Math 54-1 Your name: Key

Quiz 13, August 10, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) (a) Write a fundamental system of solutions for the ordinary differential equation (note the **four** derivatives)

$$y'''' - y = 0.$$

(You do not need to prove that the functions you wrote solve the equation or that they are linearly independent, as long as you write them correctly.)

(b) Verify that $y = e^{2x}/3$ is a solution to the equation

$$y'''' - y = 5e^{2x}$$

and write the general solution of this inhomogeneous equation.

(a) Auxiliary equation: $r^4 - 1 = 0 \rightarrow$ roots $r = 1, -1, \pm i$.

$(r^2-1)(r^2+1)=0$

Fundamental system: $\{e^x, e^{-x}, \cos x, \sin x\}$

(b) $y = \frac{e^{2x}}{3} \rightarrow y'''' = \frac{16e^{2x}}{3} \rightarrow y'''' - y = 5e^{2x}$

General solution: $\cancel{5e^{2x}}$ $\frac{e^{2x}}{3} + c_1 e^x + c_2 e^{-x} + c_3 \cos x + c_4 \sin x,$

$c_1, c_2, c_3, c_4 \in \mathbb{R}.$

2. (5 pt) Find the solution to the system

$$x_1'(t) = x_1(t) + 2x_2(t),$$
$$x_2'(t) = -x_2(t),$$

satisfying the initial conditions

$$x_1(0) = 0, \;\; x_2(0) = 2.$$

Bonus (no points): sketch the trajectory of $(x_1(t), x_2(t)) \in \mathbb{R}^2$.

$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \rightarrow \vec{x}'(t) = \underbrace{\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}}_{A} \vec{x}(t)$

A is upper triangular → eigenvalues $1, -1$; diagonalizable

$\lambda = 1 \rightarrow \text{Nul}(A - \lambda I) = \text{Nul} \begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix}$; basis $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\lambda = -1 \rightarrow \text{Nul}(A - \lambda I) = \text{Nul} \begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix}$; basis $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

Fundamental system: $\left\{ e^{t} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \rightarrow$

→ general solution $\vec{x}(t) = c_1 e^{t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$;

Initial condition: $\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \vec{x}(0) = c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$;

solving for c_1, c_2, we get $c_1 = 2, c_2 = -2$ →

→ $\vec{x}(t) = 2e^{t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 2e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Trajectory: $e^{t} \cdot e^{-t} = 1$ →