Math 54-1

Quiz 7, July 20, 2010

Your name: _____Key_____

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) Assume that A is a 6×8 matrix with $\dim \operatorname{Nul} A = 2$. Find the rank of A. Are the **rows** of A linearly independent?

By Rank Theorem, rank $A = 8 - \dim \operatorname{Nul} A = 6$.

Then rank $A^T =$ rank $A = 6$.

A^T has 6 pivot positions $\Rightarrow$ since A^T is 8×6, it has a pivot in each column. Therefore, the columns of A^T are lin. ind.

Since the rows of A are the columns of A^T, they are linearly independent.

2. (5 pt) Find the matrix of the transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$ given by the formula $(T(f))(t) = f(t-1)$ in the basis $\{1, t, t^2\}$ of $\mathbb{P}_2$ (and itself). Is the transformation T 1-to-1? Is it onto?

Bonus (no points): is T invertible? If so, find the inverse transformation.

We have

$$T(1) = 1 \longrightarrow [T(1)]_B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$T(t) = t-1 \longrightarrow [T(t)]_B = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$T(t^2) = (t-1)^2 = t^2 - 2t + 1 \longrightarrow [T(t^2)]_B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

3 pivots $\longrightarrow$ T is both 1-to-1 and onto.

Bonus: T is invertible, and T^{-1} is given by

$$(T^{-1}(f))(t) = f(t+1).$$