PROBLEM SET 19: TAYLOR AND MACLAURIN SERIES

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Find the Taylor series representation for $f(x)$ centered at the given value of a. Also, find the associated radius of convergence.*

a) $f(x) = x^5 + 2x^3 + x, \ \ a = 2$;

b) $f(x) = \ln x, \ a = 5$;

c) $f(x) = 1/x, \ a = 3$;

d) $f(x) = \tan^{-1}(x^3), \ a = 0$;

e) $f(x) = \sqrt{x}, \ a = 16$;

f) $f(x) = \frac{x^2}{\sqrt{2+x}}, \ a = 0$;

g) $f(x) = \sqrt[3]{8+x}, \ a = 0$;

h) $f(x) = x^2 \ln(1+x^3), \ a = 0$;

i) $f(x) = \sin^2 x, \ a = 0$.

Problem 2. *Use Maclaurin series to find the following limits.*

a) $\lim_{x\to 0} \frac{x-\ln(1+x)}{x^2}$;

b) $\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$;

c) $\lim_{x\to 0} \frac{\sqrt{1+x}-1-\frac{1}{2}x}{x^2}$;

d) $\lim_{x\to 0} \frac{x^3-3x+3\tan^{-1} x}{x^5}$.

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.