Errata

p. 7, 5 up:

$$e^{tf} = e^{t\alpha}\sum_{j=0}^{\infty}\frac{(t\alpha)^j}{j!}\widehat{\mu^{*j}} \quad \text{where } \alpha = M(\mathbb{Z}_n) \text{ and } \mu = \frac{M}{\alpha}.$$

p. 8, 6 down: $\Lambda_{\delta_0} = (\tau_{-x})_*\Lambda_{\delta_x} \in T_{\delta_0}\mathbf{M}_1(\mathbb{Z}_n)$.

p. 41, 5 down:

$$\hat{\lambda}(\xi) = f^{(a,b,M)}(\xi) \equiv e^{\ell^{(a,b,M)}}(\xi). \tag{2.1.10}$$

p. 46, 1 up: definition of the tangent space

p. 50, 2 up:

(**c**)
$$\lim_{R\to\infty}\sup_{(x,\mu)} M\big((x,\mu), B_{\mathbb{R}^n}(0,R)\big) = 0.$$

p. 51, 6 up:

$$L^x_\mu\eta_m(z) \le B\eta_{m-1}(z) \quad \text{where } B \equiv \sup_m\sup_{(x,\mu)}\big\|L^x_\mu\eta_m\big\|_{\mathrm{u}} < \infty.$$

p. 55, 12 down: $\{\mu_{\cdot} \restriction [0,T] : \mu_0 \in K\}$ is compact as a subset of $C\big([0,\infty);\mathbf{M}_1(\mathbb{R}^n)\big)$.

p. 61, 12 down:
Proof: Because the case when $M(\mathbb{R}^n) = 0$ is trivial, we assume that $M(\mathbb{R}^n) > 0$.

p. 63, 12 down:

$$\mathbb{Q}\left(\big\|q^{(N)} - q^{(m)}\big\|_{[0,T]} \ge \frac{2T}{m^2}\right) \le \mathbb{Q}\left(\big\|\bar{q}^{(N)} - \bar{q}^{(m)}\big\|_{[0,T]} \ge \frac{T}{m^2}\right) \le \frac{Tm^4}{2^m},$$

p. 63, 6 up: $\mathbb{P}^{(0,0,M)}$.

p. 67, 10 up:

$$\eta(\,\cdot\,,\Delta,p^1+p^2) = \eta(\,\cdot\,,\mathbb{R}^n,p^1) \quad \mathbb{Q}\text{-almost surely,}$$

p. 71, 14 down: for some $q \in (1,\infty)$

p. 78, 2–1 up:

$$\hat{b}(x) = b(x) - \int_{\mathbb{R}^{n'}} \left(\frac{|F(x,y')|^2}{1+|F(x,y')|^2} - \mathbf{1}_{[1,\infty)}(|y'|) \right) F(x,y') M'(dy') \tag{3.1.7}$$

p. 93, 11 up:

$$+\mathbb{E}^{\mathbb{P}}\left[\int_{t_1 \le \tau < t \le t_2} \Big(\partial_\tau L v\big(\tau, p(t)\big) - L\partial_\tau v\big(\tau, p(t)\big)\Big)\, d\tau dt, A \right].$$

p. 94, 16 up:

$$\big\langle \varphi, P_t\big(p(s),\,\cdot\,\big)\big\rangle = u_\varphi\big(t, p(s)\big) = \mathbb{E}^{\mathbb{P}}\Big[\varphi\big(p(s+t)\big)\,\Big|\,\mathcal{B}_s\Big] \text{ for } \varphi \in C_{\mathrm{c}}^\infty(\mathbb{R}^n;\mathbb{R}).$$

p. 94, 3 up: is continuous for all bounded, $\mathcal{B}$-measurable, continuous Φ's.

p. 97, 5 down:

$$(\partial_\xi)_x a^{\frac{1}{2}} = \beta^{-\frac{1}{2}} \sum_{\ell=1}^\infty \binom{\frac{1}{2}}{\ell} \sum_{k=1}^{\ell} m(x)^k \big((\partial_\xi)_x a\big) m(x)^{\ell-k}.$$

p. 99, 14–13 up: *Assume that* $n' \ge 2$, *and let* M' *be the Lévy measure on* $\mathbb{R}^{n'}$ *given by* $M'(dy') = \mathbf{1}_{\mathbb{R}^{n'}\setminus\{\mathbf{0}\}}(y')|y|^{n'-1}\,dy'$.

