Polar and Cylindrical Coordinates

Problem 1 (Stewart, Exercise 15.3.(15,17))**.** *Use double integrals to find the areas of the following regions.*

(1) *One loop of the rose* $r = \cos 3\theta$*.*

(2) *The region inside the circle* $(x-1)^2 + y^2 = 1$ *and outside the circle* $x^2 + y^2 = 1$*.*

Problem 2 (Stewart, Exercise 15.3.27)**.** *Use polar coordinate to find the volume of the solid inside both the cylinders* $x^2 + y^2 = 4$ *and the ellipsoid* $4x^2 + 4y^2 + z^2 = 64$*.*

Problem 3 (Stewart, Exercise 15.3.(29,30))**.** *Evaluate the following integrals:*

$$(1)\ \int_0^2 \int_0^{\sqrt{4-x^2}} e^{-x^2-y^2}\, dydx \qquad (2)\ \int_0^a \int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} (2x+y)\, dxdy.$$

Problem 4 (Stewart, Exercise 15.3.35)**.** *A swimming pool is circular with a* 40*-feet diameter. The depth is constant along east-west lines and increases linearly from* 2 *feet at the south end to* 7 *feet at the north end. Find the volume of water in the pool.*

Problem 5 (Stewart, Exercise 15.7.(15,16))**.** *In each case, sketch the solid whose volume is given by the following integrals:*

(1) $\int_{-\pi/2}^{\pi/2} \int_0^2 \int_0^{r^2} r\, dzdrd\theta$,

(2) $\int_0^2 \int_0^{2\pi} \int_0^r r\, dzd\theta dr$.

Problem 6 (Stewart, Exercise 15.7.21)**.** *Evaluate* $\iiint_E x^2\, dV$*, where* E *is the solid that lies within the cylinder* $x^2 + y^2 = 1$*, above the plane* $z = 0$*, and below the cone* $z^2 = 4x^2 + 4y^2$*.*

Problem 7 (Stewart, Exercise 15.7.25)**.** *Find the volume of the region* E *that lies between the paraboloid* $z = 24 - x^2 - y^2$ *and the cone* $z = 2\sqrt{x^2+y^2}$*. Find the centroid of* E *(i.e., the center of mass in constant density).*

Problem 8 (Stewart, Exercise 15.7.29)**.** *Evaluate*

$$\int_{-2}^2 \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^2 xz\, dzdxdy.$$

[Hint: Use cylindrical coordinates.]

References

[1] J. Stewart: *Calculus* 8th Edition, Cengage Learning, Boston 2016.