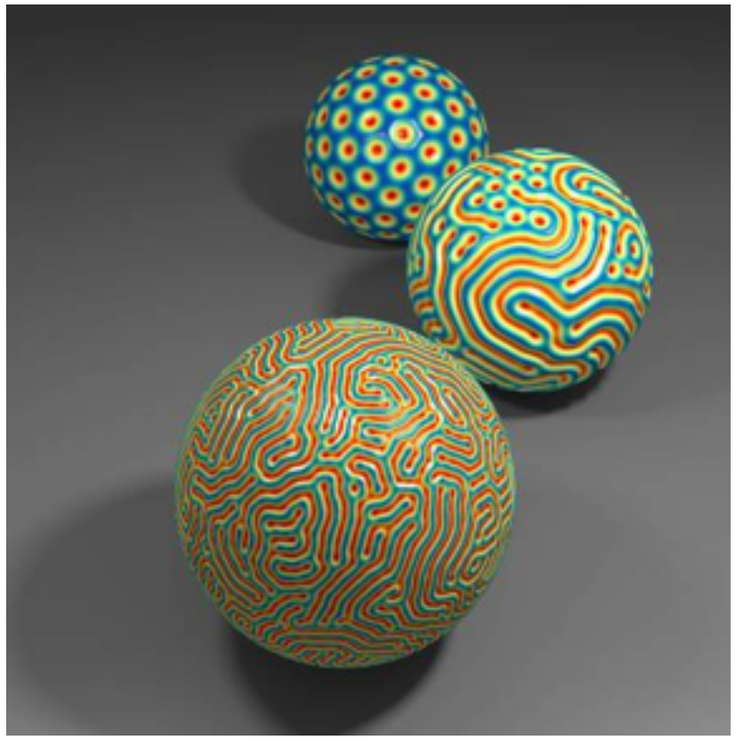
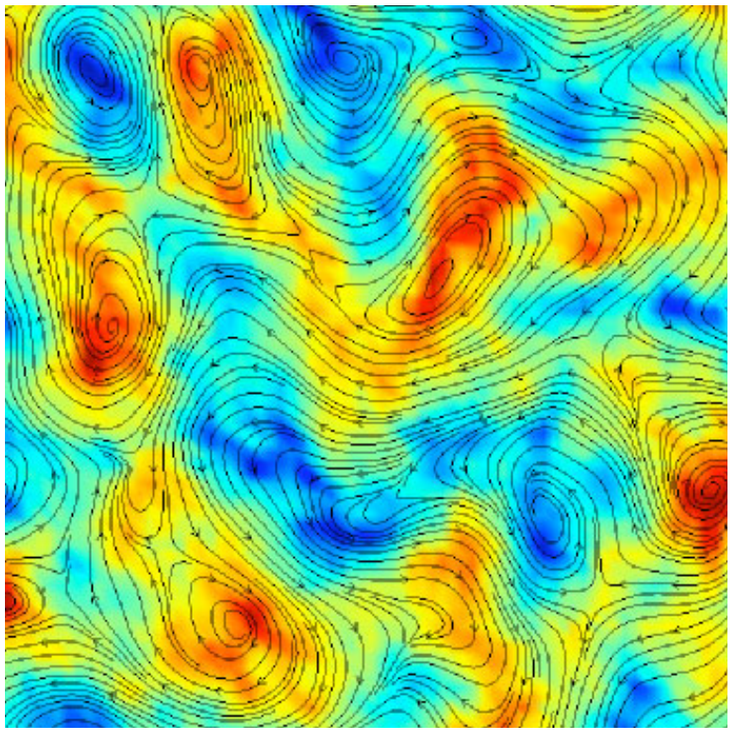
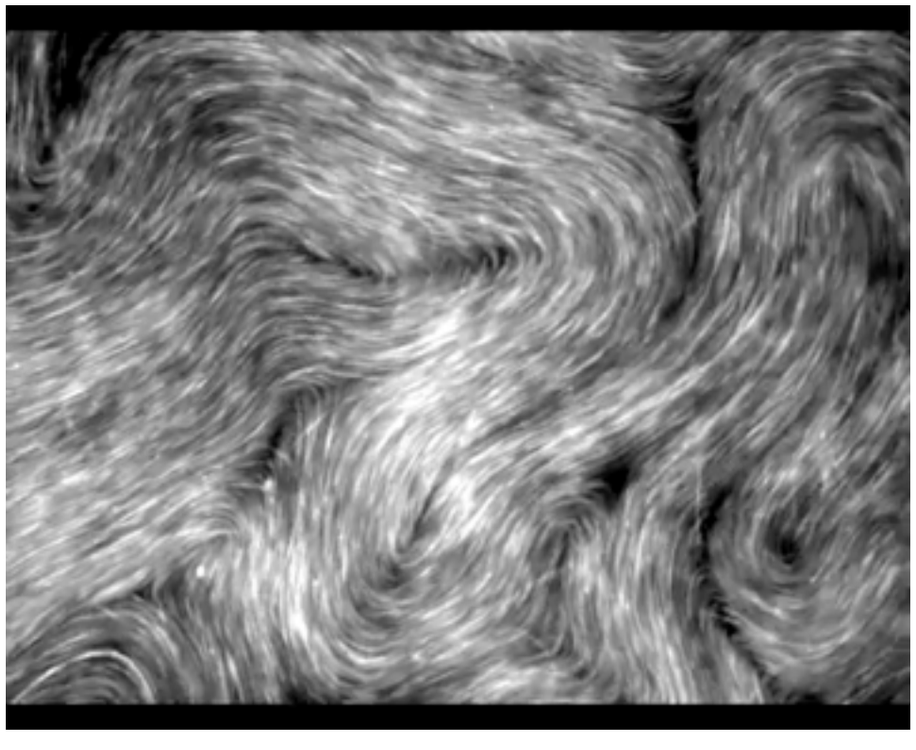
dunkel@math.mit.edu

Pattern formation in soft and biological matter

Jörn Dunkel







Dogic Lab, Brandeis
Nature 2012

Thanks to

Norbert Stoop (MIT)
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Sebastian Heidenreich (PTB Berlin)
Markus Bär (PTB Berlin)

Rik Wensink (CNRS Paris)
Hartmut Löwen (Düsseldorf)







