A practical framework for infinite-dimensional linear algebra

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We describe a framework for solving a broad class of infinite-dimensional linear equations, consisting of almost banded operators, which can be used to resepresent linear ordinary differential equations with general boundary conditions. The framework contains a data structure on which row operations can be performed, allowing for the solution of linear equations by the adaptive QR approach. The algorithm achieves O(n) complexity, where n is the number of degrees of freedom required to achieve a desired accuracy, which is determined adaptively. In addition, special tensor product equations, such as partial differential equations on rectangles, can be solved by truncating the operator along one dimension and using a generalized Schur decomposition. The framework is implemented in the ApproxFun.jl package written in the Julia programming language.