Fall 2002, Course 18.238: Mathematical ideas and notions of quantum field theory

TR 11-12.30, Rm. 4-146

Instructor Pavel Etingof

The development of quantum field theory and string theory in the last two decades led to an unprecedented level of interaction between physics and mathematics, incorporating into physics such "pure" areas of mathematics as algebraic topology, algebraic geometry, and even number theory. This interaction has been highly fruitful in both directions, and led to a necessity for physicists to know basic mathematics and for mathematicians to know basic physics. Physicists have been quick to learn, and nowadays good physicists know relevant areas of mathematics as deeply as professional mathematicians. On the other hand, mathematicians have been more slow, intimidated by the absence of rigor in physical papers, and more importantly by a totally different manner of presentation. In particular, even the basic setting of quantum field theory, necessary for understanding its more advanced (and mathematically exciting) parts, is already largely unknown to mathematicians. Nevertheless, many of the basic ideas of quantum field theory can in fact be presented in a completely rigorous and mathematical way. Doing this will be the main goal of this course.
Topics of the course (time permitting) include:
1. 0-dimensional QFT: Stationary phase formula. Calculus of Feynman diagrams with applications to combinatorics. Matrix models, large N limits.
2. 1-dimensional QFT: Formalism of classical mechanics. Lagrangians, Hamiltonians, least action principle. Path integral approach to quantum mechanics. Perturbative expansion using Feynman diagrams. Operator formalism. Feynman-Kac formula.
3. d-dimensional QFT, $d > 1$: Formalism of classical field theory. Currents, charges, Noether's theorem. Path integral approach to QFT. Perturbative expansion. Divergences. Renormalization theory.
4. Supergeometry and supersymmetry.
5. Introduction to conformal field theory.

The textbook for the course is "Quantum fields and strings: a course for mathematicians", AMS, 1998 (but I won't closely follow it). I plan to have typed lecture notes available, and to give homeworks.
Prerequisites: advanced calculus of several variables, basic differential geometry. Knowledge of physics is not required.
Warning: This is a course primarily for mathematicians. Physicists will not learn much in it, except how to present the ideas of QFT (which they already know) in a mathematical way. It is important to note that the instructor knows less QFT than a graduate student specializing in QFT or string theory.