

18.03 RECITATION SHEET WEEK 8

1. Consider the system of ODEs

$$y_1' = y_1 + y_2, \quad y_2' = y_2 - 4y_1.$$

- a.) Find the general solution.
- b.) Draw the phase portrait. What kind of phase portrait is it?
- c.) Draw the system on the trace-determinant plane.

2. The romantic dynamics of Romeo and Juliet is modeled by the system of ODEs

$$R' = -R + 3J, \quad J' = -3R + cJ.$$

Here c is an arbitrary real parameter and $R(t), J(t)$ are the affection of Romeo for Juliet and the affection of Juliet for Romeo at time t . (Romeo's affection for Juliet, whether positive or negative, tends to decline by itself. However, it is also positively influenced by Juliet's affection for Romeo. On the other hand, the more Romeo likes Juliet, the more she tends towards disliking him.)

a.) Plot this system of ODEs on the trace-determinant plane for all choices of c . Mark the intersection of the resulting set of points with the critical parabola.

b.) What kind of phase portrait does the system have? Your answer will depend on c . You may skip the cases of double eigenvalue or when one of the eigenvalues equals 0, since we did not cover these portraits in class.

c.) For which values of c is the system stable? For which values of c is it semistable?

3. Bring the following systems of linear equations to reduced row echelon form and find the general solution to each of them.

a.)

$$\begin{aligned} -3x_1 + x_2 &= -1, \\ 2x_1 + x_2 &= 4, \\ x_2 - x_1 &= 1. \end{aligned}$$

b.)

$$x_1 + x_2 + x_3 + x_4 = 4,$$

$$x_1 + x_2 - x_3 - x_4 = 2.$$

c.)

$$x_1 + 2x_2 + x_3 = 4,$$

$$x_1 + 2x_2 + 2x_3 = c,$$

$$x_1 + 2x_2 = 3,$$

where c is any number (your answer will depend on c).