

18.03 RECITATION SHEET WEEK 7

1. Consider the ODE $y'' + y' - 2y = 0$.
 - (a) Write the corresponding companion system (which will be a 2×2 first order system of ODEs).
 - (b) Find the general solution of the companion system using the method of Section 6.2 in the lecture notes.
 - (c) Find the solutions of the system satisfying the following initial conditions:
(i) $y(0) = 1, y'(0) = 1$, (ii) $y(0) = 6, y'(0) = 0$
 - (d) Plot the trajectories of the solutions from part (c)
 - (e) Draw the phase portrait of the system. What type is it? Be sure to include all the special trajectories (the ones which are straight rays).
 - (f) Plot the companion system in the trace-determinant plane.

2. Consider a system of 2 connected tanks with one of them leaking (see Section 6.1 of the lecture notes) which is described by the ODEs $y_1' = y_2 - y_1, y_2' = y_1 - 2y_2$.
 - (a) Write the system in vector form.
 - (b) Find the general solution to the system.
 - (c) Draw the phase portrait of the system. What type is it? Be sure to include all the special trajectories (the ones which are straight rays).
 - (d) Plot the system in the trace-determinant plane.