p. 101, 9 up:

$$\int_{\mathbb{R}^n\setminus\{\mathbf{0}\}} \varphi(y)\, M(x,dy) = \int_{[0,1)} \left(\int_{(0,\infty)} \varphi\big(G(x,r,t)\big)\, \frac{dr}{r^2} \right) dt.$$

p. 101, 7–4 up: *Replace* $\omega_{n'-1}$ *by* $\boldsymbol{\omega}_{n'-1}$ *four times.*

p. 103, 6 up:

$$|F(x_1, r\omega) - F(x_0, r\omega)| \le |x_1 - x_0| \int_0^1 \big|\mathrm{grad}_{x_t} \rho(\,\cdot\,,\omega, r)\big|\, dt,$$

p. 109, 11 down: In particular, $|X^N(t,x,p)| \ge |x|$.

p. 111, 15 down: $\mathbb{P}_x = \mathbb{P}_x^{(\sigma,b,F,M')}$

p. 125, 6 down: *replace* However *by* Nevertheless

p. 130, 6 down: $\psi \equiv 0$ on $\mathbb{R}\setminus(0,1)$ and $\int_{\mathbb{R}} \psi(t)\,dt = 1$, and set

p. 131, statement of Theorem 5.1.11: *Replace* $\Theta^2(\mathbb{P};\mathbb{R})$ *by* $\Theta^2(\mathbb{P};\mathbb{R}^n)$ *three times*

p. 133, 1 down: Then, by Hunt's version (cf. Theorem 7.1.14 in [36]) of Doob's Stopping Time Theorem and (5.1.12),

p. 145, 4 up: *Replace* assuming0 *by* assuming

p. 146, 3 down: Replace by

$$\zeta_{m+1}^N = \inf\left\{\tau \geq \zeta_m^N : \big|Z(\tau) - Z(\zeta_m^N)\big| \vee \max_{1\leq j\leq \ell} \int_{\zeta_m^N}^{\tau} |\theta_j(\sigma)|^2 \, d\sigma \geq 4^{-N}\right\} \wedge t.$$

p. 146, 2–1 up:

$$F\big(Z(t)\big) - F\big(Z(0)\big) = \sum_{i=1}^{k} \int_0^t F_i^N(\tau)\, dX_i(\tau) + \sum_{j=1}^{\ell} \int_0^t \big(\tilde{\theta}_j^N(\tau), d\beta(\tau)\big)_{\mathbb{R}^n}$$
$$+ \frac{1}{2} \sum_{m=0}^{\infty} \sum_{j,j'=1}^{\ell} \partial_{y_j}\partial_{y_{j'}} F(Z_m^N)\Delta_{m,j}^N \Delta_{m,j'}^N + \sum_{m=0}^{\infty} E_m^N,$$

p. 147. 2–1 up:

$$\mathbb{E}^{\mathbb{P}}\big[\Delta_{m,j}^N \Delta_{m,j'}^N \,\big|\, \mathcal{F}_{\zeta_m^N}\big] = \mathbb{E}^{\mathbb{P}}\Big[Y_j(\zeta_{m+1}^N)Y_{j'}(\zeta_{m+1}^N) - Y_j(\zeta_m^N)Y_{j'}(\zeta_m^N) \,\Big|\, \mathcal{F}_{\zeta_m^N}\Big]$$
$$= \mathbb{E}^{\mathbb{P}}\left[\int_{\zeta_m^N}^{\zeta_{m+1}^N} \big(\theta_j(\tau), \theta_{j'}(\tau)\big)_{\mathbb{R}^n} \, d\tau \,\middle|\, \mathcal{F}_{\zeta_m^N}\right],$$

p. 148, 7 down:

