**Errata**

p. 2, 15 up: replace $\frac{\ell!}{k|(\ell-k)!}$ by $\frac{\ell!}{k!(\ell-k)!}$.

p. 3, 5 dn: replace $(C^n)_{\ell-1} + (C^n)_{\ell-1}$ by $(C^n)_{\ell-1} + (C^n)_{\ell}$

p. 3, 6 up: replace “numbers” by “number”

p. 3, 7 up: replace $S_\ell - S_{m-1}$ by $S_m - S_{m-1}$

p. 3, footnote: replace “infemum” by “infimum”

p. 4, 9 dn: remove “is”

p. 4, 7 up: remove “of.”

p. 7, eq. (1.1.15): replace “$2p \vee q$” by “$2(p \vee q)$”

p. 8, 3 dn: remove “and using (1.1.16) with $a = 1$”

p. 8, 6 dn: should be “$\mathbb{E}\left[s^{\zeta_2 \circ \Sigma^1}\right]$”

p. 9, 5 up: replace “if” by “if and only if.”

p. 10, 4 up: remove “by (1.3.13)”

p. 11, bottom half: replace “$\ell$” by “$\mathbf{l}$”

p. 13, 2 dn: remove “only”

p. 13, 17 up: “$\exp\left(-\ell \log \frac{\ell}{n+1}\right)$” should be “$\exp\left(\ell \log \frac{\ell}{n+1}\right)$”

p. 13, 4 up: replace “which” by “with”

p. 14, 12 dn: replace “$\frac{n}{d}$” by “$\frac{n}{2d}$”

p. 15, 11 up: replace “see” by “sees”

p. 17, 15 dn: replace “assume” by “assumed.”

p. 17, 17 dn: replace $\sum_{m=1}^{n} =$ by $\sum_{m=1}^{n} Y_m =$.

p. 18, 7 dn: replace "Starting again from (*)" by "Starting from (bf c)"

p. 20, 1 up: replace $\frac{\mathbb{E}[|X_n|]}{n}$ by $\frac{\mathbb{E}[|X_m|]}{n}$

p. 21, 4 dn: replace "(1.3.10)" by "(1.2.3)"

p. 26, 11 up: change "$\mu$ on $\{1, \ldots, N\}$" to "$\mu$ on $\mathbb{S}$"

p. 36, 3 up: replace $= \lim_{N\to\infty}$ by $\geq \lim_{N\to\infty}$.

p. 40, 15 up: replace "$\frac{2(N-1)}{n\epsilon}$" by "$\frac{N-1}{n\epsilon}$"

p. 47, 5 dn: should be $\sum_{\ell,n=1}^{\infty}$.

p. 48, 5 dn: replace $= \mathbb{P}(\rho_j < \infty \,|X_0 = i)$ by $= \mathbb{P}(\rho_j < \rho_i \,|\, X_0 = i)$.

p. 49, 11 dn: change to

$$\mathbb{E}\big[u(X_{n\wedge\rho_\Gamma})\,\big|\,X_0 = i\big] \leq u(i) \quad \text{for } n \geq 0 \text{ and } i \in \mathbb{S} \setminus \Gamma.$$

p. 49, 13 up: change "any $i$" to "any $i \notin \Gamma$"

p. 49, 12–11 up: replace $\sum_{k\notin S}$ by $\sum_{k\notin\Gamma}$.

p. 50, 14 up to p. 51, 14 dn: change to:

3.1.8 LEMMA. *If* $\mathbf{P}$ *is irreducible on* $\mathbb{S}$*, then, for any finite subset* $F \neq \mathbb{S}$*,* $\mathbb{P}(\rho_{\mathbb{S}\setminus F} < \infty | X_0 = i) = 1$ *for all* $i \in F$*.*

PROOF: Set $\tau = \rho_{\mathbb{S}\setminus F}$. By irreducibility, $\mathbb{P}(\tau < \infty | X_0 = i) > 0$ for each $i \in F$. Hence, because $F$ is finite, there exists a $\theta \in (0,1)$ and an $N \geq 1$ such that $\mathbb{P}(\tau > N | X_0 = i) \leq \theta$ for all $i \in F$. But this means that, for each $i \in F$, $\mathbb{P}\big(\tau > (\ell+1)N \,\big|\, X_0 = i\big)$ equals

$$\begin{aligned}
&\sum_{k\in F} \mathbb{P}\big(\tau > (\ell+1)N \;\&\; X_{\ell N} = k \,\big|\, X_0 = i\big) \\
&= \sum_{k\in F} \mathbb{P}\big(X_n \in F \text{ for } \ell N + 1 \leq n \leq (\ell+1)N,\ \tau > \ell N,\ \&\ X_{\ell N} = k \,\big|\, X_0 = i\big) \\
&= \sum_{k\in F} \mathbb{P}\big(\tau > N \,\big|\, X_0 = k\big)\mathbb{P}\big(\tau > \ell N \;\&\; X_{\ell N} = k \,\big|\, X_0 = i\big) \\
&\leq \theta \mathbb{P}\big(\tau > \ell N \,\big|\, X_0 = i\big).
\end{aligned}$$

