Math 54, Section 214 Your name: Key

Quiz 2, February 5, 2009

Please write your name **on each sheet**. Show your work clearly and in order, including intermediate steps in the solutions and the final answer.

1. (7 pt) Do the vectors

$$\underset{\vec{v}_1}{\begin{bmatrix}1\\3\\4\end{bmatrix}}, \underset{\vec{v}_2}{\begin{bmatrix}2\\-3\\-1\end{bmatrix}}, \underset{\vec{v}_3}{\begin{bmatrix}3\\0\\3\end{bmatrix}}$$

form a linearly independent set? Explain.

They do not, because $\vec{v}_3 = \vec{v}_1 + \vec{v}_2$.

Alternatively, use row reduction:

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & -3 & 0 \\ 4 & -1 & 3 \end{bmatrix} \xrightarrow[R_3 \leftarrow R_3 - 4R_1]{R_2 \leftarrow R_2 - 3R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -9 & -9 \\ 0 & -9 & -9 \end{bmatrix} \xrightarrow{R_3 \leftarrow R_3 - R_2} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -9 & -9 \\ 0 & 0 & 0 \end{bmatrix},$$

no pivot in column 3.

2. (6 pt) Consider the linear transformation from $\mathbb{R}^a$ to $\mathbb{R}^b$ whose standard matrix is

$$\begin{bmatrix} -2 \\ 1 \end{bmatrix}.$$

(a) Find $a$ and $b$. Explain.

(b) Is the transformation one-to-one? Explain.

(c) Is the transformation onto? Explain.

(a) $a = 1$ and $b = 2$: $\begin{bmatrix} -2 \\ 1 \end{bmatrix} \cdot x_1 = \begin{bmatrix} -2x_1 \\ x_1 \end{bmatrix}$

(b) It is, because $\begin{bmatrix} -2 \\ 1 \end{bmatrix} x = 0 \Rightarrow \begin{bmatrix} -2x \\ x \end{bmatrix} = 0 \Rightarrow x = 0$

Alternatively, row reduction:

$\begin{bmatrix} -2 \\ 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 \\ -2 \end{bmatrix} \xrightarrow{R_2 \leftarrow R_2 + 2R_1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, pivot in every column.

(c) It is not, because there is no pivot in row 2 (see the row reduction above)

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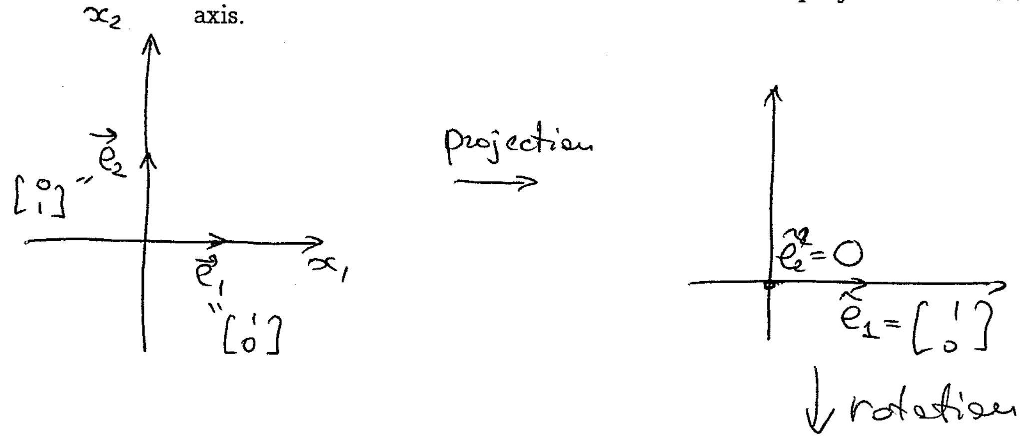
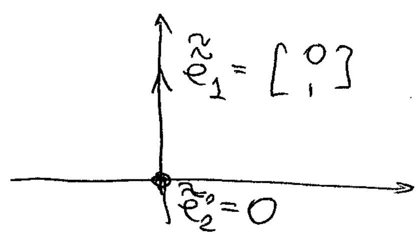
3. (7 pt) (a) Write the standard matrix of a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ which first projects onto the $x_1$ axis and then rotates 90 degrees counterclockwise.

(b) Write the standard matrix of a linear transformation $\mathbb{R}^2 \to \mathbb{R}^2$ which first rotates 90 degrees counterclockwise and then projects onto the $x_1$ axis.

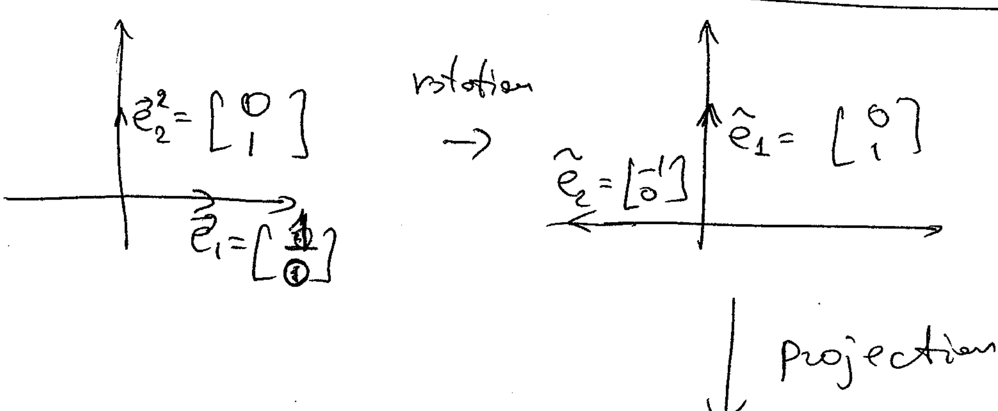
(a)




$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \Leftarrow$$




(b)




$$\begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \Leftarrow$$