18.03 RECITATION SHEET WEEK 4

(Questions with * are optional)

1. Consider the initial value problem $y'' + 4y = \cos(\omega t)$, $y(0) = 0$, $y'(0) = 0$ where $\omega > 0$ is a real parameter.
   
   (a) Find the solution of this initial value problem when $\omega \neq 2$.
   (b) Find the complex gain, amplitude gain, and phase lag as functions of $\omega$ when $\omega \neq 2$.
   (c) Find the solution of this initial value problem when $\omega = 2$.
   (d) (*) Show that as $\omega \to 2$, the solution in part (a) converges to the solution in part (b) for any fixed $t$.
   (e) For your entertainment, the graphs of $y(t)$ for several values of $\omega$ are included on the next page.

2. Which of the following ODEs are stable? For those that are not, give an example of a solution that does not go to 0 as $t \to \infty$.
   
   (a) $y'' + 7y' + 8y = 0$
   (b) $y'' + y' - 2y = 0$

3. Compute the linear combination $2\vec{a} - 3\vec{b}$ where

\[
\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}
\]

4. Are the following sets of vectors linearly independent? For those which are linearly dependent, find a nontrivial linear combination which gives the zero vector. (Hint: write the equation that a linear combination of those is equal to 0 and solve for the coefficients.)

   (a) $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$
   (b) $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$
   (c) $\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, $\vec{c} = \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix}$