erc

ALFRED P. SLOAN FOUNDATION 1934

Soft surface patterns

Fingerprints



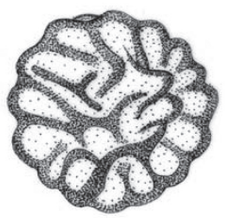
Brain corals

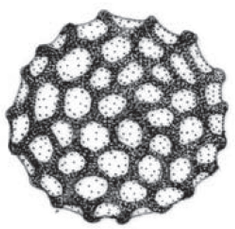




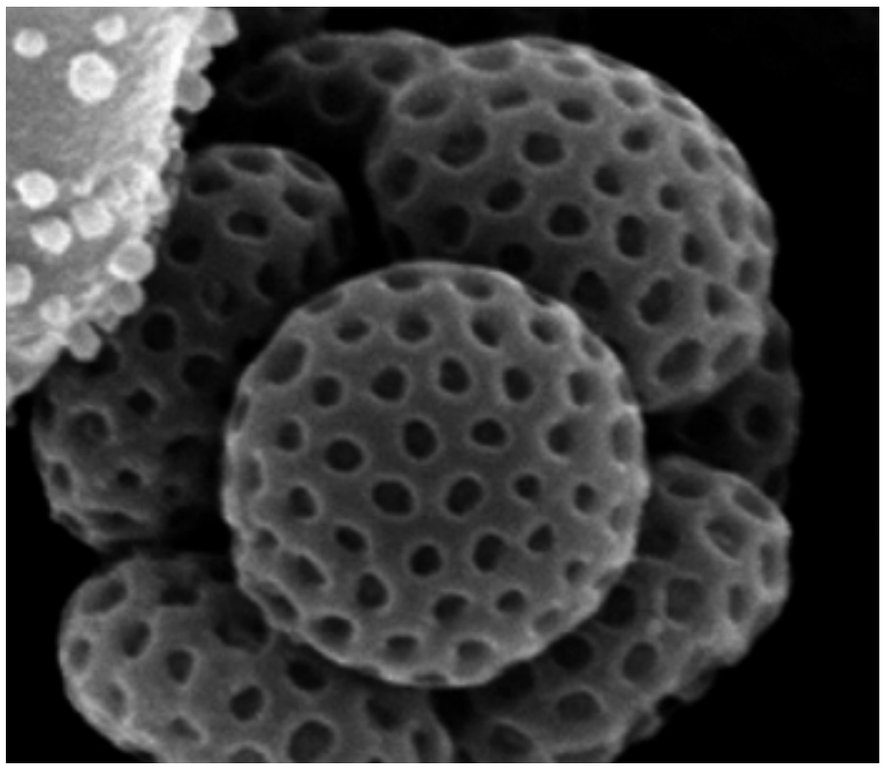
Fungal sporulation





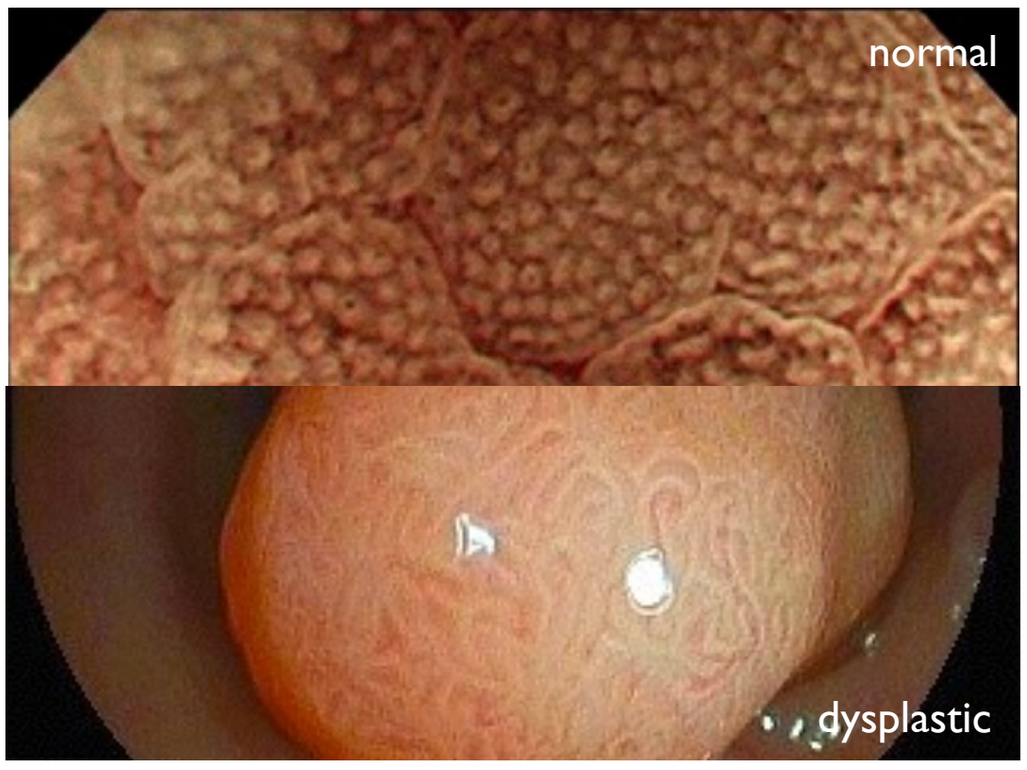


Aphanoascus



Courtesy of Nico Dam
Netherlands' Mycological Society

Colon cancer



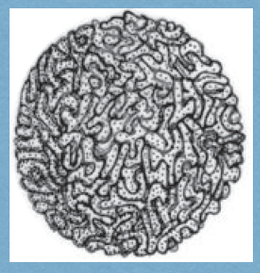
Courtesy of Bas Oldenburg
University Medical Center Utrecht

