Practice Midterm 3

Problem 1 Consider the simple graph $G$ with

$$V(G) = \{1, 2, \ldots, 30\} \quad \text{and} \quad E(G) = \{ij \mid i \leq 10 < j\}.$$  

Find the number of Hamiltonian cycles of $G$.

Problem 2 Prove the following statements.

1. In any tree, any two longest paths cross each other.

2. A tree with no vertex of degree 2 has more leaves than non-leaf vertices.

Problem 3 Let $G$ be a tree on the set of vertices $[10]$. In how many ways can we add to $G$ edges to obtain a tree on the set of vertices $[30]$?

Problem 4 Let $G$ be a simple connected graph with weight function $\omega: E(G) \to \mathbb{R}_+$, and assume that $\omega$ is injective. If $C$ is a cycle in $G$ and $e$ is the heaviest edge in $C$, prove that no minimum-weight spanning tree of $G$ contains $e$.

Problem 5 Explain how to count the number of 3-cycles of a simple graph using its adjacency matrix.