## ERRATA

This is perpetually growing list of errata, many of which were found by Robert Koirala.

**p. 3, line 3up**: Change $B(\mathbf{0}, 2^{-m+1})$ to $\overline{B(\mathbf{0}, 2^{-m+1})}$

**p. 7, line 3up**: Replace $\lambda_t(d\mathbf{y})$ by $\lambda_t$

**p. 10, line 12up**: Insert "in" after "rays"

**p. 11, line 2dn**: Replace $\mu_t$ by $\lambda_t$

**p. 11, line 12dn**: Change $C_{\mathrm{b}}(\mathbb{R}^N; \mathbb{C})$ to $C^2_{\mathrm{b}}(\mathbb{R}^N; \mathbb{C})$

**p. 15, line 7up**: Replace $(1 + \|\mathbf{x}|)^{\Lambda t}$ by $(1 + |\mathbf{x}|)e^{t6L}$

**p. 19, line 8dn**: Delete $\int[t]$ from this line

**p. 22, line 10up**: Replace $\delta_x$ by $\delta_{\mathbf{x}}$

**p. 28, line 7up**: Replace $|\mathbf{y} - \mathbf{x}| \leq$ by$|\mathbf{y} - \mathbf{x}|^2 \leq$

**p. 31, line 10dn**: Replace $e^{-\frac{\xi^2}{2}\mathbb{E}^{\mathbb{P}}[X_n]^2}$ by $e^{-\frac{\xi^2}{2}\mathbb{E}^{\mathbb{P}}[X_n^2]}$

**p. 33, line 12up**: Replace $2^{\frac{n}{2}-1}$ by $2^{-\frac{n}{2}-1}$

**p. 34, line 3up**: Change $R^{\frac{n}{p}+r-\alpha}$ to $R^{\frac{N}{p}+r-\alpha}$

**p. 35, footnote**: Replace $\|\mathbf{x}\|_\infty = \max_{1\leq j\leq N} |x_j|$ by $\|\mathbf{x}\|_1 = \sum_{j=1}^N |x_j|$

**p. 35, lines 2, 5, & 3dn and 7up**: Replace $\|_\infty = 1$ by $\|_1 = 1$

**p. 35, lines 3up and 1up**: Replace $(2^{N+1}N)^{\frac{1}{p}}$ by $4^{\frac{N}{p}}$

**p. 36, lines 5dn and 6, & 3up**: Replace $(2^{N+1}N)^{\frac{1}{p}}$ by $4^{\frac{N}{p}}$

**p. 37, line 3dn**: Change $\sup_{\mathbf{x}\in[2^{n-1},2^n]}$ to $\sup_{2^{n-1}\leq|\mathbf{x}|\leq 2^n}$

**p. 43, line 2dn**: Replace $\mathbb{E}^{\mathbb{P}}$ by $\mathbb{E}^{\mathcal{W}}$

**p. 43, line 7dn**: Change $\|_{uH.S.}$ to $\|_{\mathrm{HS}}$

**p. 45, line 9dn**: Change $\mathbb{E}^W$ to $\mathbb{E}^{\mathcal{W}}$ and replace $2^{\frac{n}{2}}$ by $2^{-\frac{n}{2}}$

**p. 46, line 10up**: Change $\int_0^t \langle \varphi, P(\tau, \mathbf{x})\rangle\, d\tau$ to $\int_0^t \langle L\varphi, P(\tau, \mathbf{x})\rangle\, d\tau$

**pp. 50–51 Lemma 2.2.2**: Change Lemma 2.2.2 and its proof to:

**Lemma 2.2.2.** *Assume that $a \geq \epsilon\mathbf{I}$. If $a$ is continuously differentiable in a neighborhood of $\mathbf{x}$, then so is $a^{\frac{1}{2}}$ and*

$$\max_{1\leq i\leq n} \left\|\partial_{x_i} a^{\frac{1}{2}}(\mathbf{x})\right\|_{\mathrm{op}} \leq \frac{\|\partial_{x_i} a(\mathbf{x})\|_{\mathrm{op}}}{2\epsilon^{\frac{1}{2}}}.$$

