

On low-rank approximability of solutions to Kronecker-structured operator equations

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November 6, 2014

Let A and B be invertible, and let C have low rank. Then it is trivial that the rank of the solution to the matrix equation $AXB^T = C$ is not larger than the rank of C . Formally speaking, this matrix equation admits a low-rank solution. There is some numerical evidence that the solution of frequently occurring linear equations like $A_1XB_1^T + \dots + A_RX_R^T = C$, where the number of terms is still considerably smaller than the matrix dimension (which might be infinite in an Hilbert space setting), has good low-rank approximability, that is, fast decaying singular values. Exponential decay, as sometimes observed in practice, seems very challenging to prove. It is, however, rather easy to obtain a nontrivial algebraic decay rate by relating the convergence speed of some polynomial approximation to the Kronecker rank of the corresponding operator polynomial. For an eigenvalue problem $AXB^T = \lambda X$ a similar argumentation is possible, although with considerable restrictions. This simple method of estimating the low-rank approximation error for matrices has important consequences for the low-rank approximability of solutions to Kronecker-structured operator equations in higher-order tensor product spaces, as it provides estimates on the singular value decay of certain matrix unfoldings of the solution tensor, which in turn govern the error in higher-order SVD truncations. The talk is based on joint-work with Daniel Kressner.