Practice Midterm 2

Problem 1 *For each $n \in \mathbb{N}$, let r_n be the number of permutations $\pi \in S_n$ such that π^2 is the identity permutation (here π^2 means π composed with itself as a function). Prove that $r_{n+1} = r_n + nr_{n-1}$ for every $n \geq 2$.*

Problem 2 *In how many ways can we roll a die 8 consecutive times such that all six faces appear at least once?*

Problem 3 *For $n \in \mathbb{N}_0$, let c_n be the number of ways we can have n cents into pennies, nickels, quarters using at most five nickels. Find the explicit ordinary generating function for $(c_n)_{n\geq 0}$.*

Problem 4 *The sequence $(a_n)_{n\geq 0}$ satisfies $a_0 = 1$ and $a_{n+1} = 3a_n + 2n$ for every $n \in \mathbb{N}_0$. Find an explicit formula for a_n.*

Problem 5 *For each $n \in \mathbb{N}$, let c_n be the number of ways to subdivide a group of n delegates into committees of sizes at least 3, and then select a leader in each committee. Assume that $c_0 = 1$, and find the exponential generating function of $(c_n)_{n\geq 0}$.*