Consider the function

$$f(x) = 1, \quad 0 \leq x \leq \pi.$$ 

The corresponding sine Fourier series is

$$\frac{4}{\pi} \sum_{j=0}^{\infty} \frac{\sin((2j+1)x)}{2j+1} = \frac{4}{\pi} \sum_{k \geq 1 \text{ odd}} \frac{\sin(kx)}{k} = \frac{4}{\pi} \left( \frac{\sin x}{x} + \frac{\sin(3x)}{3} + \frac{\sin(5x)}{5} + \cdots \right).$$

The figure below shows the partial sum of the above series over $j = 0, \ldots, n$ for $n = 5$ and $n = 50$:

On the top next page we plot the partial sum of the same series for a larger range of values of $x$. Note that each term in the series (and thus the partial sum) is an odd $2\pi$-periodic function. The sum of the series is the $2\pi$-periodic odd extension of the function $f(x)$ except at the points $x = \pi n$ where it is equal to 0.
Next we look at the solution of the initial-boundary value problem for the heat equation
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad t \geq 0, \quad 0 \leq x \leq \pi;
\]
\[
u(0, t) = u(\pi, t) = 0, \quad t \geq 0;
\]
\[
u(x, 0) = 1, \quad 0 < x < \pi
\]
which is given by
\[
u(x, t) = \frac{4}{\pi} \sum_{j=0}^{\infty} \frac{e^{-(2j+1)^2t} \sin((2j+1)x)}{2j+1}.
\]
Note that the profile of the solution approaches an (exponentially decaying) sine wave because the term with \( j = 0 \) dominates the series when \( t \to \infty \).