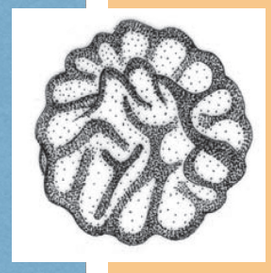
Soft surface patterns

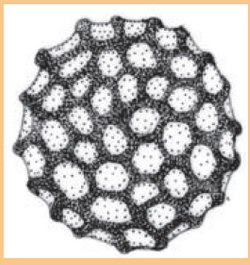
Labyrinthine







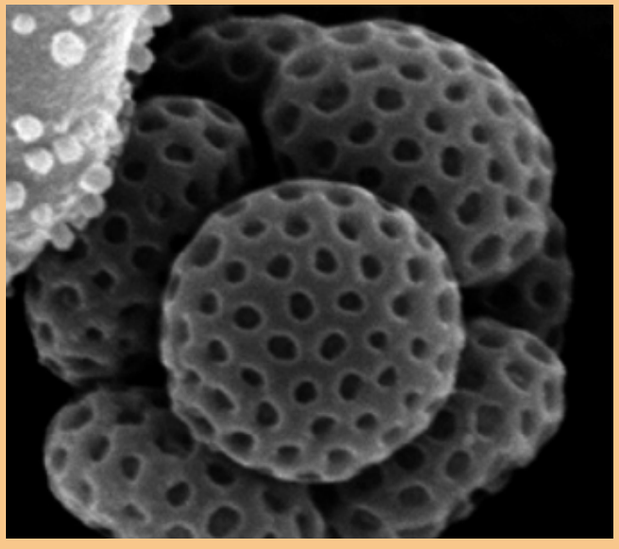


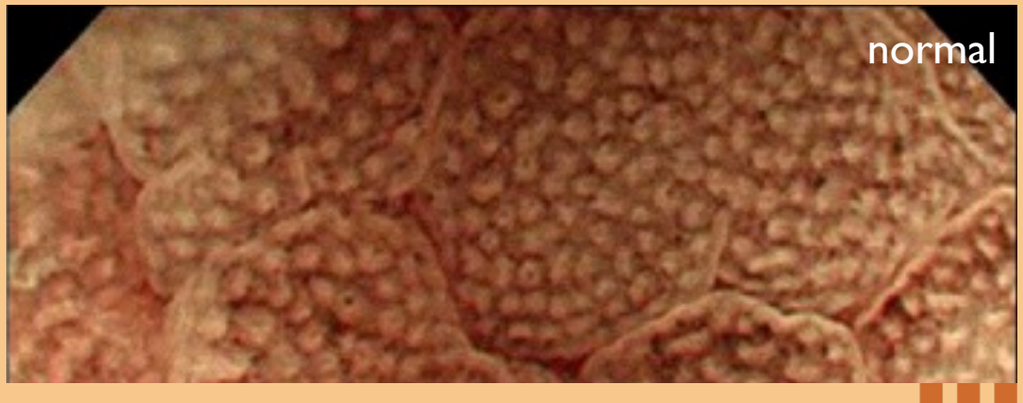




Crystalline

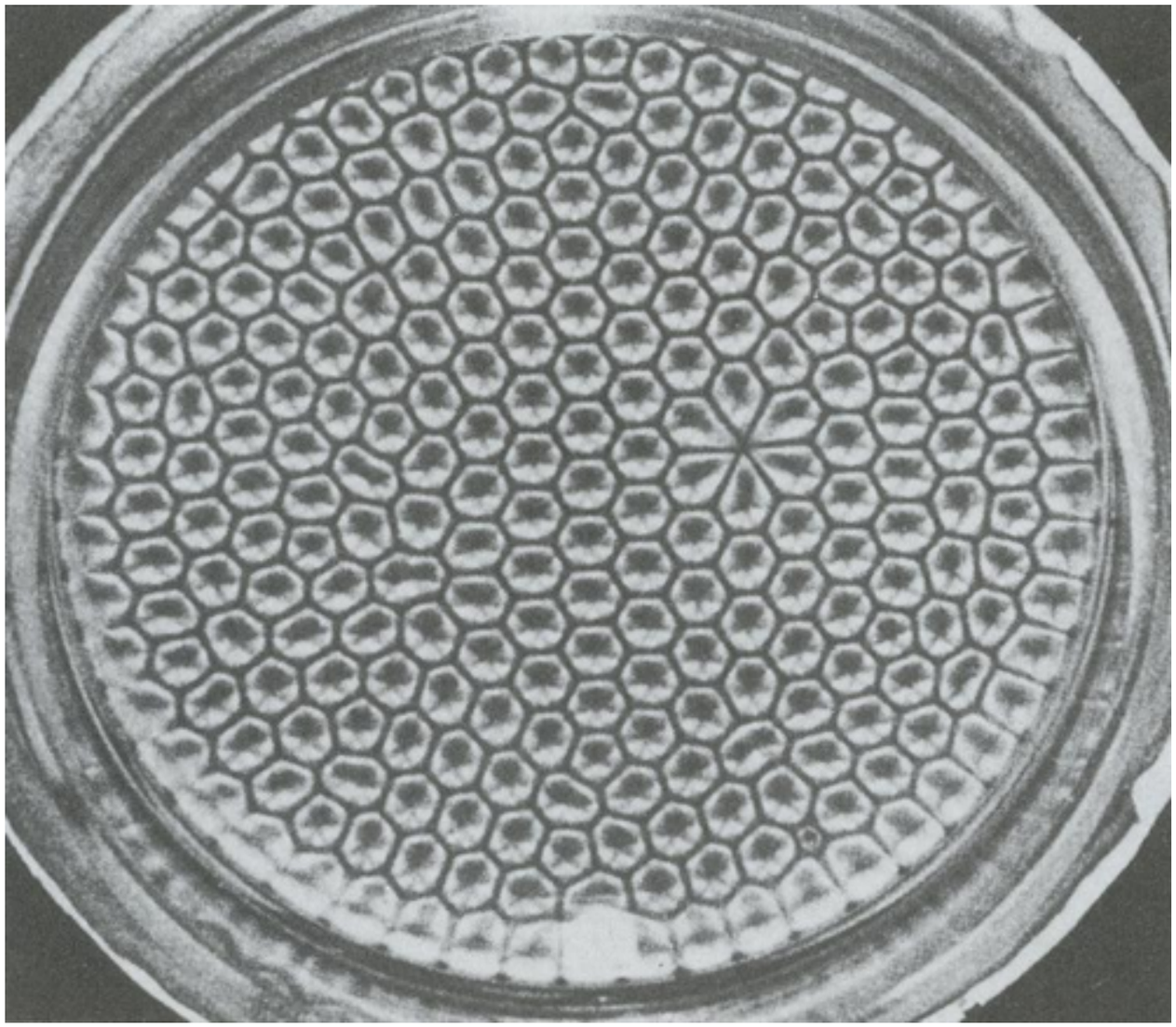






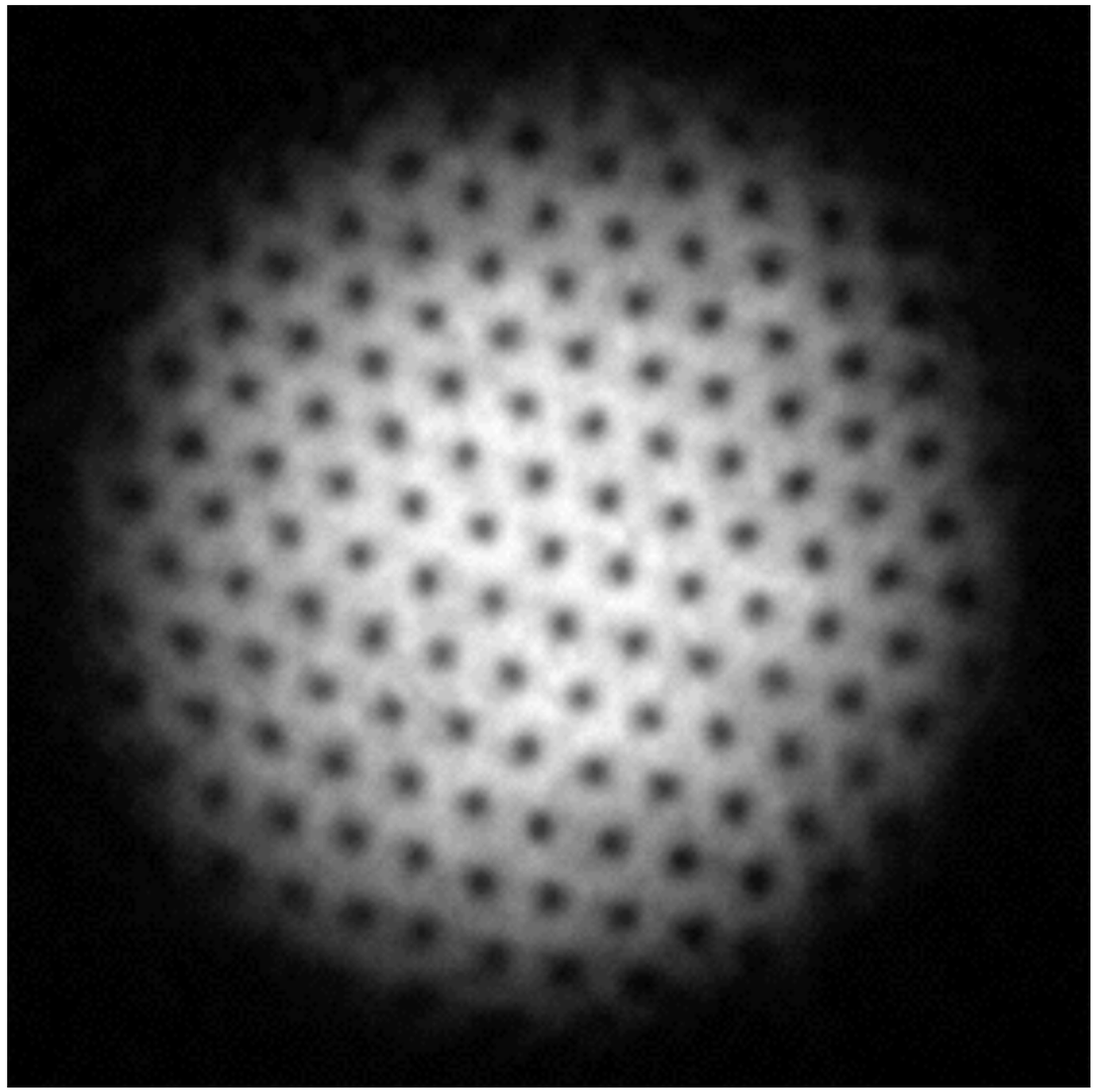
Fluid patterns

Rayleigh-Benard convection



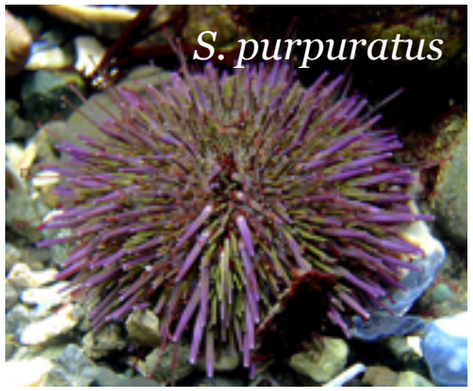
Van Dyke (1982) *An Album of Fluid Motion*

