Spherical Coordinates

Problem 1 (Stewart, Exercise 15.8.(17,18)). *In each case, sketch the solid whose volume is given by the following integrals:*

(1) $\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2 \sin\phi \, d\rho d\theta d\phi$,

(2) $\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec\phi} \rho^2 \sin\phi \, d\rho d\theta d\phi$.

Problem 2 (Stewart, Exercise 15.8.27). *Find the volume of the part of the ball $\rho \leq a$ that lies between the cones $\phi = \pi/6$ and $\phi = \pi/3$.*

Problem 3 (Stewart, Exercise 15.8.28). *Find the average distance from a point inside a ball of radius a to its center.*

Problem 4 (Stewart, Exercise 15.8.30). *Find the volume of the solid that lies within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$.*

Problem 5 (Stewart, Exercise 15.8.41). *Evaluate*

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz dy dx.$$

[Hint: Use spherical coordinates.]

Problem 6 (Cal Final). *Let E be the region defined by the inequalities*

$$x^2 + y^2 + z^2 \leq 4, \quad 0 \leq y \leq x, \quad z \geq 0.$$

Calculate the total mass of E if the mass density is given by z^2.

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.