Math 54-1 Your name: Key
Quiz 10, July 29, 2010

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (5 pt) Find a basis for $(\text{Nul}\, A)^{\perp}$, where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 3 & 3 \end{bmatrix}.$$

We have $(\text{Nul}\, A)^{\perp} = \text{Col}\, A^T$;

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \\ 1 & 0 & 3 \end{bmatrix} \xrightarrow[R_2 \leftarrow R_2 - R_1]{R_3 \leftarrow R_3 - R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 0 \\ 0 & -2 & 0 \end{bmatrix} \rightarrow$$

$$\rightarrow \begin{bmatrix} \boxed{1} & 0 & 3 \\ 0 & \boxed{1} & 0 \\ 0 & 0 & 0 \end{bmatrix};$$

Basis for $(\text{Nul}\, A)^{\perp}$: $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

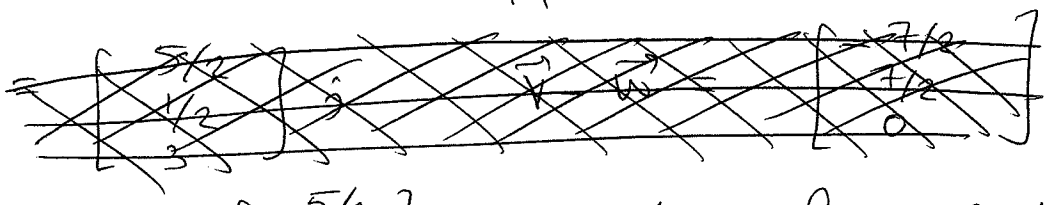
2. (5 pt) Verify that $\{(1,1,1),(1,-3,2)\}$ is an orthogonal system; let V be the space spanned by these two vectors. Find the orthogonal projection of $(-1,4,3)$ onto V. Find the distance from $(-1,4,3)$ to V.

$\vec{u_1} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\vec{u_2} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}$.

$\vec{u_1} \cdot \vec{u_2} = 1 \cdot 1 + 1 \cdot (-3) + 1 \cdot 2 = 0 \rightarrow \{\vec{u_1}, \vec{u_2}\}$ is an orthogonal system. Now,

$$\vec{w} = \text{proj}_{\vec{u_1},\vec{u_2}} \vec{v} = \frac{\vec{v} \cdot \vec{u_1}}{\vec{u_1} \cdot \vec{u_1}} \vec{u_1} + \frac{\vec{v} \cdot \vec{u_2}}{\vec{u_2} \cdot \vec{u_2}} \vec{u_2} =$$

$$= \frac{6}{3} \vec{u_1} - \frac{7}{14} \vec{u_2} = 2\vec{u_1} - \frac{1}{2}\vec{u_2} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} + \begin{bmatrix} -1/2 \\ 3/2 \\ -1 \end{bmatrix} =$$



$$= \begin{bmatrix} 3/2 \\ 7/2 \\ 1 \end{bmatrix};$$

$\vec{v} - \vec{w} = \begin{bmatrix} -5/2 \\ 1/2 \\ 2 \end{bmatrix}$; distance from $\vec{v}$ to V =

$$= \|\vec{v} - \vec{w}\| = \sqrt{\frac{25}{4} + \frac{1}{4} + 4} = \frac{\sqrt{42}}{2}.$$