BEC/Fermi condensates

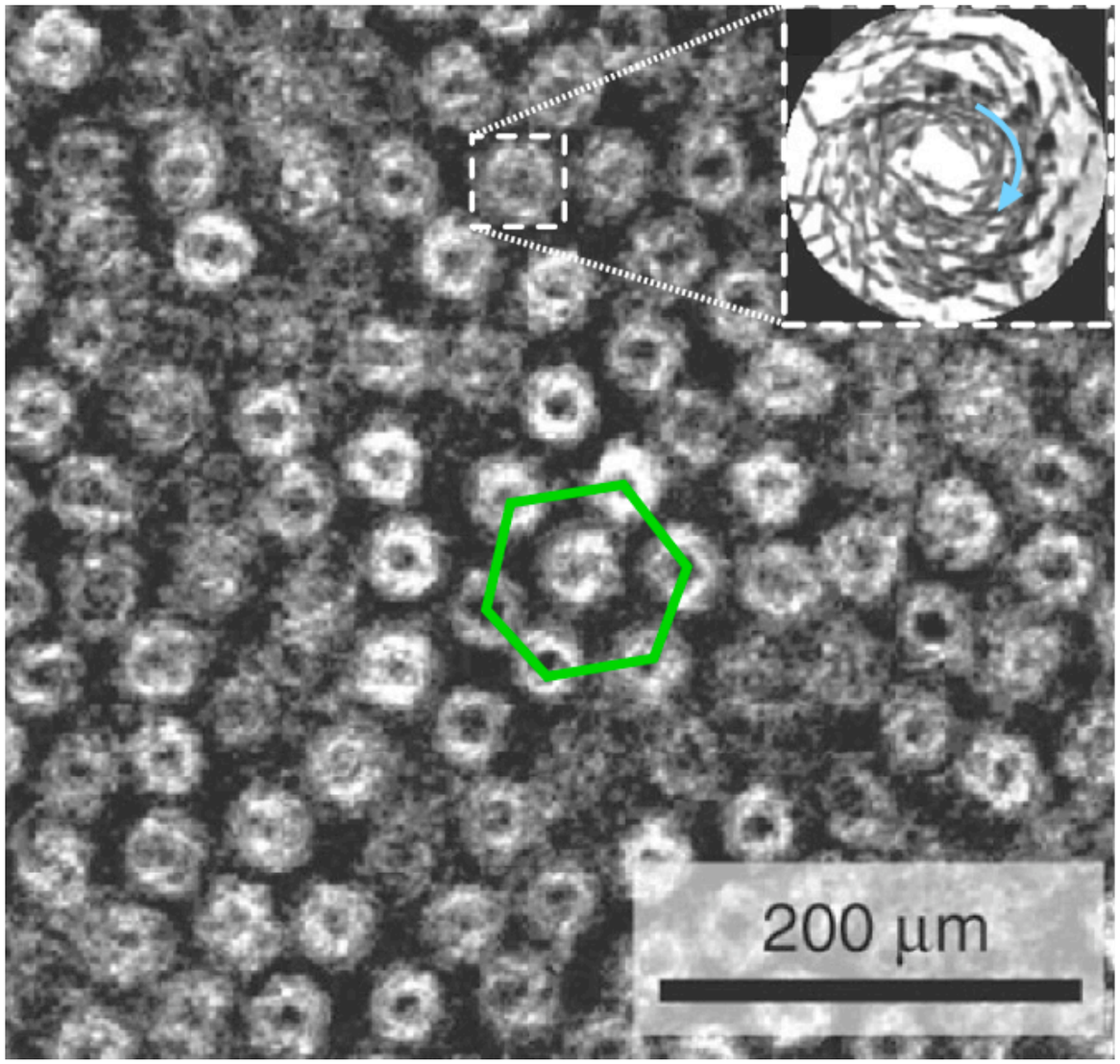


Zwierlein et al (2005) Nature

Active fluids

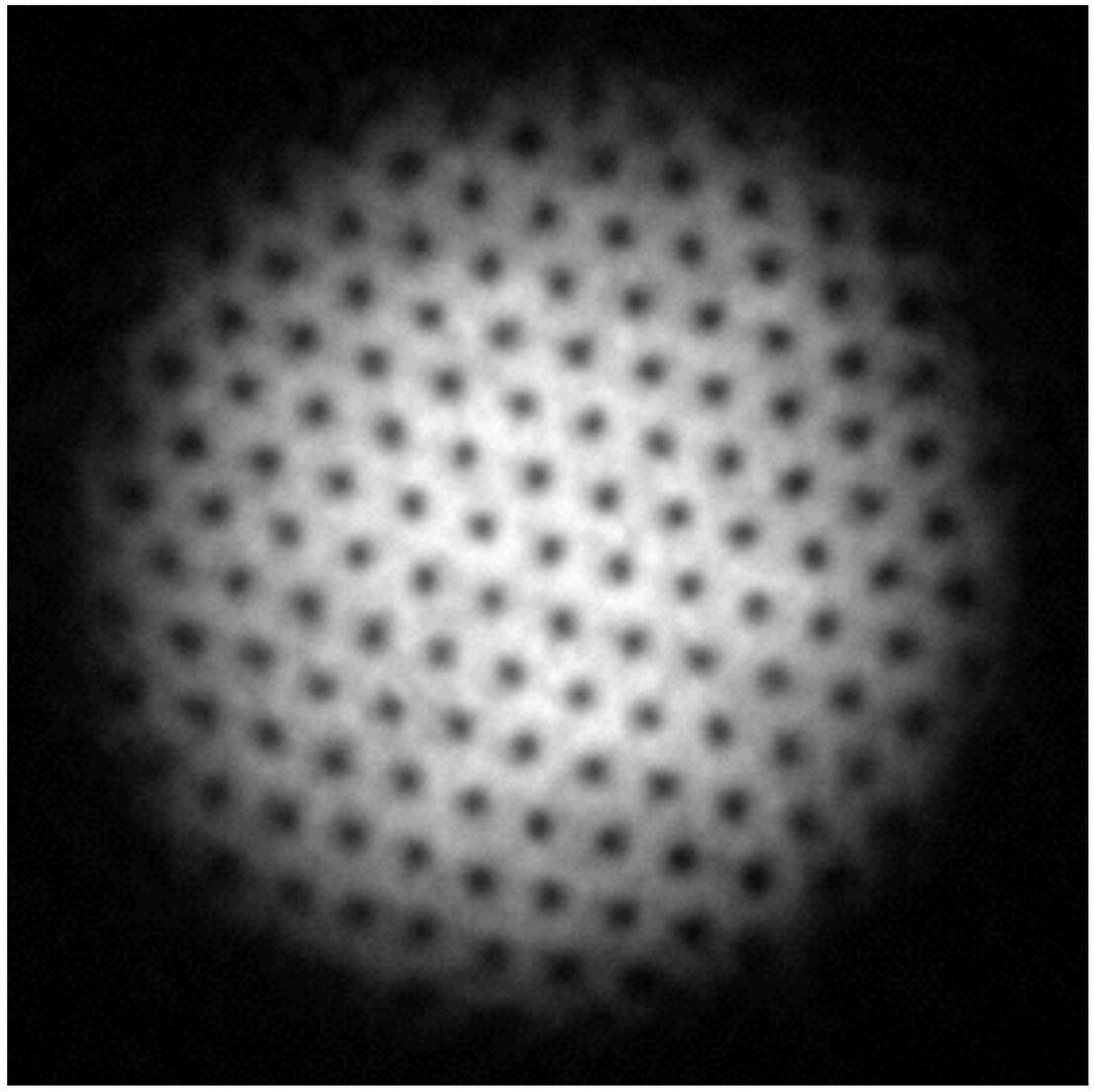


Sea urchin sperm



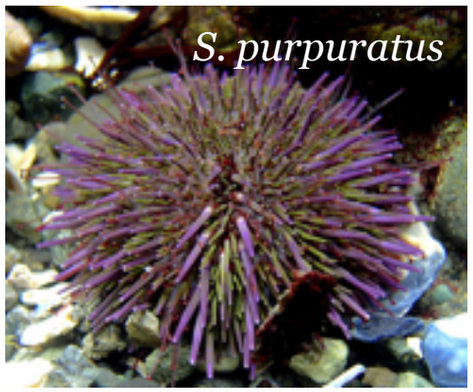
Riedel et al (2005) Science

BEC/Fermi condensates

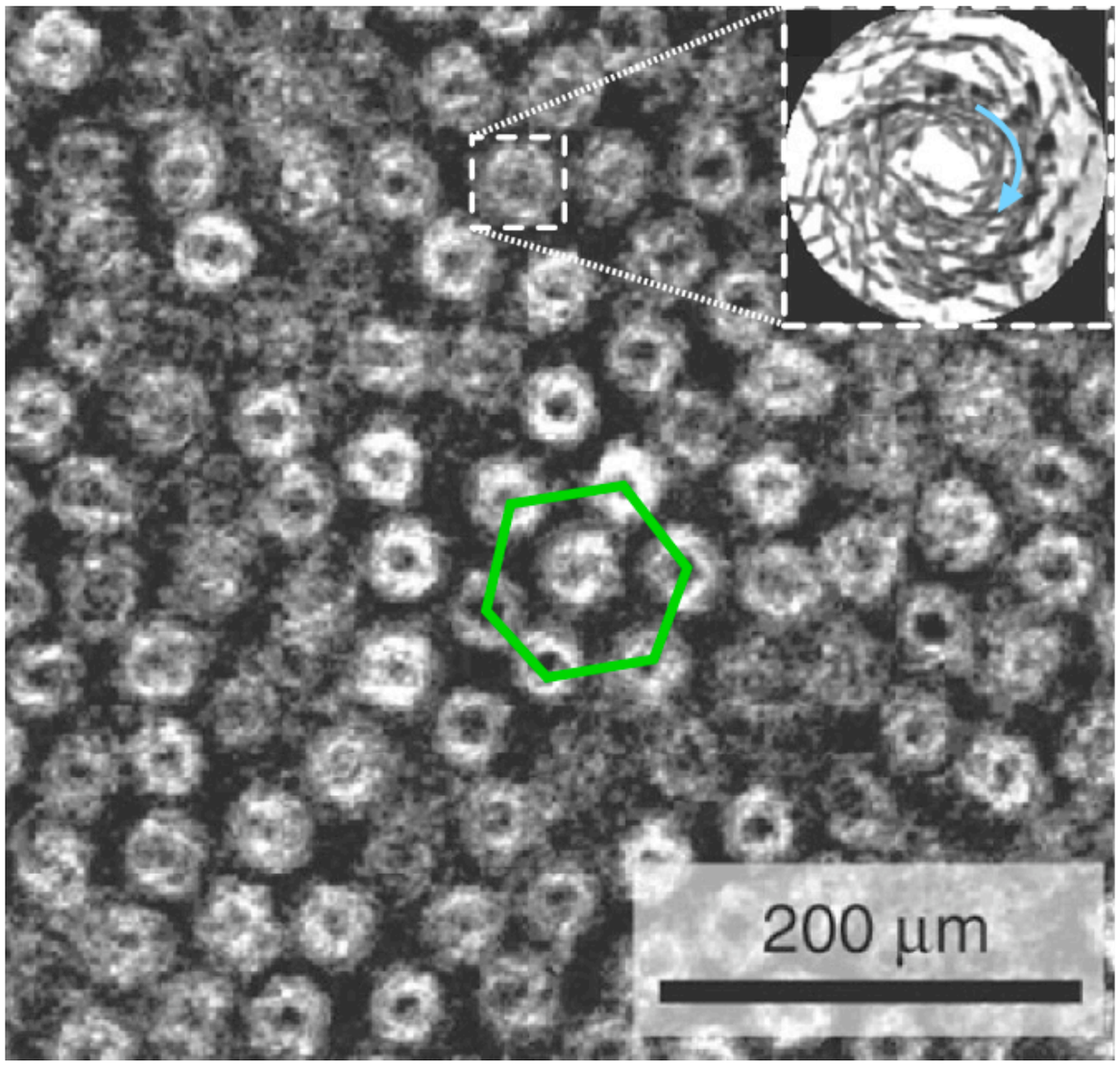


Zwierlein et al (2005) Nature

Active fluids

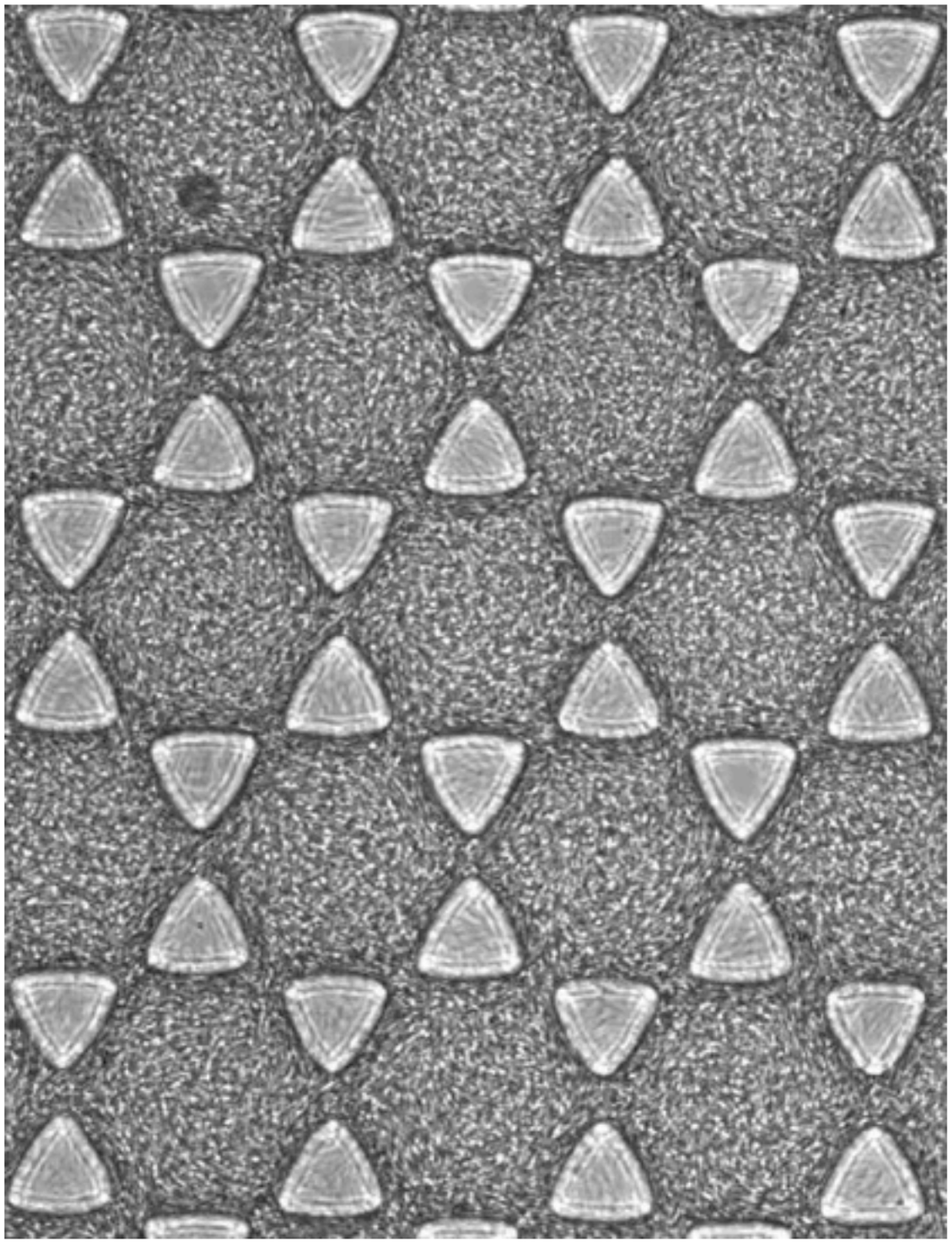


Sea urchin sperm



Riedel et al (2005) Science

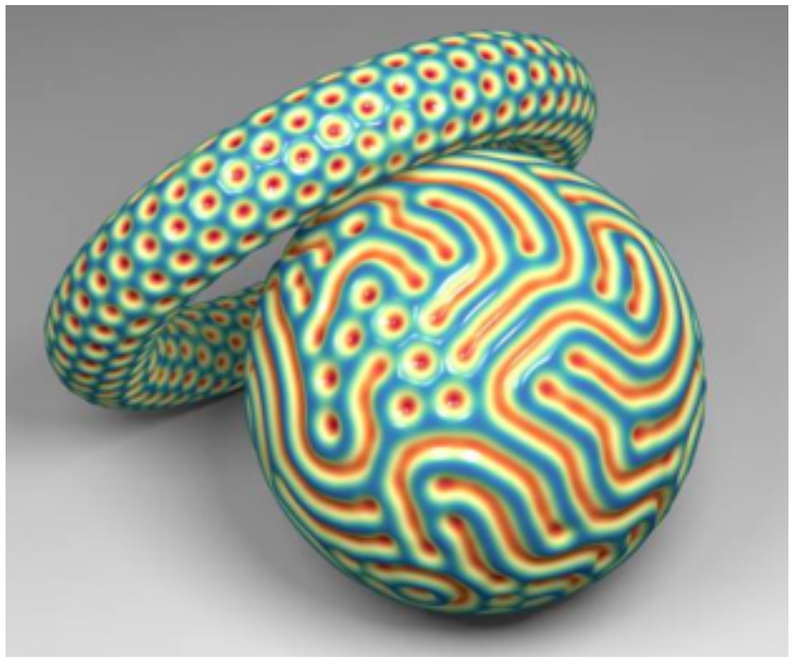
Bacterial suspension



Nature Physics (2016)

