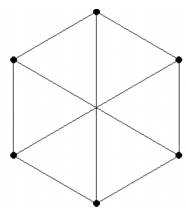
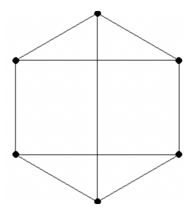
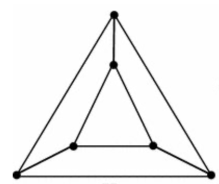
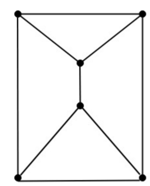
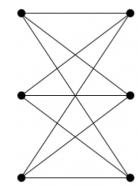
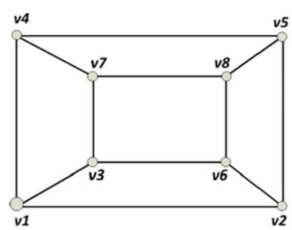
Problem Set 6 (Due on Wednesday, 12/08)

Problem 1 *Recall that two graphs G_1 and G_2 are called isomorphic if there exists a bijective function $f\colon V(G_1) \to V(G_2)$, called an isomorphism, such that $uv \in E(G_1)$ if and only if $f(u)f(v) \in E(G_2)$. Which of the following graphs are isomorphic? Justify your answer.*

Problem 2 *Let G be a graph. If $f\colon V(G) \to V(G)$ is an isomorphism of graphs, then we call f an* ***automorphism*** *of G. How many automorphisms does the following graph have? Justify your answer.*



Problem 3 *Let G be a simple graph. We say that $e \in E(G)$ is a* bridge *if the graph $(V(G), E(G) \setminus \{e\})$ has more connected components than G. Let G be a bipartite k-regular graph for $k \geq 2$. Prove that G has no bridge.*

Problem 4 *For every $n \in \mathbb{N}$ with $n \geq 3$, find the chromatic polynomial of C_n, the cycle graph on $[n]$.*

Problem 5 *For $n \in \mathbb{N}$, prove that the chromatic polynomial of the complete bipartite graph $K_{n,2}$ is $x(x-1)^n + x(x-1)(x-2)^n$.*

Problem 6 *Let G be a simple connected k-regular graph (with $k \geq 3$) that is neither an odd cycle nor a complete graph, and assume that G has no cut-vertices. Prove that if the subgraph $G \setminus \{v\}$ of G contains a cut-vertex for some $v \in V(G)$, then $\chi(G) \leq \Delta(G)$.*

Problem 7 *Is it possible to subdivide a square into finitely many concave quadrilateral?*

Problem 8 *Let P be a convex polyhedron with triangular faces. Suppose that the edges of P are oriented. A* ***singularity*** *of P is a face whose edges form an oriented cycle or a vertex v with* $\operatorname{indeg}(v) \cdot \operatorname{outdeg}(v) = 0$. *Prove that P has at least two singularities.*