Math 54-1, H/W 3

1.8 (8.) We need for $\vec{x} \in \mathbb{R}^4$, $A\vec{x}$ to be well defined and $A\vec{x} \in \mathbb{R}^5$. Therefore, A has to be a 5×4 matrix.

(9) We need to find all $\vec{x}$ such that $A\vec{x}=0$; in other words, to solve the system of linear equations with the augmented matrix

$$[A\ \vec{0}] = \begin{bmatrix} 1 & -4 & 7 & -5 & 0 \\ 0 & 1 & -4 & 3 & 0 \\ 2 & -6 & 6 & -4 & 0 \end{bmatrix} \xrightarrow[\text{red}]{\text{row}} \begin{bmatrix} \boxed{1} & 0 & -9 & 7 & 0 \\ 0 & \boxed{1} & -4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, $x_1 = 9x_3 - 7x_4$, $x_2 = 4x_3 - 3x_4$, x_3, x_4 free;

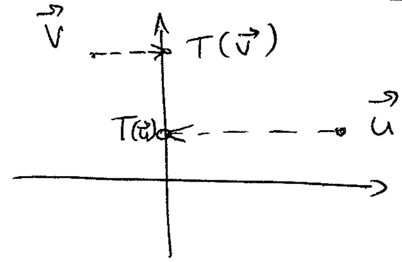
in parametric vector form, $\vec{x} = x_3 \begin{bmatrix} 9 \\ 4 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ -3 \\ 0 \\ 1 \end{bmatrix}$.

(11.) $\vec{b}$ is in the range of the linear transformation $\vec{x} \mapsto A\vec{x}$ iff for some $\vec{x}$, $A\vec{x}=\vec{b}$; that is, iff the system $A\vec{x}=\vec{b}$ is consistent. Row reduce: $[A\ \vec{b}] \longrightarrow$

$$\longrightarrow \begin{bmatrix} \boxed{1} & -4 & 7 & -5 & -1 \\ 0 & \boxed{1} & -4 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ consistent; so, $\vec{b} \in$ range A.

(15) $T(\vec{u}) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$, $T(\vec{v}) = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$;

projection onto the x_2 axis.



(22) (a) True (see in the beginning of p. 77)
(b) False, the codomain is $\mathbb{R}^m$ (do not confuse with range)
(c) False, this is an existence question
(d) True, by definition
(e) True, (see the end of p. 77)

(33) We have $T(0) = (0,4,0) \neq 0$, while $T(0)=0$ for any linear transformation.

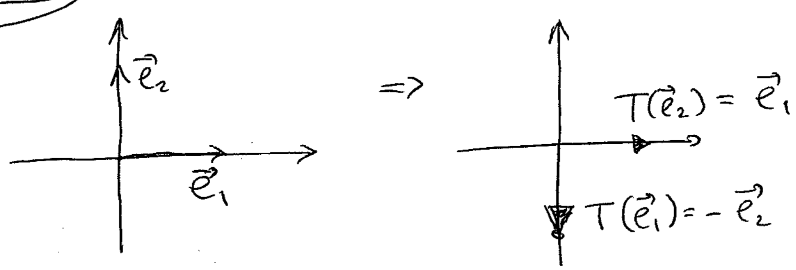
(35.) Take $\vec{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$. Then

(1) $T(\vec{u}+\vec{v}) = T\left(\begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \end{bmatrix}\right) = \begin{bmatrix} u_1+v_1 \\ u_2+v_2 \\ -u_3-v_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ -u_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ -v_3 \end{bmatrix} =$

$= T(\vec{u}) + T(\vec{v})$;

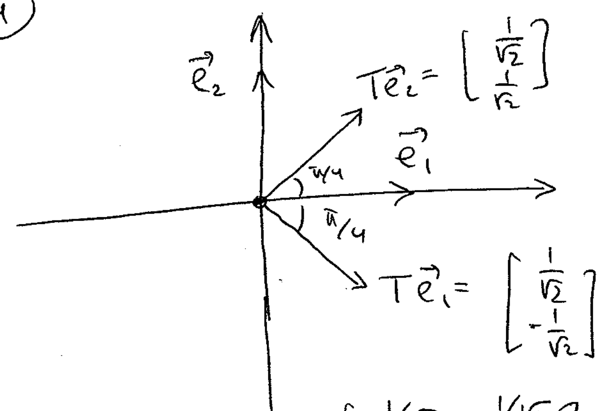
(2) $T(c\vec{u}) = T\left(\begin{bmatrix} cu_1 \\ cu_2 \\ cu_3 \end{bmatrix}\right) = \begin{bmatrix} cu_1 \\ cu_2 \\ -cu_3 \end{bmatrix} = c\begin{bmatrix} u_1 \\ u_2 \\ -u_3 \end{bmatrix} = cT(\vec{u})$.