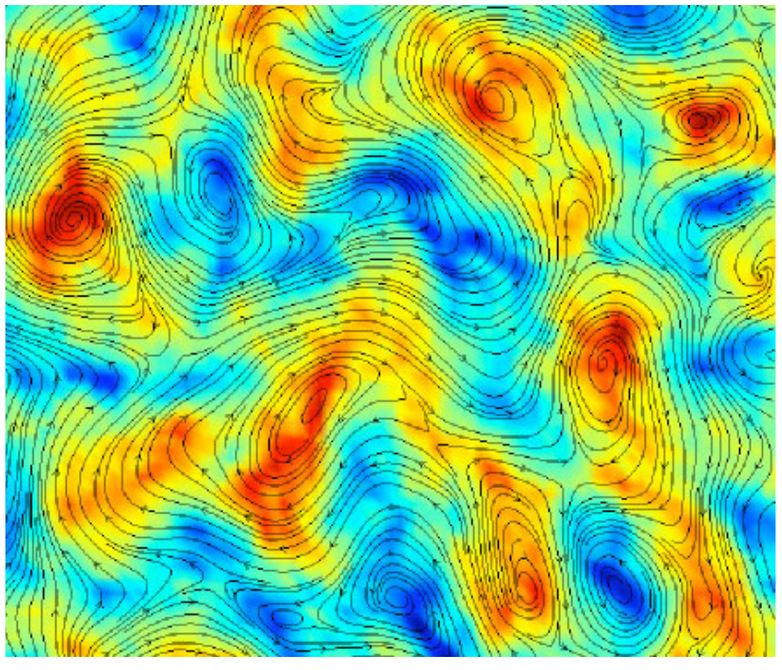
Can we describe complex patterns through generic (minimal) PDE models ?

scalar fields



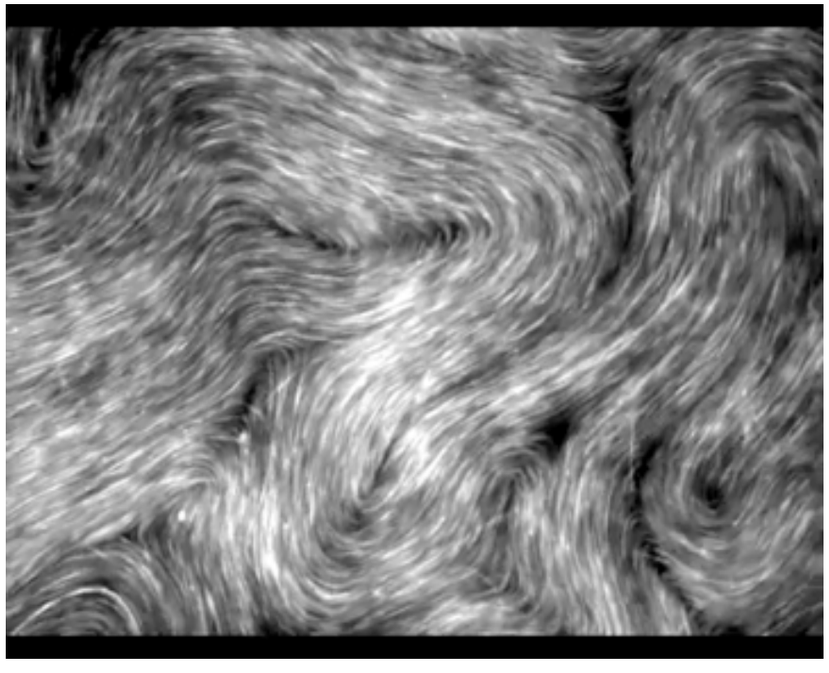
curved elastic materials

vector fields



bacterial suspensions

matrix fields



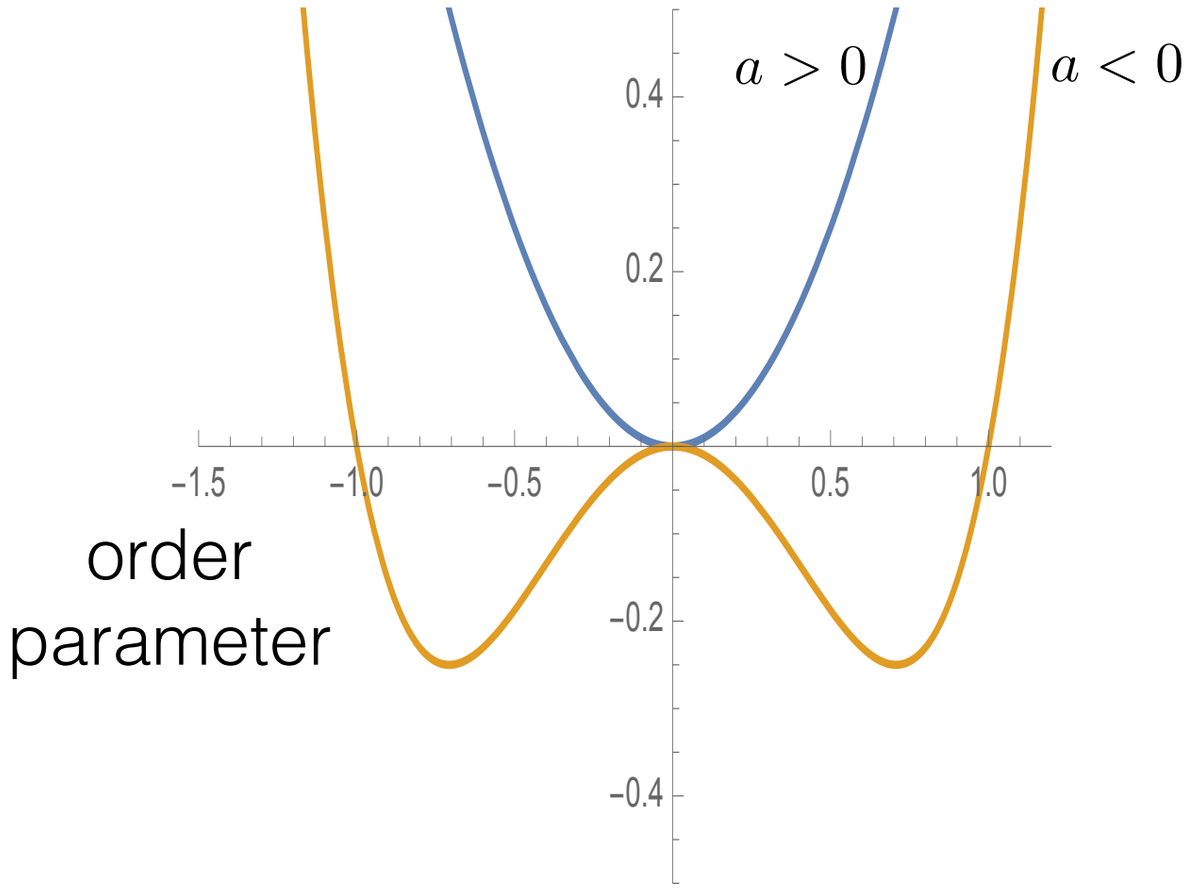
ATP driven active nematics (Dogic lab)

