18.04 PROBLEM SET 1
DUE IN CLASS, FEBRUARY 19^{TH}, 2019

1.1) (30: 10,10,10 points)

(a) Let $z_1 = 1 + i$, $z_2 = 1 + 3i$. Compute $z_1 z_2$, z_1/z_2, $z_1^{z_2}$ (use the principal branch of log).

(Give $z_1 z_2$ and z_1/z_2 in standard rectangular form and $z_1^{z_2}$ in polar form.)

(b) Compute all the values of i^i. Say which one comes from the principal branch of log.

(Give all your answers in standard form.)

Is it surprising that i^i is real?

(c) Let $z = 1 + i\sqrt{3}$.

(i) Compute z^8. *(Give your answer in standard form.)*

(ii) Find all the 4th roots of z.

1.2) (15: 5,5,5 points)

(a) Show $\overline{\mathrm{e}^z} = \mathrm{e}^{\overline{z}}$.

(b) Show that if $|z| = 1$ then $z^{-1} = \overline{z}$.

(c) Let $\frac{x+iy}{x-iy} = a + ib$. Show that $a^2 + b^2 = 1$.

Hint: This takes one line if you look at it right. Think polar form.

1.3) (15: 5,10 points)

(a) Sketch the curve $z = \mathrm{e}^{t(1+i)}$, where $-\infty < t < \infty$.

(b) Consider the mapping $z \to w = z^2$. Draw the image in the w-plane of the triangular region in the z-plane with vertices 0, 1 and i.

1.4) (10 points)

Let $z_k = \mathrm{e}^{2\pi i/n}$. Show

$$1 + z_k + z_k^2 + z_k^3 + \ldots + z_k^{n-1} = 0$$

Hint: The polynomial $z^n - 1$ has one easy root. Use that to factor it into a linear term and a degree $n - 1$ term.

1.5) (20 points)

Consider the mapping $w = \mathrm{e}^z$.
(i) Sketch in the w-plane the image under this mapping of vertical lines in the z-plane.

(ii) On the same graph sketch the image of horizontal lines.

Show enough lines to give a good idea of what's happening.

(iii) Show (argue either geometrically or analytically) that the images of a vertical and a horizontal lines meet at right angles.