1.9 (3)



$A = [-\vec{e}_2 \ \ \vec{e}_1] = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(4)

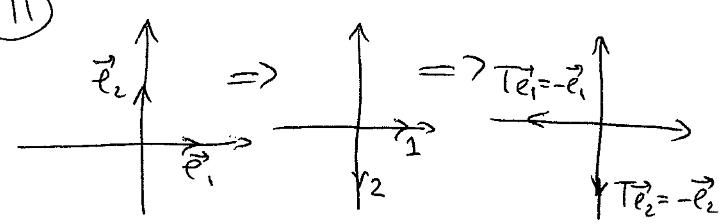


$A = [T\vec{e}_1 \ \ T\vec{e}_2] = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

(5) $T(\vec{e}_1) = \vec{e}_1 - 2\vec{e}_2$, $T(\vec{e}_2) = \vec{e}_2$

$A = [T(\vec{e}_1) \ \ T(\vec{e}_2)] = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

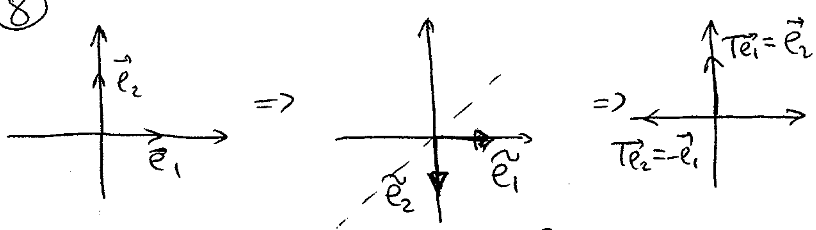
(11)



$A = [-\vec{e}_1 \ \ -\vec{e}_2] = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} =$

= the matrix of rotation by π.

(8)



$A = [\vec{e}_2 \ \ -\vec{e}_1] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(17) $T(\vec{e}_1) = (0,1,0,0)^T$
$T(\vec{e}_2) = (0,1,1,0)^T$
$T(\vec{e}_3) = (0,0,1,1)^T$
$T(\vec{e}_4) = (0,0,0,1)^T$

So, $A = [T(\vec{e}_1)\, T(\vec{e}_2)\, T(\vec{e}_3)\, T(\vec{e}_4)] =$

$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(12) $A = \begin{bmatrix} \cos \pi/2 & -\sin \pi/2 \\ \sin \pi/2 & \cos \pi/2 \end{bmatrix}$;

rotation by $\pi/2$ CCW

2.1 (11) $AD = \begin{bmatrix} 2 & 3 & 5 \\ 2 & 6 & 15 \\ 2 & 12 & 25 \end{bmatrix}$ Multiplies each <u>column</u> of A by the corresponding diagonal element of D.

$DA = \begin{bmatrix} 2 & 2 & 2 \\ 3 & 6 & 9 \\ 5 & 20 & 25 \end{bmatrix}$ Multiplies each <u>row</u> of A by the corresponding diagonal element of D.

~~To have $AD = DA$, we need to have no off diagonal elements in A. For example, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 7 \end{bmatrix}$ works.~~

An example of B; $AB = BA$:
$B = 2I_2$ or $B = A$.

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(23) <u>Given:</u> $CA = I_n$

<u>Need to prove:</u> $A\vec{x} = 0$ has only the trivial solution

Take $\vec{x}$ a solution to $A\vec{x} = 0$. Then

$(CA)\vec{x} = I_n \vec{x} = \vec{x}$. On the other hand,

~~Cl~~ $(CA)\vec{x} = C(A\vec{x}) = C \cdot 0 = 0$. So, $\vec{x} = 0$.

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(24) <u>Given:</u> $AD = I_m$.

<u>Need to prove:</u> for each $\vec{b}$, there exists $\vec{x}$: $A\vec{x} = \vec{b}$.

~~Fix b~~ Take $\vec{b}$ and put $\vec{x} = D\vec{b}$. Then

$A\vec{x} = A(D\vec{b}) = (AD)\vec{b} = I_m \vec{b} = \vec{b}$. So,

$\vec{x} = D\vec{b}$ solves $A\vec{x} = \vec{b}$.