*Moreover, for each $n \geq 2$, there is a $C_n < \infty$ such that*

$$\max_{\|\boldsymbol{\alpha}\|=n} \left\|\partial^{\boldsymbol{\alpha}}_{\mathbf{x}} a^{\frac{1}{2}}(\mathbf{x})\right\|_{\mathrm{op}} \leq C_n \frac{\max_{\|\alpha\|\leq n} \|\partial^\alpha a(\mathbf{x})\|^n_{\mathrm{op}}}{\epsilon^{n-\frac{1}{2}}}$$

*when $a$ is $n$-times continuously differentiable in a neighborhood of $\mathbf{x}$. Hence, if $a \in C^n_{\mathrm{b}}\big(\mathbb{R}^N;\mathrm{Hom}(\mathbb{R}^N;\mathbb{R}^N)\big)$, then so is $a^{\frac12}$.*

*Proof.* Without loss in generality, assume that $\mathbf{x}=\mathbf{0}$ and that there is a $\Lambda<\infty$ such that $a\le \Lambda\mathbf{I}$ on $\mathbb{R}^N$.

In order to express $a^{\frac12}$ in an analytically tractable way in terms of $a$, we will use the power series expansion

$$(1-t)^{\frac12}=\sum_{m=0}^\infty(-1)^m\binom{\frac12}{m}t^m \text{ where } \binom{\frac12}{m}=\frac{\prod_{\ell=0}^{m-1}\big(\frac12-\ell\big)}{m!}.$$

We will make use of the fact that $(-1)^m\binom{\frac12}{m}\le 0$ for $m\ge1$.

Set $d=\mathbf{I}-\frac{a}{\Lambda}$. Obviously $d$ is symmetric, $0\mathbf{I}\le d\le\big(1-\frac{\epsilon}{\Lambda}\big)\mathbf{I}$, and $a=\Lambda(\mathbf{I}-d)$. Thus, if $\binom{\frac12}{0}=1$ and

$$\binom{\frac12}{m}=\frac{\prod_{\ell=0}^{m-1}\big(\frac12-\ell\big)}{m!}\quad\text{for } m\ge1$$

are the coefficients in the Taylor expansion of $x\rightsquigarrow(1+x)^{\frac12}$ around 0, then

$$\sum_{m=0}^\infty(-1)^m\binom{\frac12}{m}d^m$$

converges in the operator norm uniformly on $\mathbb{R}^N$. In addition, if $\lambda$ is an eigenvalue of $a(\mathbf{y})$ and $\boldsymbol{\xi}$ is an associated eigenvector, then $d(\mathbf{y})\boldsymbol{\xi}=\big(1-\frac{\lambda}{\Lambda}\big)\boldsymbol{\xi}$, and so

$$\left(\Lambda^{\frac12}\sum_{m=0}^\infty(-1)^m\binom{\frac12}{m}d^m(\mathbf{y})\right)\boldsymbol{\xi}=\lambda^{\frac12}\boldsymbol{\xi}.$$

Hence,

$$a^{\frac12}=\Lambda^{\frac12}\sum_{m=0}^\infty(-1)^m\binom{\frac12}{m}d^m. \tag{2.2.7}$$

Now assume that $a$ is continuously differentiable. Because $\partial_{x_i}d^m=\sum_{j=1}^m d^{j-1}(\partial_{x_i}d)d^{m-j}$ and therefore $\|\partial_{x_i}d^m\|_{\mathrm{op}}\le m\|d\|_{\mathrm{op}}^{m-1}\|\partial_{x_i}d\|_{\mathrm{op}}$,