$$(S_{m,j,j'}^N)^2 \leq C_2^2 4^{-N}\Big((\Delta_{m,j}^N)^2 + (\Delta_{m,j'}^N)^2 + \int_{\zeta_m^N}^{\zeta_{m+1}^N} \big(|\theta_j(\tau)|^2 + |\theta_{j'}(\tau)|^2\big)\, d\tau\Big).$$

p. 152, 2 up: change to

$$\leq 4\mathbb{E}^{\mathbb{P}}\left[\int_0^t \big|F'\big(\beta(\tau)\big) - \mathbf{1}_{[y,\infty)}\big(\beta(\tau)\big)\big|\, d\tau\right]$$

p. 155, 3 down:

$$\frac{1}{2}\int_{\mathbb{R}} \varphi(y)\ell(t,y)\, dy = \lim_{N\to\infty} 2^{-N-1} \sum_{m\in\mathbb{Z}} \varphi(\xi_m^N)\ell(t, m2^{-N})$$

p. 161, 15 down: *Furthermore, if* η *is a* $\{\mathcal{B}_t : t \geq 0\}$*-progressively measurable,* $\mathbb{R}^n$*-valued map satisfying*

p. 161, 7up: Replace by

$$H_\varphi(t) \equiv \Big(\big(\eta(t), \mathrm{grad}_x\varphi\big)_{\mathbb{R}^m} + \tfrac{1}{2}\Delta\varphi + \big(\theta(t), \mathrm{grad}_y\varphi\big)_{\mathbb{R}^n}\Big)\big(X(t), \beta(t)\big).$$

p. 161, 3 up: *Replace* $\mathbb{Q}(A)$ *by* $\mathbb{Q}^T(A)$

p. 164, 14–13 up:

$$S(t\wedge\zeta_R) = S^R(t) \equiv \exp\left(-\int_0^t \big(\theta^R(\tau), d\beta(\tau)\big)_{\mathbb{R}^n} - \frac{1}{2}\int_0^t \big|\theta^R(\tau)\big|^2\,d\tau\right),$$

where $\theta^R(t) \equiv \mathbf{1}_{[0,\zeta_R]}(t)\theta(t)$, and so

p. 168, 4 down:

$$\mathbb{E}^{\mathbb{P}^0_x}\big[F\,\big|\,p(T)=y\big] = \mathbb{E}^{\mathbb{P}^0_x}\left[F\frac{\gamma_{T-t}\big(y-p(t)\big)}{\gamma_T(y-x)}\right],$$

p. 169, 6 down:

$$\mathbb{E}^{\mathbb{P}^0_x}\big[F\,\big|\,\sigma\big(p(T)\big)\big] = \mathbb{E}^{\mathbb{P}^0_{T,x,p(T)}}[F] \quad \mathbb{P}^0_x\text{-almost surely.}$$

p. 169, 14–13 up: application of (*), for any $m \geq 1$ and $0 < t_1 < \cdots < t_m < T$,

$$\begin{aligned}&\mathbb{P}^0_x\big(X_y(t_1)\in d\xi_1,\dots X_y(t_m)\in d\xi_m\big)\\ &\quad= \frac{\gamma_{t_1}(\xi_1-x)\cdots\gamma_{t_m-t_{m-1}}(\xi_m-\xi_{m-1})\gamma_{T-t_m}(y-\xi_m)}{\gamma_T(y-x)}\,d\xi_1\cdots d\xi_m\\ &\quad= \mathbb{P}^0_y\big(X_x(T-t_m)\in d\xi_m,\dots X_x(T-t_1)\in d\xi_1\big).\end{aligned}$$

p. 170, 7–9 down:

$$\int_{(1-\delta)T}^T \frac{\mathbb{E}^{\mathbb{P}^0}\big[|X_y(\tau)-y|\big]}{T-\tau}\,d\tau = \int_0^{\delta T}\frac{\mathbb{E}^{\mathbb{P}^0_y}\big[|X_0(\tau)-y|\big]}{\tau}\,d\tau,$$

and it is an easy matter to check that

$$\sup_{|y|\leq R}\ \sup_{\tau\in(0,\frac{T}{2}]}\frac{\mathbb{E}^{\mathbb{P}^0_y}\big[|X_0(\tau)-y|\big]}{\sqrt{\tau}} < \infty.$$