Basic ‘universal’ recipe



Order-parameter space

potential

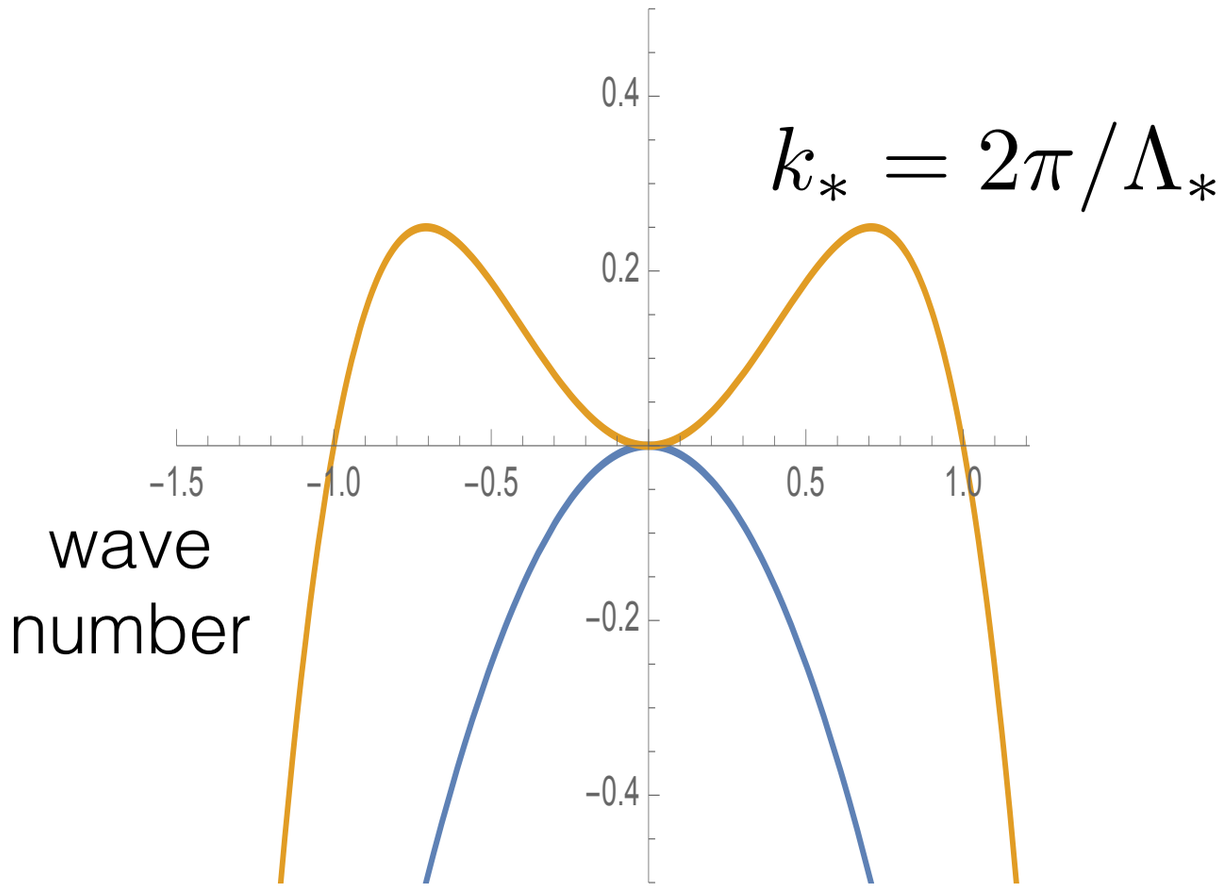


order parameter

$$a\phi^2 + b\phi^4$$

Fourier space

dispersion



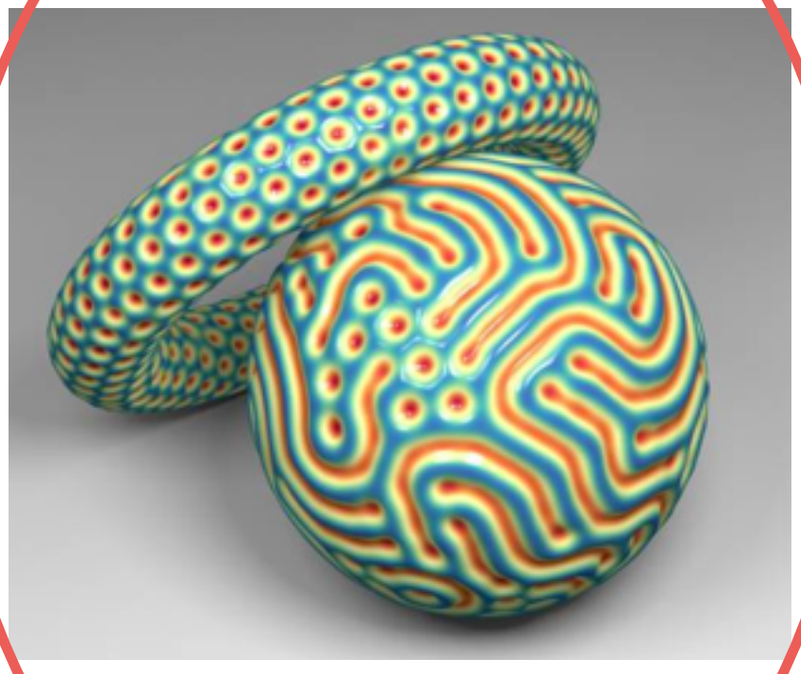
wave number

$$\alpha k^2 + \beta k^4$$

$$-\alpha\nabla^2 + \beta(\nabla^2)^2$$

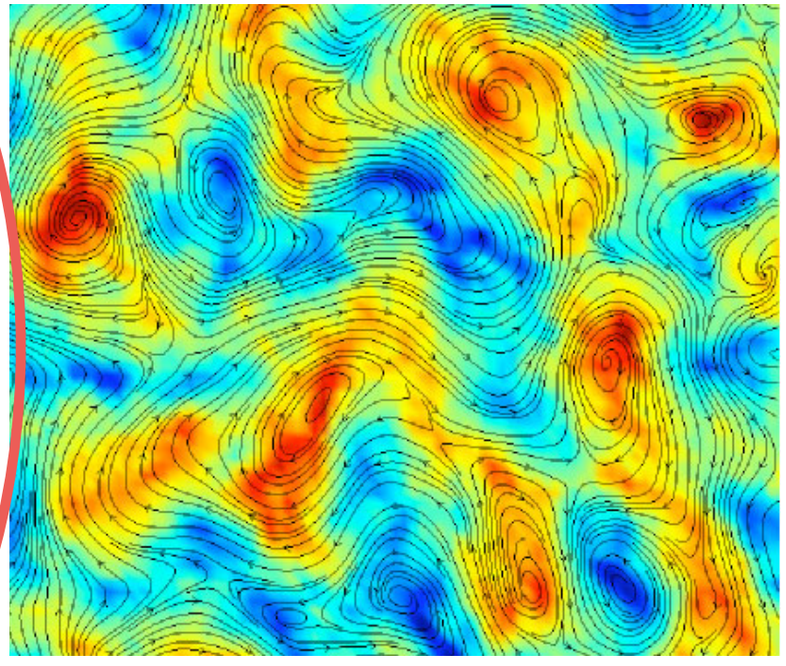
Can we describe complex patterns through generic (minimal) PDE models ?

scalar fields



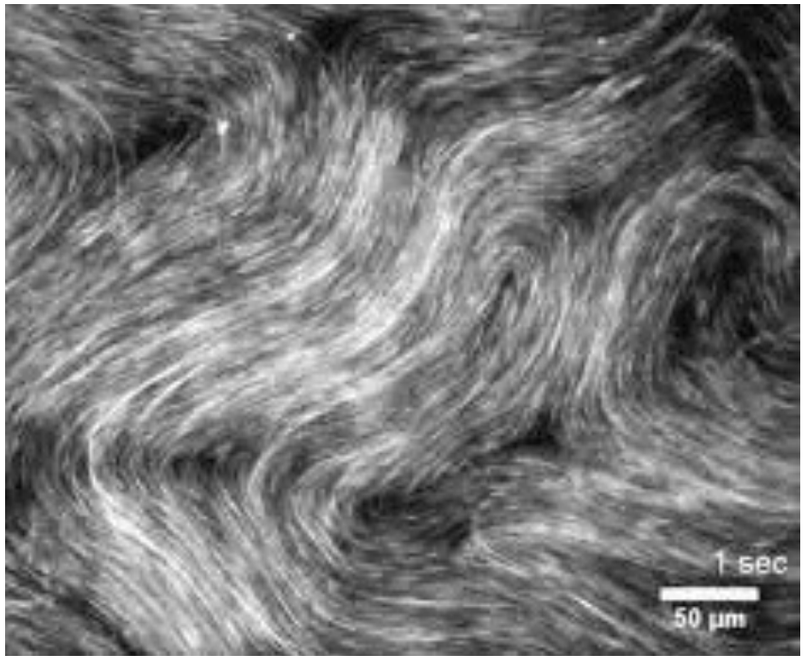
curved elastic materials

vector fields



bacterial suspensions

matrix fields



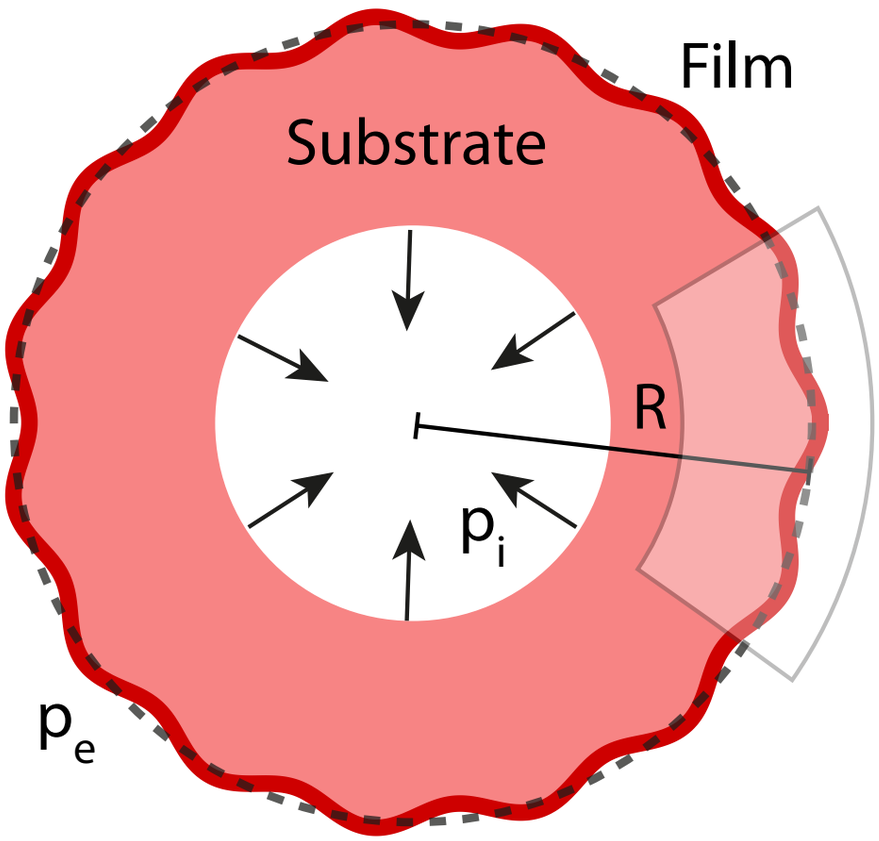
ATP driven active nematics (Dogic lab)