$$\begin{aligned}\Lambda^{-\frac12}\big\|\partial_{x_i}a^{\frac12}\big\| &\le \sum_{m=1}^\infty(-1)^{m-1}\binom{\frac12}{m}\|\partial_{x_i}d^m\|_{\mathrm{op}}\\ &\le\left|\sum_{m=1}^\infty m(-1)^m\binom{\frac12}{m}\|d\|_{\mathrm{op}}^{m-1}\right|\|\partial_{x_i}d\|_{\mathrm{op}}\\ &=\frac{\|\partial_{x_i}d\|_{\mathrm{op}}}{2(1-\|d\|_{\mathrm{op}})^{\frac12}}\le\Lambda^{-\frac12}\frac{\|\partial_{x_i}a\|_{\mathrm{op}}}{2\epsilon^{\frac12}},\end{aligned}$$

where the final inequality follows from $d\le\big(1-\frac{\epsilon}{\Lambda}\big)\mathbf{I}$. Hence, starting from (2.2.7), one sees that $a^{\frac12}$ is continuously differeniable in a neighborhood of $\mathbf{0}$ and that the asserted estimate for $\|\partial_{x_i}a^{\frac12}(\mathbf{x})\|_{\mathrm{op}}$ holds.

Next, assume that $a$ is $n$-times differentiable for some $n\ge2$, let $\|\alpha\|=n$, and, for $m\ge1$, define

$$B_\alpha(m)=\left\{(\beta^1,\dots,\beta^m)\in(N^N)^m:\sum_{\ell=1}^m\beta^\ell=\alpha\right\}.$$

Then

$$\partial^\alpha d^m = \sum_{(\beta^1,\dots,\beta^m)\in B_\alpha(m)} (\partial^{\beta^1} d)\cdots(\partial^{\beta^m} d).$$

If $1 \le m < n$, then

$$\|\partial^\alpha d\|_{\mathrm{op}} \le \operatorname{card}\big(B_\alpha(m)\big)\Lambda^{-m} \max_{\beta:\,\|\beta\|\le n} \|\partial^\beta a\|^m.$$

If $m \ge n$ and $(\beta^1,\dots,\beta^m) \in B_\alpha(m)$, then the number of $1 \le \ell \le m$ for which $\beta^\ell \ne \mathbf{0}$ is at most $n$, and so

$$\|\partial^\alpha d^m\|_{\mathrm{op}} \le \operatorname{card}\big(B_\alpha(m)\big) d^{m-n}\Lambda^{-m} \max_{\beta:\,\|\beta\|\le n} \|\partial^\beta a\|^n.$$

and

$$\operatorname{card}\big(B_\alpha(m)\big) = \operatorname{card}\big(B_\alpha(n)\big)\prod_{\ell=0}^{n-1}(m-\ell)$$

Hence, since

$$\left|\sum_{m\ge n}(-1)^m\binom{\frac12}{m}\prod_{\ell=0}^{n-1}(m-\ell)\|d\|_{\mathrm{op}}^{m-n}\right| = \frac12\prod_{\ell=1}^{n-1}\big(\ell-\tfrac12\big)\big(1-\|d\|_{\mathrm{op}}\big)^{\frac12-n},$$

it is clear that asserted estimate for $n \ge 2$ holds. □

**p. 51, line 3up**: Change $\sqrt{K}$ to $\sqrt{2K}$.

**p. 52, lines 11dn 6 & 5up**: Change $K$ to $2K$ in the expressions there.

**p. 53, line 14dn**: Replace $\int_{\mathbb{R}^N}$ by $\int_\Gamma$

**p. 59, line 13up**: Change $e^{-\frac{y^2}{2}}$ to $e^{-\frac{y^2}{2t}}$

**p. 65, lines 8dn–17dn**: Change to
such that

$$\mathfrak{H} := \big\{(t,\mathbf{y}) : t \in [0,s] \text{ and } |\mathbf{y}-p(t)| < 2r\big\} \subseteq \mathfrak{G},$$