Thus, $\mathbb{P}(\tau > \ell N | X_0 = i) \le \theta^\ell$, and so $\mathbb{P}(\tau_m = \infty | X_0 = j) = 0$ for all $i \in F$. $\square$

3.1.9 THEOREM. *Assume that* $\mathbf{P}$ *is irreducible on* $\mathbb{S}$, *and let* $u : \mathbb{S} \longrightarrow [0,\infty)$ *be a function with the property that* $\{k : u(k) \le L\}$ *is finite for each* $L \in (0,\infty)$. *If, for some* $j \in \mathbb{S}$, $(\mathbf{Pu})_i \le u(i)$ *for all* $i \ne j$, *the chain determined by* $\mathbf{P}$ *is recurrent on* $\mathbb{S}$.

PROOF: If $\mathbb{S}$ is finite, then (cf., for example, Exercise 2.4.2) at least one state is recurrent, and therefore, by irreducibililty, all are. Hence, we will assume that $\mathbb{S}$ is infinite.

Given $i \ne j$, set $F_L = \{k : u(k) \le u(i) + u(j) + L\}$ for $L \in \mathbb{N}$, and denote by $\rho_L$ the first return time $\rho_{(\mathbb{S}\setminus F_L)\cup\{j\}}$ to $(\mathbb{S} \setminus F_L) \cup \{j\}$. By Lemma 3.1.6,

$$u(i) \ge \mathbb{E}\big[u(X_{n\wedge\rho_L}) \,\big|\, X_0 = i\big] \ge \big(u(i) + u(j) + L\big)\mathbb{P}\big(\rho_{\mathbb{S}\setminus F_L} < n \wedge \rho_j \,\big|\, X_0 = i\big)$$

for all $n \ge 1$. Hence, after letting $n \to \infty$, we conclude that, for all $L \in \mathbb{N}$,

$$\begin{aligned} u(i) &\ge \big(u(i) + u(j) + L\big)\mathbb{P}\big(\rho_{\mathbb{S}\setminus F_L} < \rho_j \,\big|\, X_0 = i\big) \\ &\ge \big(u(i) + u(j) + L\big)\mathbb{P}\big(\rho_j = \infty \,\big|\, X_0 = i\big), \end{aligned}$$

since, by Lemma 3.1.8, we know that $\mathbb{P}(\rho_{\mathbb{S}\setminus F_L} < \infty | X_0 = i) = 1$. Thus, we have now shown that $\mathbb{P}(\rho_j < \infty | X_0 = i) = 1$ for all $i \ne j$. Since $\mathbb{P}(\rho_j < \infty | X_0 = j) = (\mathbf{P})_{jj} + \sum_{i\ne j} \mathbb{P}(\rho_j < \infty | X_0 = i)(\mathbf{P})_{ji}$, it follows that $\mathbb{P}(\rho_j < \infty | X_0 = j) = 1$, which means that $j$ is recurrent. $\square$

p. 57, 12–11 up: changle to: If $j$ is transient, then $\pi_{ij} = 0$ for all $i$ and therefore $\mu_j = 0$. If $j$ is recurrent, then either $i$ is transient, and so $\mu_i = 0$, or $i$ is recurrent, in which case, by Theorem 3.1.2, either $\pi_{ij} = 0$ or $i \leftrightarrow j$ and $\pi_{ij} = \pi_{jj}$.

p. 57, 3 up: change to "and, for each $i \in C$ and $s \in (0,1)$, $\big(\mathbf{R}(s)\big)_{ik} > 0 \iff k \in C$."

p. 57, 2 up: change "In particular," to "In addition,"

p. 58, 16 up: change to $b = a = a'$

p. 65: Here is a more conceptual way to prove that

$$\sum_{r=1}^{n} \mathbb{P}(\rho_j \ge r \,|\, X_0 = j)(\mathbf{P}^{n-r})_{jj} = 1.$$

Take $\rho_j^{(0)} = 0$ and, for $m \ge 1$, $\rho_j^{(m)}$ to be the time of the $m$th return to $j$. In addition, set $T_j^{(n-1)} = \sum_{\ell=0}^{n-1} \mathbf{1}_{\{j\}}(X_\ell)$. Then

$$X_0 = j \implies \{T_j^{(n-1)} = m + 1\} = \{\rho_j^{(m)} < n \le \rho_j^{(m+1)}\}.$$