Experiment



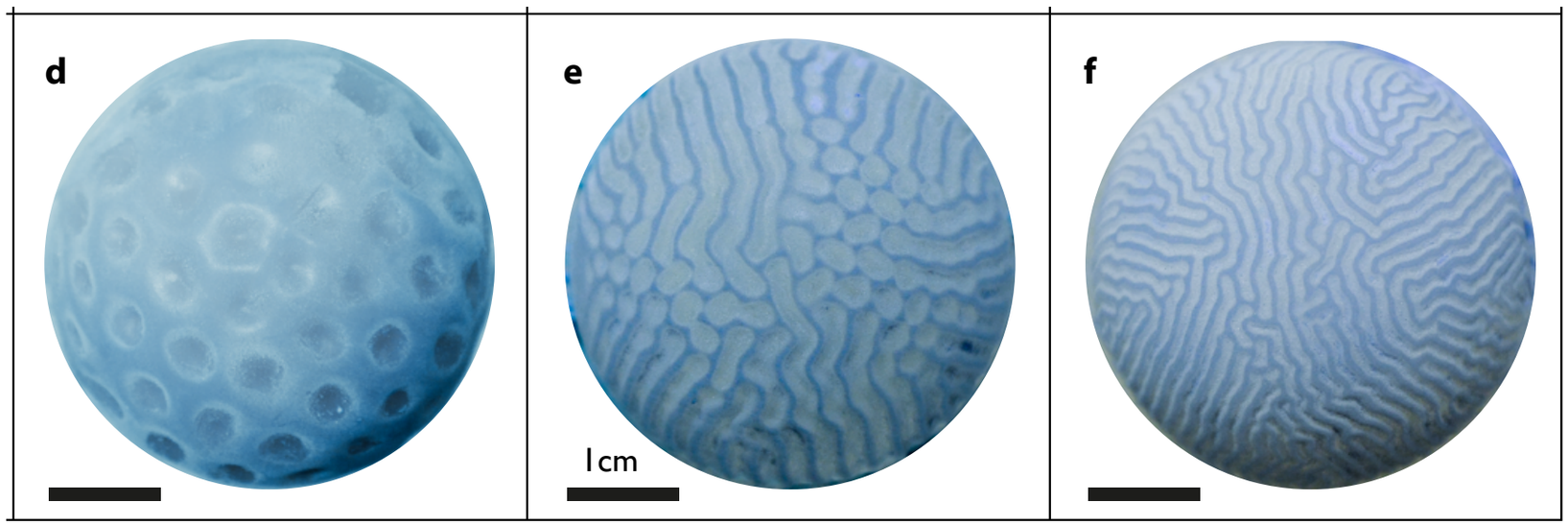


Denis Terwagne & Pedro Reis

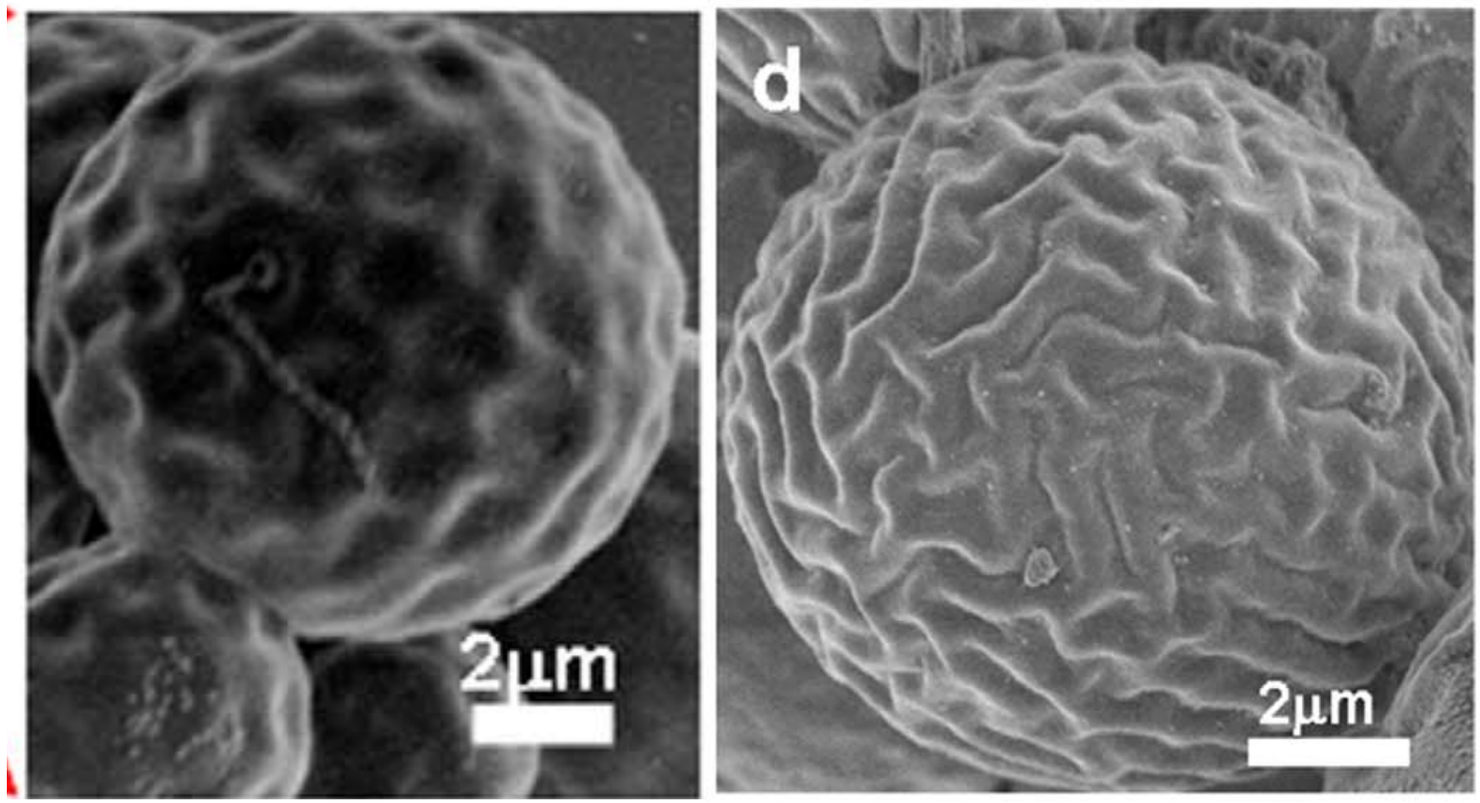




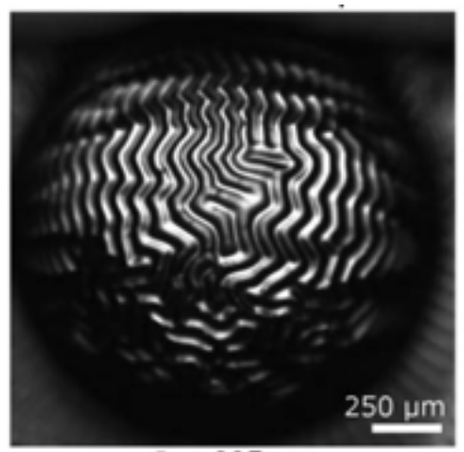
Curvature-induced wrinkling transitions



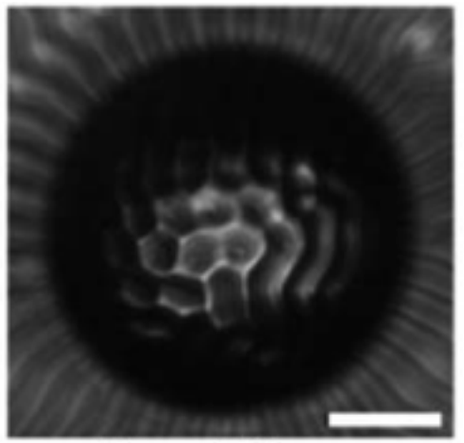
Yin et al (2014) Sci Rep



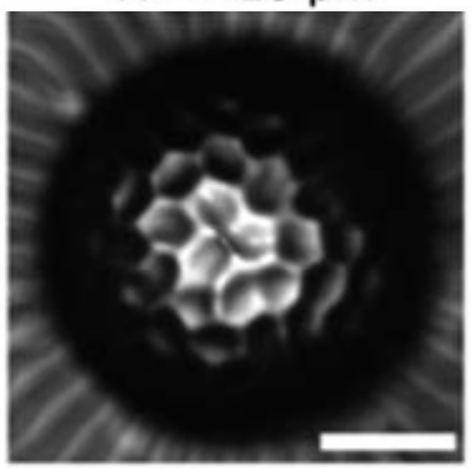
Breid & Crosby (2013) Soft Matter



R = 805 µm



R = 428 µm



R=381 µm

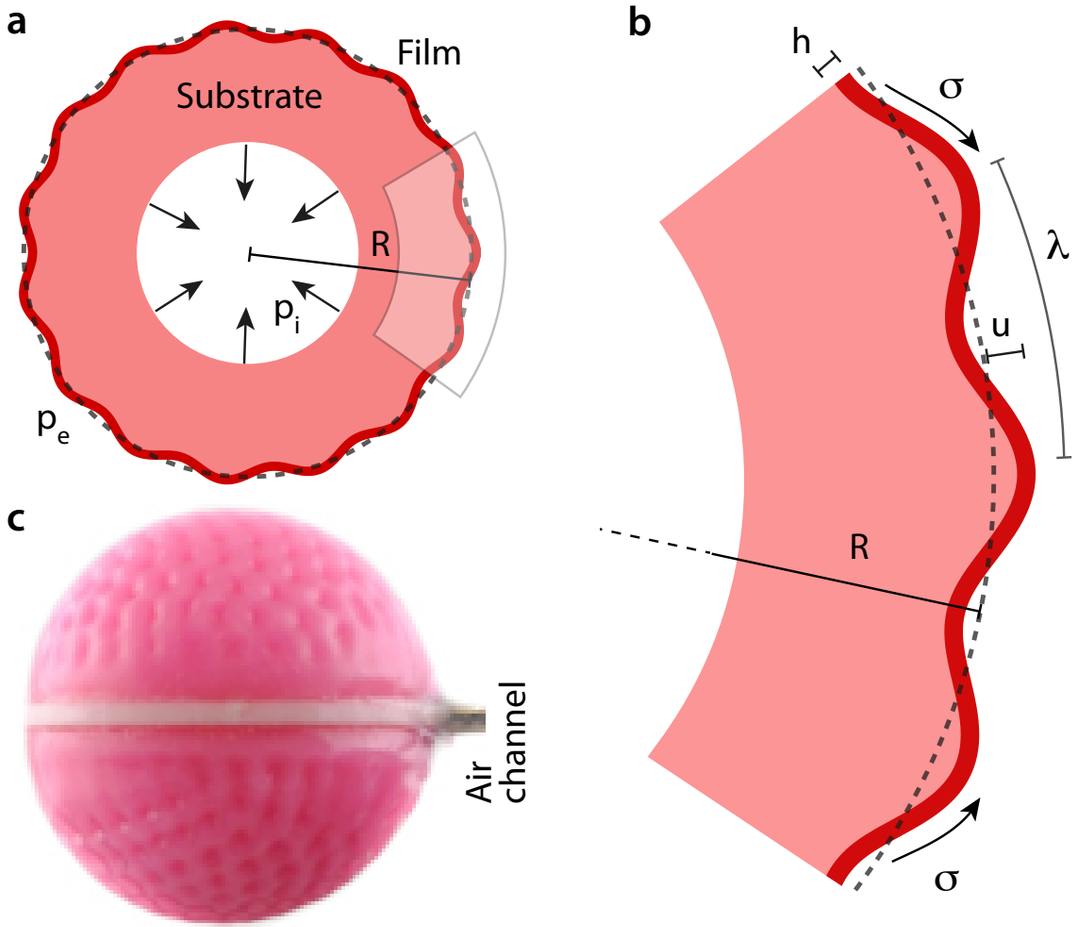
5 orders of magnitude in length … similar patterns & transitions

