

## SUPPLEMENTARY GRAPH FOR §4.6.2

SEMYON DYATLOV

Consider the damped harmonic oscillator with sinusoidal forcing

$$y'' + by' + 9y = \cos(\omega t)$$

where  $b \geq 0$  and  $\omega \geq 0$ . The characteristic polynomial is

$$P(z) = z^2 + bz + 9.$$

Similarly to the computation in §4.6.2 the amplitude gain is given by

$$r(\omega) = \frac{1}{|P(i\omega)|} = \frac{1}{|9 - \omega^2 + bi\omega|} = \frac{1}{\sqrt{(9 - \omega^2)^2 + b^2\omega^2}}.$$

Below we plot the graph of  $r$  as a function of  $\omega$  for different values of the parameter  $b$ . Note that as the damping parameter  $b$  approaches 0, the peak in the graph becomes more and more pronounced. When  $b = 0$ , the value of  $r$  becomes infinite at the resonant frequency  $\omega = 3$ .