Hence,

$$\sum_{r=1}^{n}\mathbb{P}(\rho_j \ge r \,|\, X_0=j)(\mathbf{P}^{n-r})_{jj} = \sum_{r=0}^{n-1}\mathbb{P}(\rho_j \ge n-r \,|\, X_0=j)(\mathbf{P}^{r})_{jj}$$
$$= \sum_{r=0}^{n-1}\sum_{m=0}^{r}\mathbb{P}(\rho_j \ge n-r \,|\, X_0=j)\mathbb{P}(\rho_j^{(m)}=r \,|\, X_0=j)$$
$$= \sum_{r=0}^{n-1}\sum_{m=0}^{r}\mathbb{P}(\rho_j^{(m+1)} \ge n \;\&\; \rho_j^{(m)}=r \,|\, X_0=j)$$
$$= \sum_{m=0}^{n-1}\sum_{r=m}^{n-1}\mathbb{P}(\rho_j^{(m+1)} \ge n \;\&\; \rho_j^{(m)}=r \,|\, X_0=j)$$
$$= \sum_{m=0}^{n-1}\mathbb{P}\big(\rho_j^{(m)} < n \le \rho_j^{(m+1)} \,\big|\, X_0=j\big)$$
$$= \sum_{m=0}^{n-1}\mathbb{P}(T_j^{(n-1)}=m+1) = \mathbb{P}\big(T_n^{(n-1)} \le n \,|X_0=j) = 1.$$

p. 67, 7 dn: change to "$i \in \mathbb{S}_{r+n} \implies 1 = \sum_{r'=0}^{d-1}(\mathbf{P}^n\mathbf{1}_{\mathbb{S}_{r'}})_i = (\mathbf{P}^n\mathbf{1}_{\mathbb{S}_r})_i$."

p. 67, 17 dn: replace $\pi_j^{(r)}$ by $\pi_{jj}^{(r)}$.

p. 68, Exercise 3.3.3: One needs to add that assumtion that $(\mathbf{P}u)_j < \infty$.

p. 70, 4 dn: replace $\alpha > 0$ by $\alpha \ge 1$.

p. 72, 3 dn: change "Exercise (3.3.8)" to "Exercise (3.3.7)"

p. 72, 12 up: replace "to see ... as $n \to \infty$." by "to see that $(\boldsymbol{\nu})_j \ge (\mu)_j$ for all $j \in \mathbb{S}$. Now consider $\boldsymbol{\omega} \equiv \boldsymbol{\nu} - \boldsymbol{\mu}$, and conclude that $\boldsymbol{\omega} = \mathbf{0}$."

p 73, 15 dn: replace $= \pi_{ii}\pi_{jj}$ by $\frac{\pi_{jj}}{\pi_{ii}}$.

p. 74, 9 up: replace $\epsilon > 0$ by $0 < \epsilon < 1$.

p. 74, 7 up: replace $\sup_{m \ge N\epsilon}$ by $\sup_{m \ge n\epsilon}$.

p. 76, 9 dn: replace $N(t) - N(s)$ by $N(s+t) - N(s)$

p. 76, 1 up: replace "of positive" by "are positive"

p. 77, 10 up: insert "$e^{-t}$" into the integrand of both integrals

p. 81, 9 dn: replace $E_n = \frac{J_n - J_{n-1}}{R_{n-1}}$ by $E_n = R_{X_{n-1}}(J_n - J_{n-1})$

p. 81, replace $R_{j_{m-1}}$ in the second line of (4.2.3) by $R_{X_{m-1}}$

p. 81, 16 & 13 up: replace $\Phi^{(\mathfrak{R},\mathbf{P})}$ by $\Phi^{\mathfrak{R}}$

p. 81, 1 up: should be $\{\bar{X}(t \wedge \zeta) : t \geq 0\}$

p. 82, 15 dn: replace $\Phi^{(\bar{\mathfrak{R}},\mathbf{P})}$ by $\Phi^{\bar{\mathfrak{R}}}$

p. 83, 9 & 8 up: replace $\Phi^{(\mathfrak{R},\mathbf{P})}$ by $\Phi^{\mathfrak{R}}$

p. 83, 4 up: replace $\{N(s) = m\}$ by $\{J_m \leq s < J_{m+1}\}$

p. 84, 3 dn: replace $\Phi^{\mathfrak{R},\mathbf{P}}$ by $\Phi^{\mathfrak{R}}$

p. 92, 10 up: Starting here and running through 8 dn on p. 93, the proof of Theorem 4.3.2 can be replaced by: In particular, for any $T > 0$ and $m \geq 1$,

$$\mathbb{P}\big(J_n \leq T \,\big|\, X(0) = i\big) \leq \mathbb{P}\big(J_n \leq T \;\&\; \rho_i^{(m)} \leq n \,\big|\, X(0) = i\big) + \mathbb{P}\big(\rho_i^{(m)} > n \,\big|\, X(0) = i\big),$$

