# PROBLEM SET 11: SERIES

Note: Most of the problems were taken from the textbook [1].

**Problem 1.** *Decide whether each series is convergent or divergent. If it is convergent, compute its value.*

*a)* $\sum_{n=1}^{\infty} \frac{5}{\pi^n}$;

*b)* $\sum_{n=0}^{\infty} \frac{3^{n+1}}{(-2)^n}$;

*c)* $\sum_{n=1}^{\infty} (\sin 100)^n$;

*d)* $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

*e)* $\sum_{n=0}^{\infty} (\sqrt{3})^{-n}$;

*f)* $\sum_{n=1}^{\infty} \tan^{-1}(n)$;

*g)* $\sum_{n=0}^{\infty} \frac{1}{n^2+3n+2}$

*h)* $\sum_{n=1}^{\infty} \left( \frac{1}{e^n} + \frac{1}{n(n+1)} \right)$.

**Problem 2.** *Determine the values of $x$ for which the following series converges.*

*a)* $\sum_{n=0}^{\infty} \frac{(x-2)^{n+1}}{3^n}$;

*b)* $\sum_{n=0}^{\infty} \frac{\sin^n x}{3^n}$;

*c)* $\sum_{n=0}^{\infty} e^{nx}$;

**Problem 3.** *Show that the following series is divergent and also show that its sequence of terms converges to zero:*

$$\sum_{n=1}^{\infty} \ln \left( 1 + \frac{1}{n} \right)$$

## References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.