Applications of the Cauchon Algorithm

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During the last years, the Cauchon algorithm (also called deleting derivation algorithm [5] and Cauchon reduction algorithm [6]) has been applied in algebra, e.g., to tours invariant prime ideals in the quantum matrices. In [5] and [6] this algorithm was employed for the recognition of totally nonnegative cells. A real matrix is called *totally nonnegative* if all its minors are nonnegative. Such matrices arise in a variety of ways in mathematics and its applications, e.g., in differential and integral equations, numerical mathematics, combinatorics, statistics, and computer aided geometric design. For background information we refer to the recently published monographs [3], [7]. In [2] we apply the Cauchon algorithm [5], [6] for a proof of a conjecture posed by the speaker in 1982 [4], see also [3, Section 3.2] and [7, Section 3.2]. This conjecture originated in the interpolation of interval-valued data by using B-splines. It concerns the checkerboard ordering which is obtained from the usual entry-wise ordering in the set of the square real matrices of fixed order by reversing the inequality sign for each entry in a checkerboard fashion. The conjecture concerns the *interval property* of the nonsingular totally nonnegative matrices, i.e., if the two bound matrices of an interval with respect to this ordering are nonsingular and totally nonnegative then all matrices lying between the two bound matrices are nonsingular and totally nonnegative, too.

In our talk we also briefly report on some very recent applications of the Cauchon algorithm, viz.

- new determinantal tests for testing a given matrix for total nonnegativity and related properties which require much fewer minors to be checked as the tests known so far,
- finding for each entry of a nonsingular totally nonnegative matrix the largest amount by which this entry can be perturbed without losing the property of total nonnegativity,
- identifying other subclasses exhibiting the interval property of the sign regular matrices, i.e., of matrices with the property that all their minors of fixed order have one specified sign or are allowed also to vanish.

References and Literature for Further Reading

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