

Problem 1. Determine whether the sequence $a_n = \sin\left(\sqrt[n]{\pi^{209+n}}\right)$ converges or diverges. If it is convergent, find the limit.

Problem 2. Determine whether the series

$$\sum_{n=1}^{\infty} \sqrt[5]{\frac{n^2}{n^2 + 20n + 9}}$$

is convergent or divergent. If it is convergent, find the sum.

Problem 3. Find the exact area of the surface of revolution obtained by rotating the curve $x = 1 + 2y^2$, $2 \leq y \leq 3$ about the x -axis.

Problem 4. Use the Integral Test to determine whether the series $\sum_{n=1}^{\infty} n^2 \pi^{-n^3}$ is convergent or divergent.

Problem 5. *Determine whether the series*

$$\sum_{n=1}^{\infty} \left(\frac{1}{n^2 + 5n + 6} + \frac{2}{e^{n+1}} \right)$$

is convergent or divergent. If it is convergent, find the sum.

A Few Standard Integrals

$$(1) \int a^x dx = \frac{a^x}{\ln a} + C;$$

$$(2) \int \sec^2 x dx = \tan x + C;$$

$$(3) \int \csc^2 x dx = -\cot x + C;$$

$$(4) \int \tan x dx = \ln |\sec x| + C;$$

$$(5) \int \cot x dx = \ln |\sin x| + C;$$

$$(6) \int \sec x dx = \ln |\sec x + \tan x| + C;$$

$$(7) \int \csc x dx = -\ln |\csc x + \cot x| + C;$$

$$(8) \int \frac{1}{x^2+a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C;$$

$$(9) \int \frac{1}{\sqrt{a^2-x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0.$$

Formula for the Area of a Surface of Revolution

$$A_f([a, b]) = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$