *Find the volume of the region bounded by the surface* $\sqrt{x}+\sqrt{y}+\sqrt{z}=1$ *and the coordinate planes. [Hint: Use the change of coordinates* $x=u^2$, $y=v^2$, *and* $z=w^2$*.]*

Problem 5 (Stewart, Exercise 5.57). *Evaluate* $\iint_R xy\,dA$, *where* R *is the square with vertices* $(0,0)$, $(1,1)$, $(2,0)$, *and* $(1,-1)$. *[Hint: Use an appropriate change of coordinates.]*

Problem 6 (UC Berkeley Final). *Let* D *be the region in the plane where* $x^2+2y^2\leq 1$. *Compute the double integral*

$$\iint_D (x^2+2y^2)^{2018}\,dA.$$

[Hint: Use $u=x$ *and* $y=\sqrt{2}y$*. Answer:* $\sqrt{2}\pi/4038$*.]*

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.