Practice Midterm 3

Problem 1 *Consider the simple graph G with*

$$V(G) = \{1, 2, \ldots, 30\} \quad and \quad E(G) = \{ij \mid i \leq 10 < j\}.$$

Find the number of Hamiltonian cycles of G.

Problem 2 *Prove the following statements.*

1. *In any tree, any two longest paths cross each other.*

2. *A tree with no vertex of degree 2 has more leaves than non-leaf vertices.*

Problem 3 *Let G be a tree on the set of vertices $[10]$. In how many ways can we add to G edges to obtain a tree on the set of vertices $[30]$?*

Problem 4 *Let G be a simple connected graph with weight function $\omega \colon E(G) \to \mathbb{R}_+$, and assume that ω is injective. If C is a cycle in G and e is the heaviest edge in C, prove that no minimum-weight spanning tree of G contains e.*

Problem 5 *Explain how to count the number of 3-cycles of a simple graph using its adjacency matrix.*