1. Consider the matrix \( A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & -1 \end{pmatrix} \).

(a) Find a basis for and the dimension of the null space of \( A \).
(b) Find a basis for and the dimension of the column space of \( A \).
(c) What is the rank of \( A \)? Explain why the rank-nullity theorem holds for \( A \).
(d) Are the columns of \( A \) linearly independent? Do they span \( \mathbb{R}^3 \)?

2. Assume that a \( 5 \times 3 \) matrix \( A \) has rank 3.

(a) What are the dimensions of the nullspace of \( A \) and the column space of \( A \)?
(b) Are the columns of \( A \) linearly independent? Do they span \( \mathbb{R}^5 \)?
(c) Could it be that the equation \( A\vec{x} = \vec{b} \) has no solution for some \( \vec{b} \)? Could it be that this equation has more than one solution for some \( \vec{b} \)?

*3. Let \( A \) be an \( n \times n \) matrix and assume that the null space of \( A \) is equal to the column space of \( A \). Show that \( A^2 = 0 \).

4. For which values of the real parameter \( c \) is the matrix \( A_c = \begin{pmatrix} 1 & c \\ 2c & 8 \end{pmatrix} \) invertible?
Find a formula for the inverse \( A_c^{-1} \).

5. Consider the matrix \( A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \).

(a) Is the matrix \( A \) invertible? If so, find the inverse.
(*b) Find the eigenvalues and eigenvectors of \( A \)
(*c) Diagonalize \( A \), i.e. write it as \( A = SDS^{-1} \) where \( D \) is a diagonal matrix.

6. Consider the matrix \( A = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \).

(a) Find the eigenvectors and eigenvalues of \( A \).
(b) Diagonalize \( A \), i.e. write it as \( A = SDS^{-1} \) where \( D \) is a diagonal matrix.
(c) Compute the 10th power \( A^{10} \). (Hint: use the diagonalization. To compute the 10th power of \( D \) use the polar form of the complex eigenvalues of \( A \).)
*7. Assume that a diagonalizable $n \times n$ matrix $A$ has only eigenvalues 1 and $-1$. Show that $A^2 = I$. 