COMBINATORIAL ANALYSIS

Practice Midterm 1 (MIT, Fall 2021)

This practice midterm will give you an idea about the format, length, and difficulty level of the first midterm. As the actual midterm, this practice midterm should be taken as a closed-book exam with a duration of 50 minutes.

Problem 1. *Prove that there exists* $n \in \mathbb{N}$ *such that* 211 *divides* $18^n - 1$.

Problem 2. *Using a combinatorial argument, prove that*

$$\sum_{k=0}^{m} \binom{n}{k}\binom{n-k}{m-k} = 2^m \binom{n}{m}$$

for every $m, n \in \mathbb{N}$ *with* $m \leq n$.

Problem 3. *Prove that the number of compositions of* n *into parts greater than* 1 *is* F_{n-1}*, the* $(n-1)$*-th Fibonacci term.*

Problem 4. *Using only the combinatorial definition of Stirling numbers of the second kind, find a formula for* $S(n, n-2)$.

Problem 5. *Prove that the number of partitions of* n *into at most* k *parts equals the number of partitions of* $n+k$ *into* k *parts.*