$[s-r,s]\times\overline{B(\mathbf{x},2r)} \subseteq \mathfrak{G}$, $|p(t)-\mathbf{x}| < r$ for $t \in [s-r,s]$,and $u(t,\mathbf{y}) \ge u(0,\mathbf{0})+\delta$ for $(t,\mathbf{y}) \in [s-r,s]\times\overline{B(\mathbf{x},2r)}$. Next, set

$$\zeta^{\mathfrak{H}}(w) = \inf\big\{t \ge 0 : \big(t,w(t)\big) \notin \mathfrak{H}\big\} \text{ and } \zeta(w) = \inf\big\{t \ge s-r : w(t) \in \overline{B(\mathbf{x},2r)}\big\},$$

and observe that $\|w-p\|_{[0,s]} < r \implies \zeta(w) < \zeta^{\mathfrak{H}}(w)$. Hence, since

$$\begin{aligned} u(0,\mathbf{0}) &= \mathbb{E}^{\mathcal{W}}\big[u\big(\zeta\wedge\zeta^{\mathfrak{H}}, w(\zeta\wedge\zeta^{\mathfrak{H}})\big)\big] \ge u(0,\mathbf{0})\mathcal{W}(\zeta^{\mathfrak{H}} \le \zeta) + \big(u(0,\mathbf{0})+\delta\big)\mathcal{W}(\zeta < \zeta^{\mathfrak{H}}) \\ &= u(0,\mathbf{0}) + \delta\mathcal{W}(\zeta < \zeta^{\mathfrak{H}}) \end{aligned}$$

and $\mathcal{W}(\zeta < \zeta^{\mathfrak{H}}) \ge \mathcal{W}\big(\|w-p\|_{[0,s]} < r\big) > 0$, we would have the contradiction that $u(0,\mathbf{0}) > u(0,\mathbf{0})$.

**p. 68, line 1up**: Insert "to" after "respect"

**p. 70, 1up**: Change $I_{\eta_3}$ to $I_{\eta_2}$

**p. 75, line 7dn**: Change $\varphi\big(I_\sigma(\tau)\big)$ to $\varphi\big(V(\tau), I_\sigma(\tau)\big)$

**p. 76, lines 7up & 4up; p. 77, 1dn**: Change $I_\sigma$ to $I_{\sigma_n}$

**p. 77, lines 10dn & 11dn**: Change $m < 2^n$ to $m < 2^n t$

**p. 79, line 6up**: Change to:

$$\mathbb{E}^{\mathbb{P}}\big[\|I_\sigma(\,\cdot\,)\|_{0,t\wedge\zeta_R}^p\big]^{\frac{1}{p}} \le \frac{p}{\sqrt{2(p-1)}}\mathbb{E}^{\mathbb{P}}\big[A(t)^{\frac{p}{2}}\big]^{\frac{1}{p}}$$

**p. 79, line 3up**: Change to epresion for $K_p$ to $K_p = \left(\frac{p}{\sqrt{2(p-1)}}\right)^{\frac{p}{2}} \le (2p)^{\frac{p}{2}}$

**p. 80, line 7dn**: Change "a is" to "is a"

**p. 80, line 13up**: Change $\mu_t$ to $\mu(t,\,\cdot)$

**p. 80, line 11up**: Change $\mathbf{1}_{[a,t]}$ to $\mathbf{1}_{[p(a),p(t)]}$

**p. 87, line 9up**: Change $2^{\frac{n}{2}}$ to $2^{-\frac{n}{2}}$

**pp. 89–91**: Change $2^{\frac{n}{2}}$ to $2^{-\frac{n}{2}}$ in line 2 & 12dn on p. 89, 7up on p. 90, and 8dn on p. 91

**p. 96**: Delete this page.

**p. 97**: Delete lines 1 through 13, replace equation (3.5.4) by
(3.5.4).

$Z^{(m)}$is the closure in $L^2(\mathcal{W};\mathbb{R})$ of span of $\{I^{(m)}_{f^{\otimes m}}(\infty) : f \in L^2([0,\infty);\mathbb{R}^M)\}$

and replace line 14 by "Finally, $Z^{(m)} \perp Z^{(m')}$ when"

