PROBLEM SET 10: SEQUENCES

Note: Most of the problems were taken from the textbook [1].

Problem 1. *Decide whether each of the following sequences converges:*

a) $\{a_n\}$, *where* $a_n = \frac{3\sqrt{n}}{\sqrt{n}+2}$;

b) $\{a_n\}$, *where* $a_n = \ln(n+1) - \ln n$;

c) $\left\{\frac{\ln n}{\ln 2n}\right\}$;

d) $\{a_n\}$, *where* $a_n = n\sin(1/n)$;

e) $\{a_n\}$, *where* $a_n = \left(1 + \frac{2}{n}\right)^n$;

f) $\{a_n\}$, *where* $a_n \sqrt[n]{n}$.

Problem 2. *Find the limit of the sequence* $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$.

Problem 3. *Consider the sequence* $\{a_n\}$ *recurrently defined as* $a_1 = 2$ *and* $a_{n+1} = \frac{1}{3-a_n}$.

(1) Assuming that $0 \leq a_n \leq 2$ *and that the sequence* $\{a_n\}$ *is decreasing, argue that* $\{a_n\}$ *is convergent and find its limit.*
(2) Can you show that $0 \leq a_n \leq 2$ *for every natural* n*?*
(3) Can you show that $\{a_n\}$ *is decreasing?*

References

[1] J. Stewart: *Single Variable Calculus* 8th Edition, Cengage Learning, Boston 2015.