p. 176, 14–10 up: Finally, given $\mu\in\mathcal{A}$ with $\mu_m = 0$ for $m > M$, set $G_\mu = g_1^{\otimes\mu_1}\otimes\cdots\otimes g_M^{\otimes\mu_M}$, where the meaning here is determined by the convention that $g_1^{\otimes\mu_1}\otimes\cdots\otimes g_\ell^{\otimes\mu_\ell} = g_1^{\otimes\mu_1}\otimes\cdots\otimes g_{\ell-1}^{\otimes\mu_{\ell-1}}$ if $\mu_\ell = 0$. In particular, $G_\mu \in L^2\big(\square^{(m)};(\mathbb{R}^n)^m\big)$ and, for $\mu,\nu\in\mathcal{A}$ with $\|\mu\|_1 = m = \|\nu\|_1$,

$$\big(\tilde{I}^{(m)}_{G_\mu}, \tilde{I}^{(m)}_{G_\nu}\big)_{L^2(\mathbb{P}^0;\mathbb{R})} = \delta_{\mu,\nu}\mu!. \tag{6.3.3}$$

p. 177, 1 down: *There is a unique linear map (cf.* (5.1.1)*)*

p. 177, 8 down: *where* $\mu! \equiv \prod_1^\infty \mu_j!$ *and the convergence is in* $L^2(\mathbb{P}^0;\mathbb{R})$.

p. 177, 2 up:

$$= \int_{\Delta^{(m'-m)}} \prod_{\ell=0}^{m'-m-1} \big(f_{m-\ell}(\tau_{m-\ell}), f_{m'-\ell}(\tau_{m'-\ell})\big)_{\mathbb{R}^n}$$

p. 179, 3 up: $L^1\big([0,\infty);\mathbb{R}^n\big)$ is dense in $L^2(\mathbb{P}^0;\mathbb{R})$.

p. 186, 4 down: *Replace* the one *by* one can

p. 190, 5 up: *Replace* meant to be *by* used to indicate

p. 191, 2 down: *Replace* in [34] *by* in [35]

p. 191, 12 down:

$$\mathbb{P}\big(\exists\, t \in [0,\infty)\ 0 < |M|(t) < \infty\big) = 0.$$

p. 192, 7 down: the contradiction $\mathbb{E}^{\mathbb{P}}\big[M_R(t)^2\big] \geq \frac{1}{4}\mathbb{E}^{\mathbb{P}}\big[\|M_R\|^2_{[0,t]}\big] > 0$.

p. 193, 6 up:

$$Y_{N'}(t,\omega) - Y_N(t,\omega) = \sum_{m=1}^{\infty} \Big(M_{m-1,N'}(\omega) - M^{(N)}_{m-1,N'}(\omega)\Big)\Delta_{m,N'}(t,\omega).$$

p. 194, 15 up: *Replace* $\sigma\big(N(\tau):\,\tau\geq 0\big)$ *by* $\sigma\big(\{N(\tau):\,\tau\geq 0\}\big)$

p. 195, 16 down: *Replace* $\mathbb{E}^{\mathbb{P}}\big[\big(M(\zeta) - M90)^2\big]$ *by* $\mathbb{E}^{\mathbb{P}}\big[\big(M(\zeta) - M(0)\big)^2\big]$

p. 197, 16 up:

$$\Big|\sqrt{\langle M_2\rangle(t)} - \sqrt{\langle M_1\rangle(t)}\Big| \leq \sqrt{\langle M_2 - M_1\rangle(t)} \leq \sqrt{\langle M_2 - M_1\rangle(T)},\ 0 \leq t \leq T,$$

p. 200, 9 up: *Replace* $\theta(t,\omega)$ *by* $\theta_N(t,\omega)$

p. 201, 5–1 up: *Change to*: Indeed, (7.2.8) allows us to get the following extention of Theorem 5.3.1 by the the same argument as we used earlier.