**p. 97, line 9-8up**: Change this line to

$$\int\limits_{0<\tau_{m'-m}<\cdots<\tau_{m'}} \mathbb{E}\big[I_{f_1'\otimes f_{m'-m}'}(\tau_{m'-m})\big] \times \prod_{\ell=1}^{m}\big(f_\ell(\tau_{m'-m+\ell}), f_{m'-m+\ell}'(\tau_{m'-m+\ell}\big)_{\mathbb{R}^M}\, d\tau_{m'-m}\cdots d\tau_{m'} = 0.$$

**pp. 97-98**: Relace lines 3up on p. 97 through 1dn on p. 98 by:
When $m \ge 1$, to understand why $I^m_{f^{\otimes m}}(\infty)$ is said to be of $m$th order chaos, it is helpful to write $dw(\tau)$ as $\dot{w}(\tau)\,d\tau$ write $I^{(m)}_{f^{\otimes m}}(\infty)$ as

$$\int_{\tau_1<\cdots\tau_m}\big(f(\tau_1),\dot{w}(\tau_1)\big)_{\mathbb{R}^M}\cdots\big(f(\tau_m),\dot{w}(\tau_m)\big)_{\mathbb{R}^M}d\tau_1\dots d\tau_m.$$

In the world of engenineerting and physics,

**p. 99, lines 11 & 7-6up**: Replace these line by

$$w \rightsquigarrow F\Big(\big(\xi_1, w(t_1)\big)_{\mathbb{R}^M}, \dots, \big(\xi_L, w(t_L)\big)_{\mathbb{R}^M}\Big)$$

where $L \ge 1$, $0 < t_1 < \cdots < t_L$, and $\{\xi_1, \dots, \xi_L\} \subset \mathbb{R}^M$.

**p. 111, line 13up**: Change $M(\zeta_{m,n})| \ge 2^{-n}$ to $M(\zeta_{m,n+1})| \ge 2^{-n-1}$

**p. 112, line 8dn**: Change $2^{1-2n}$ to $4^{1-n}$

**p. 117, line 10up**: Change ${}_n(\,\cdot\,) - I_m(\,\cdot\,)$ to $I_n(t) - I_m(t)$

**p. 120, line 3dn**: Change $F(a) - F(b)$ to $F(b) - F(a)$

**p. 121, line 1dn**: Replace $-I^M_\xi(t)$ to $-I^M_\xi(t\wedge\zeta_1)$

**p. 122, line 2dn**: Change $\sigma(\tau)^\top dA(\tau)\sigma(\tau)$ to $\sigma(\tau)dA(\tau)\sigma(\tau)^\top$

**p. 122, line 1up**: Insert after $n \geq 0$: $\zeta_{m,0} = m$

**p. 123, lines 5dn & 5up**: Change $)_{\mathbb{R}^{N_2}}$ to $)_{\mathbb{R}^{N_1}}$ in 5dn and $\Big(\nabla_{(2)}\varphi(\mathbf{V}(\tau), d\mathbf{M}(\tau)_{\mathbb{R}^{N_2}}$ to $\Big(\nabla_{(2)}\varphi\big(\mathbf{V}(\tau), \mathbf{M}(\tau)\big), d\mathbf{M}(\tau)\Big)_{\mathbb{R}^{N_2}}$ in 5up

**p. 124, lines 1 & 6dn**: Change $\zeta_{m1}$ to $\zeta_{m+1}$ in line 1dn and $\nabla_{(2)}\varphi$ to $\nabla^2_{(2)}\varphi$ in 6dn

**p. 128, lines 12 & 15dn**: Change $\Pi(t)$ to $\Pi(t)^\perp$ in line 12dn and $\sigma^{-1}\boldsymbol{\xi}$ to $\sigma^{-1}(\tau)\boldsymbol{\xi}$ in 15dn

**p. 129, 3up**: Change $-x_1x_3$ to $-x_1x_2$ in second line of matrix

**p. 133, line 4dn**: Change $d(x(\tau)$ to $dX(\tau)$

**p. 133, line 1up**: After "derivatives," insert "assume that the first derivatives of $\sum_{k=1}^M \mathcal{L}_{V_k}V_k$ are bounded,"