where $\rho_i^{(m)}$ is the time of the $m$th return of $\{X_n : n \geq 0\}$ returns to $i$. Because, $i$ is recurrent for $\{X_n : n \geq 0\}$, the second term tends to 0 as $n \to \infty$. At the same time,

$$\rho_i^{(m)} \leq n \implies J_n \geq \frac{1}{R_i} \sum_{\ell=1}^{m} E_{\rho_i^{(\ell)}},$$

and so the first term in dominated by the probability that the sum of $m$ mutually indepedent, unit exponential random variable is less than or equal to $R_i T$, and this probability tends to 0 as $m \to \infty$. Hence, we have shown that, for all $T > 0$,

$$\mathbb{P}\big(J_\infty \leq T \,\big|\, X(0) = i\big) = \lim_{n \to \infty} \mathbb{P}\big(J_n \leq T \,\big|\, X(0) =\big) = 0,$$

and so

$$\mathbb{P}\big(\mathfrak{e} < \infty \,\big|\, X(0) = i\big) = \mathbb{P}\big(J_\infty < \infty \,\big|\, X(0) = i\big) = \lim_{T \to \infty} \mathbb{P}\big(J_\infty \leq T \,\big|\, X(0) = i\big) = 0.$$

p. 99, 6 dn: replace $\Phi^{\mathfrak{R},\mathbf{P}}$ by $\Phi^{\mathfrak{R}}$

p. 103, 14 dn: Change to

$$(\mathbf{P})^\top_{ij} = \frac{R_j \hat{\pi}_j}{R_i \hat{\pi}_i}$$

p. 105, 1 & 3 dn: replace $\boldsymbol{\mu}$ by $\hat{\boldsymbol{\mu}}$.

p. 105, 6 dn: replace $\boldsymbol{\nu} = \boldsymbol{\nu}\mathbf{P}$ by $\hat{\boldsymbol{\nu}} = \hat{\boldsymbol{\nu}}\mathbf{P}$.

p. 107, 1 dn: replace “This is” by “This chapter is”

p. 110, 1 up: remove “is”

p. 113, 1 up: replace “In” by “If”

p. 114, 9 up: replace $(\boldsymbol{\pi})_j)\big|$ by $(\boldsymbol{\pi})_j\big|$

p. 124, 2dn: change right hand side to $-\sum_{i,j\in F_N} (\hat{\boldsymbol{\pi}})_i (\mathbf{Q})_{ij} g(i) g(j)$

p. 125, 7dn: change to

$$\|f\|^2_{2,\hat{\boldsymbol{\pi}}} - \|\mathbf{P}(t)f\|^2_{2,\hat{\boldsymbol{\pi}}} = \sum_{m=0}^{n-1} \Big( \big\|\mathbf{P}\big(\tfrac{mt}{n}\big)f\big\|^2_{2,\hat{\boldsymbol{\pi}}} - \big\|\mathbf{P}\big(\tfrac{(m+1)t}{n}\big)f\big\|^2_{2,\hat{\boldsymbol{\pi}}} \Big),$$

p. 134, 15dn: Change to

$$\int_s^{s+\mathcal{T}(s,i,\xi)} S(\tau, i, L)\, d\tau = \xi.$$

p. 139, 5 up: replace $2\psi(s)$ by $2\psi(t)$

p. 140, 20 up: replace $(\mathbf{P}^\top\mathbf{P})_{ii}$ by $\mathbf{P}\mathbf{P}^\top)_{ii}$

p. 140 , 8 up: replace $\sum_{m=1}^\infty$ by $\sum_{k=1}^\infty$ twice

p. 141, 3 dn: replace $\theta_d^{rm}$ by $\theta_d^{-rm}$

p. 141, 6 dn: replace “Let $H$ be the subspace of ....” by “Let $H$ be the linear subspace spanned by ....”

p. 142, 18 dn: replace by "by a reversible probability vector for $\mathbf{P}$."

p. 142: change (5.6.11) to $\beta_+ \geq \frac{2\#E}{D^2 L(\mathcal{P})B(\mathcal{P})}$ and (5.6.12) to $\beta_- \geq \frac{2}{DL(\mathcal{P})B(\mathcal{P})}$

p.143, 15 up: replace by $\rho_k^{\boldsymbol{\mu}}(\boldsymbol{\omega}) \quad \text{if } \boldsymbol{\eta} = \hat{\boldsymbol{\omega}}^k$

p. 143, 10 up: change left hand side to $-\langle g, \mathbf{Q}^{\boldsymbol{\mu}} f \rangle_{\boldsymbol{\mu}}$