18.03 RECITATION SHEET WEEK 11

(Questions with * are optional)

1. Compute the exponential $\exp(tA)$, where $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$. Verify by hand that $\vec{y}(t) = \exp(tA)\vec{w}$ solves the system of ODEs $\vec{y}\,' = A\vec{y}$ for any constant vector $\vec{w}$.

2. In this problem we solve the system of ODEs $y_1' = y_2 + 1$, $y_2' = y_1 + 2$ using variation of parameters.

- **(a)** Write the system in the vector form $\vec{y}\,' = A\vec{y} + \vec{b}(t)$.
- **(b)** Compute the matrix exponential e^{tA}.
- **(c)** We will look for the solution in the form $\vec{y}(t) = e^{tA}\vec{u}(t)$. Write the equation for $\vec{u}\,'(t)$.
- **(d)** Integrate to find $\vec{u}(t)$.
- **(e)** Find the general solution $\vec{y}(t)$ to the system.

3. In this problem we follow the method of Section 9.2 in the lecture notes to find the Neumann eigenvalues and eigenfunctions for D^2 on the interval $[0, \pi]$. Namely we will find all complex numbers λ such that there exists a nonzero function $y(x)$ on the interval $[0, \pi]$ satisfying $y'' = \lambda y$, $y'(0) = y'(\pi) = 0$.

- **(a)** Use integration by parts to show that each eigenvalue λ has to be a nonpositive real number.
- **(b)** Find all λ (eigenvalues) and for each of them find a corresponding function y (eigenfunctions).