**p. 134, line 4dn**: Change $= \varphi(\mathbf{x})$ to $-\varphi(\mathbf{x})$

**p. 155, line 9up**: Change $\lfloor\tau\rfloor$ to $\lfloor\tau\rfloor_n$

**p. 163, line 5dn**: Change $E_\beta$ to $\tilde{E}_\beta$

**p. 165, line 9dn**: Replace $\sqrt{g^\Phi \circ \Phi^{-1}}$ by $\sqrt{\det g^\Phi \circ \Phi^{-1}}$

**p. 166, line 7dn**: Insert "equation" after "stochastic integral" at the end of this line

**p. 166, line 3up**: Change $(x_1^{\mathfrak{e}}, \dots, x_m^{\mathfrak{e}})$ to $(x_1^{\mathfrak{e}}, \dots, x_N^{\mathfrak{e}})$

**p. 167, line 3up**: Change $\sum_{j=m+1}^M$ to $\sum_{j=m+1}^N$

**p. 168, line 6up**: Change $L = \sum_{j=1}^N$ to $L = \frac{1}{2}\sum_{j=1}^N$

**pp. 168 & 169, lines 4up & 6dn**: Change $= \Delta_M$ to $= \frac{1}{2}\Delta_M$

**p.178, line 2dn**: Change *Riccardi equation* to *Riccati equation*

**p. 180, line 7dn**: Change $(f_\delta + \epsilon)^{\frac{1}{p-1}}$ for $(f_\delta + \epsilon)^{\frac{1}{p}-1}$

**p. 181, line 2up**: Change $\|D_h\Phi\|_{L^r(\mathcal{W};\mathbb{R})}$ to $\|D_h\Phi\|_{L^p(\mathcal{W};\mathbb{R})}$

**p. 184, lines 5 & 9dn**: Change $[0,\infty) \times \mathbb{R}^N$ to $[0,\infty) \times \mathbb{R}$ in line 5dn and $D_h(\tau, x)$ to $D_hX(\tau, x)$ in line 9dn

**p. 188, line 4dn**: Insert $dt$ before $\geq$

**p. 190, lines 3 & 4dn**: Change the right hand side of the equation to

$$\mathcal{A}(x_1)^{-1}\begin{pmatrix}\big(D(\varphi \circ X(1,x), DX_1(1,x)\big)_{H^1(\mathbb{R})} \\ \big(D(\varphi \circ X(1,x), DX_2(1,x)\big)_{H^1(\mathbb{R})}\end{pmatrix}$$

**p. 191, line 1up**: Change $e^{\epsilon_m(\alpha k^{2-2m})^{\frac{1}{5}}}$ to $e^{\epsilon_m(\alpha k^{-2m})^{\frac{1}{5}}}$

**p. 192, line 2dn**: Change to

$$\sum_{k=1}^\infty e^{-\epsilon_m(\alpha k^{-2m})^{\frac{1}{5}}} \leq e^{-\epsilon_m \alpha^{\frac{1}{m+5}}} \sum_{k \leq \alpha^{\frac{1}{2m+5}}} e^{-\epsilon_m k^2} + \sum_{k > \alpha^{\frac{1}{2m+5}}} e^{-\epsilon_m k^2}$$

**p. 192, lines 4 & 5 dn**: Change $\frac{1}{m+4}$ to $\frac{1}{m+5}$

**p. 193, line 7up**: Change $\int_s^1$ to $\int_s^1$

**p. 194, lines 1 & 2 dn**: Change $\sum_{k=1}^{n}$ to $\sum_{k=1}^{\infty}$

**p. 200, line 4up**: Change $(D\Phi_1, D\Psi_2)^2_{L^2(\mathcal{W};H^1(\mathbb{R}^N))}$ to $(D\Phi_1, D\Psi_2)_{L^2(\mathcal{W};H^1(\mathbb{R}^N))}$