p. 204, 15 down: *Replace* proof (7.2.16) *by* proof of (7.2.16)

p. 204, 7 up:

$$= \frac{(q')^q}{2}\mathbb{E}^{\mathbb{P}}\left[\int_0^T F''_\epsilon\big(M(\tau)\big)\,\langle M\rangle(d\tau)\right]$$

p, 206, 14 down:

$$\beta(s,\omega) = \begin{cases} M\big(\zeta(s,\omega),\omega\big) - M(0) & \text{when } \langle M\rangle(\infty,\omega) > s \\ 0 & \text{otherwise,} \end{cases}$$

p. 208, 4 down: $M'(s,\omega)$, and $A(t,\omega) = \langle M\rangle(t,\omega)$.

p. 210, 8 up:

$$\pi(t,\omega)=\lim_{\epsilon\searrow 0}\epsilon\big(a(t,\omega)+\epsilon I\big)^{-1}\ \&\ \alpha^{-1}(t,\omega)=\lim_{\epsilon\searrow 0}\alpha(t,\omega)\big(a(t,\omega)+\epsilon I\big)^{-1}.$$

p. 211, 3 down: Clearly, in the case when $\lambda_{[0,\infty)}\big(\{t:\ a(t,\omega)=0\}\big)=0$ and, for each $\xi\in\mathbb{R}^n\setminus\{\mathbf{0}\}$, $\int_0^\infty\big(\xi,a(t,\omega)\xi\big)_{\mathbb{R}^n}\,dt=\infty$ $\mathbb{P}$-almost surely,

p. 211, 9 down: *Replace* $\pi\big|(t)^\perp\xi\big|^2\,dt$ *by* $\big|\pi(t)^\perp\xi\big|^2$

p. 212, 4 up: *Replace* $\langle M\rangle(t,p)$ *by* $\langle\langle M\rangle\rangle(t,p)$

p. 215, 7 & 5 up: *Replace* u_φ *by* u_X

p. 217, 4 up: *Replace* (6.3.12) *by* (6.3.19)

p. 227, 7–6 up: *Replace by*:
show that, for each $\xi\in\mathbb{S}^{n-1}$, $\big(\check\beta^T_\xi(t),\check{\mathcal{F}}^T_t,\mathbb{P}\big)$ is a semimartingale with $\langle\check\beta^T_\xi,\check\beta^T_\xi\rangle(t)=t\wedge T$ and bounded variation part $t\rightsquigarrow-\int_0^t\frac{\check\beta^T_\xi(\tau)}{T-\tau}\,d\tau$ when $\check\beta^T_\xi\equiv(\xi,\check\beta^T)_{\mathbb{R}^n}$.

p. 235, 4 down: *Insert* (cf. (8.2.3))

p. 236, 11 up: *Replace* kth coordinate *by* ℓth coordinate

p. 238, 4–5 down:

$$\begin{aligned}&t\in\big(m2^{-N},(m+1)2^{-N}\big)\longrightarrow\\&\quad E\Big((t-m2^{-N})\big(2^{-N},p\big((m+1)2^{-N}\big)-p(m2^{-N})\big),X(m2^{-N},x,p)\Big)\in\mathbb{R}^n\end{aligned}$$

p. 239, 1 down: *Replace* $n\in\mathbb{N}$ *by* $N\in\mathbb{N}$

p. 239, 3 up: *Replace by*:

$$Y_N\big(t-[t]_N,Y_N([t]_N,x)\big)\qquad\text{for } t\ge 2^{-N}.$$

p. 240, 12 up: if $M\ni x$ is a closed submanifold of $\mathbb{R}^n$ with the property that

p. 240, 6 up: $C\big([0,\infty);\mathbb{R}^r\big)$, and closed intervals $I\subseteq[0,\infty)$.

p. 252, 4 up: where $\xi^L(\tau,p)\equiv\big([\tau]_L-[\tau]_{L-1},p([\tau]_L)-p([\tau]_{L-1})\big)$. Having done so, one

p. 258, 13 up:

$$\sup_{\substack{x\in\mathbb{R}^n\\ \delta\in(0,1]}}\mathbb{E}^{\mathbb{P}^0}\Big[\Big(\big\|X(\,\cdot\,,x,p)\big\|^{(\alpha)}_{[0,T]}\Big)^q\,\Big|\,\|p-g\|_{M,[0,T]}\le\delta\Big]<\infty.$$