

## SOLUTIONS FOR QUIZ 1

Note: Most of the problems were taken from the textbook [1].

**Problem 1.** Solve the integral

$$\int \cos^5 t \sin^6 t dt.$$

*Solution:* Taking  $u = \sin t$  one has that  $du = \cos t dt$  and, therefore,

$$\begin{aligned} \int \cos^5 t \sin^6 t dt &= \int (1 - \sin^2 t)^2 \sin^6 t \cos t dt \\ &= \int (1 - u^2)^2 u^6 du \\ &= \int u^6 - 2u^8 + u^{10} du \\ &= \frac{u^7}{7} - 2\frac{u^9}{9} + \frac{u^{11}}{11} + C \\ &= \frac{\sin^7 t}{7} - 2\frac{\sin^9 t}{9} + \frac{\sin^{11} t}{11} + C. \end{aligned}$$

□

**Problem 2.** Solve the integral

$$\int x^2 5^x dx.$$

*Solution:* Let us use integration by parts taking  $u = x^2$  and  $dv = 5^x dx$ . In this case,  $du = 2x dx$  and  $v = 5^x / \ln 5$ , which implies that

$$(0.1) \quad \int x^2 5^x dx = \frac{x^2 5^x}{\ln 5} - \frac{2}{\ln 5} \int x 5^x dx.$$

To compute the integral in the right-hand side of (0.1), we applying again integration by parts setting this time  $u = x$  and  $dv = 5^x dx$ . Then  $du = dx$  and  $v = 5^x / \ln 5$ , and we obtain

$$\int x^2 5^x dx = \frac{x^2 5^x}{\ln 5} - \frac{2}{\ln 5} \left( \frac{x 5^x}{\ln 5} - \frac{1}{\ln 5} \int 5^x dx \right) = \frac{x^2 5^x}{\ln 5} - \frac{2x 5^x}{(\ln 5)^2} + \frac{2 \cdot 5^x}{(\ln 5)^3} + C.$$

□

**Problem 3.** Evaluate the integral

$$\int_0^2 \frac{dt}{\sqrt{4+t^2}}.$$

*Solution:* Taking  $t = 2 \tan u$ , we get  $\sqrt{4+t^2} = 2 \sec u$  and  $dt = 2 \sec^2 u \, du$ . As result,

$$\int_0^2 \frac{dt}{\sqrt{4+t^2}} = \int_0^{\pi/4} \sec u \, du = \ln |\sec u + \tan u| \Big|_0^{\pi/4} = \ln(\sqrt{2} + 1).$$

□

**Problem 4.** Solve the integral

$$\int \frac{x^3 - 3x^2 + 9x + 14}{x^2 - 4x + 13} dx.$$

*Solution:*

$$\begin{aligned} \int \frac{x^3 - 3x^2 + 9x + 14}{x^2 - 4x + 13} dx &= \int \left( x + 1 + \frac{1}{x^2 - 4x + 13} \right) dx \\ &= \frac{x^2}{2} + x + \int \frac{dx}{(x-2)^2 + 3^2} \\ &= \frac{x^2}{2} + x + \frac{1}{3} \tan^{-1} \left( \frac{x-2}{3} \right) + C. \end{aligned}$$

□

**Problem 5.** Solve the integral

$$\int \sec^4 x \tan^3 x \, dx.$$

*Solution:* Taking  $u = \tan x$  we obtain  $du = \sec^2 x \, dx$ . Thus,

$$\begin{aligned} \int \sec^4 x \tan^3 x \, dx &= \int (1 + \tan^2 x) \tan^3 x \sec^2 x \, dx \\ &= \int (1 + u^2) u^3 \, du \\ &= \frac{u^4}{4} + \frac{u^6}{6} + C \\ &= \frac{\tan^4 x}{4} + \frac{\tan^6 x}{6} + C. \end{aligned}$$

□

**Problem 6.** Solve the integral

$$\int \frac{dx}{\sqrt{x^2 + 6x + 10}}.$$

*Solution:* Taking  $x + 3 = \tan u$ , one finds that  $dx = \sec^2 u \, du$ . Therefore

$$\begin{aligned} \int \frac{dx}{\sqrt{x^2 + 6x + 10}} &= \int \frac{dx}{\sqrt{(x + 3)^2 + 1}} \\ &= \int \sec u \, du \\ &= \ln |\sec u + \tan u| + C \\ &= \ln |\sqrt{x^2 + 6x + 10} + x + 3| + C. \end{aligned}$$

□

**Problem 7.** Solve the integral

$$\int \frac{x^2 - 7}{x^2 + 4x + 3} \, dx.$$

*Solution:*

$$\begin{aligned} \int \frac{x^2 - 7}{x^2 + 4x + 3} \, dx &= \int \left( 1 - \frac{4x + 10}{x^2 + 4x + 3} \right) \, dx \\ &= x - \int \frac{3}{x + 1} \, dx - \int \frac{1}{x + 3} \, dx \\ &= x - 3 \ln |x + 1| - \ln |x + 3| + C. \end{aligned}$$

□

#### REFERENCES

- [1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.