Generalized Swift-Hohenberg theory



Norbert Stoop

$$\partial_t u = \gamma_0 \Delta u - \gamma_2 \Delta^2 u - au - bu^2 - cu^3$$
$$+ \Gamma_1 \left[(\nabla u)^2 + 2u\Delta u \right] + \Gamma_2 \left[u(\nabla u)^2 + u^2 \Delta u \right]$$



Small deformations of a **sphere**

$$(a_{\alpha\beta}) = \begin{pmatrix} (R\sin\theta_2)^2 & 0 \\ 0 & R^2 \end{pmatrix}$$

$$\triangle\psi = \nabla_\alpha \nabla^\alpha \psi = a^{\gamma\delta}\psi_{,\gamma\delta} - a^{\gamma\delta}\Gamma^\lambda_{\gamma\delta}\psi_{,\lambda}$$

Nature Materials 2015 (joint work with Reis lab MIT MechE)

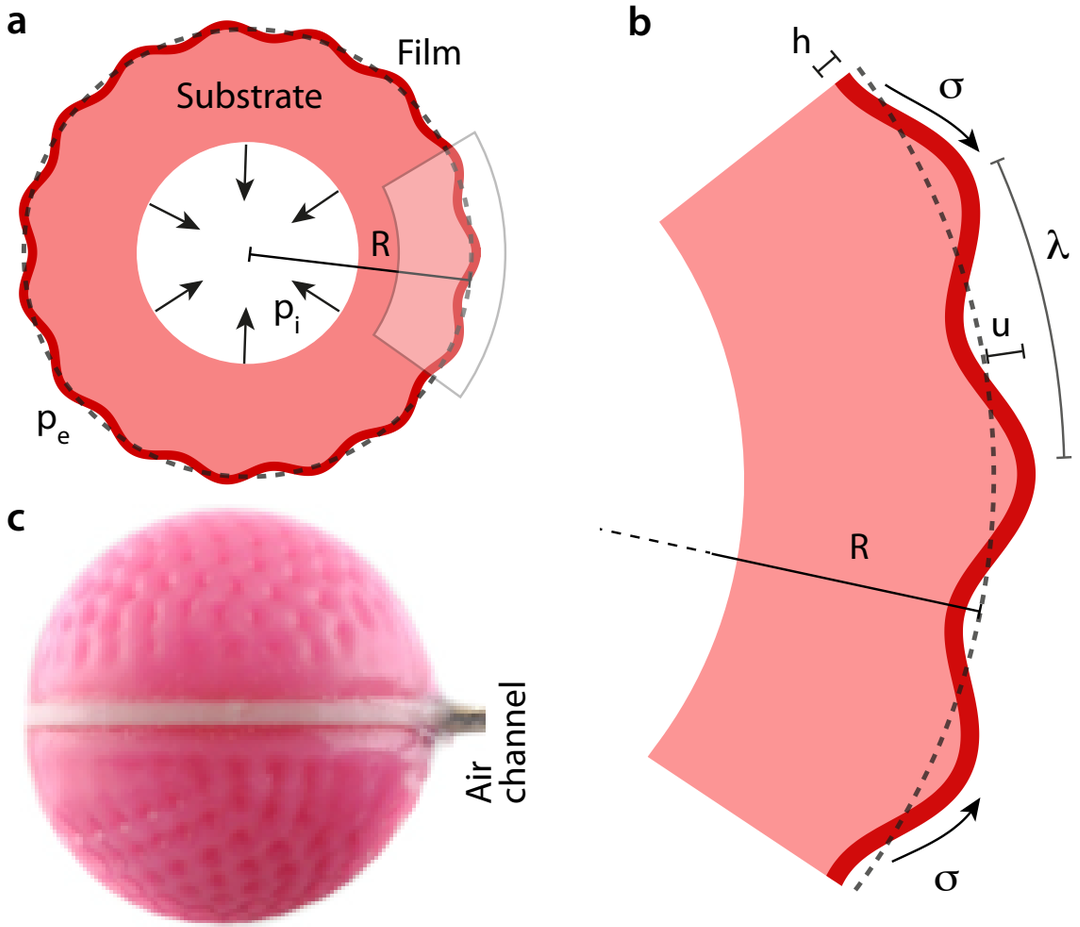
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$$+ \Gamma_1 \left[(\nabla u)^2 + 2u\Delta u \right] + \Gamma_2 \left[u(\nabla u)^2 + u^2 \Delta u \right]$$



$$\gamma_0 = \frac{\kappa^2}{3} - \frac{1}{6}\sqrt{\eta^{4/3} + 24(1+\nu)\kappa^2 + 16\kappa^4}$$

$$a = \frac{\eta^{4/3}}{12} + \frac{6(1+\nu) - \eta^{2/3}}{3}\kappa^2 + \frac{\kappa^4}{3} + \tilde{a}_2 \Sigma_e$$

$$b = 3(1+\nu)\kappa^3$$

$$c = \frac{2(1+\nu)\eta^{2/3}}{3} c_1 + (1+\nu)\kappa^4$$

$$\Gamma_1 = \frac{1+\nu}{2}\kappa$$

$$\Gamma_2 = \frac{1+\nu}{2}\kappa^2$$

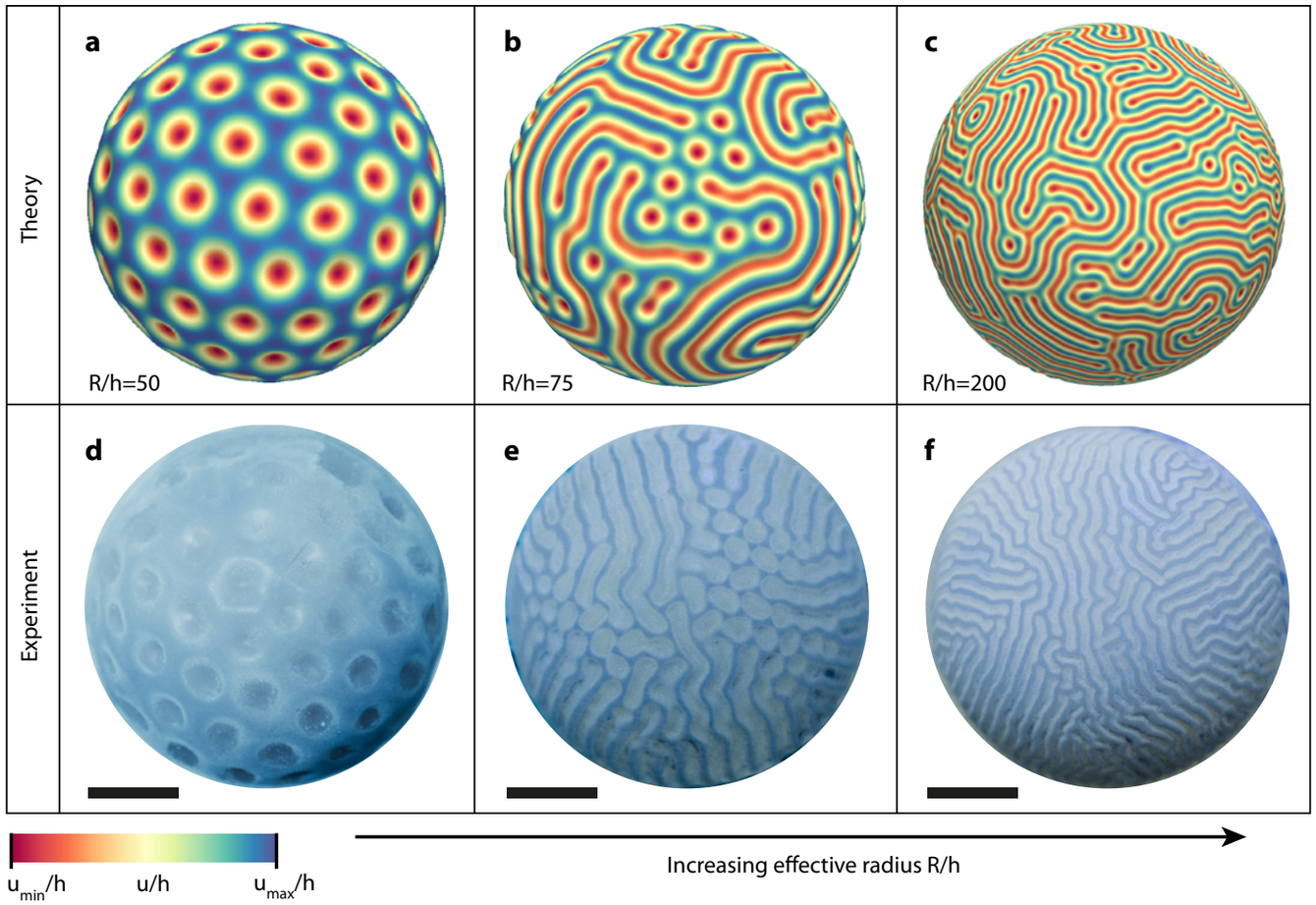
$$\tilde{a}_2 = -\frac{\eta^{4/3}(c + 3|\gamma_0|\Gamma_2)}{48\gamma_0^2}$$

TABLE I: List of parameters for Eq. (1) in units $h = 1$, with $\eta = 3E_s/E_f$, $\gamma_2 = 1/12$, $\Sigma_e = (\sigma/\sigma_c) - 1$ and $\kappa = h/R$. The only remaining fitting parameter of the model is c_1.

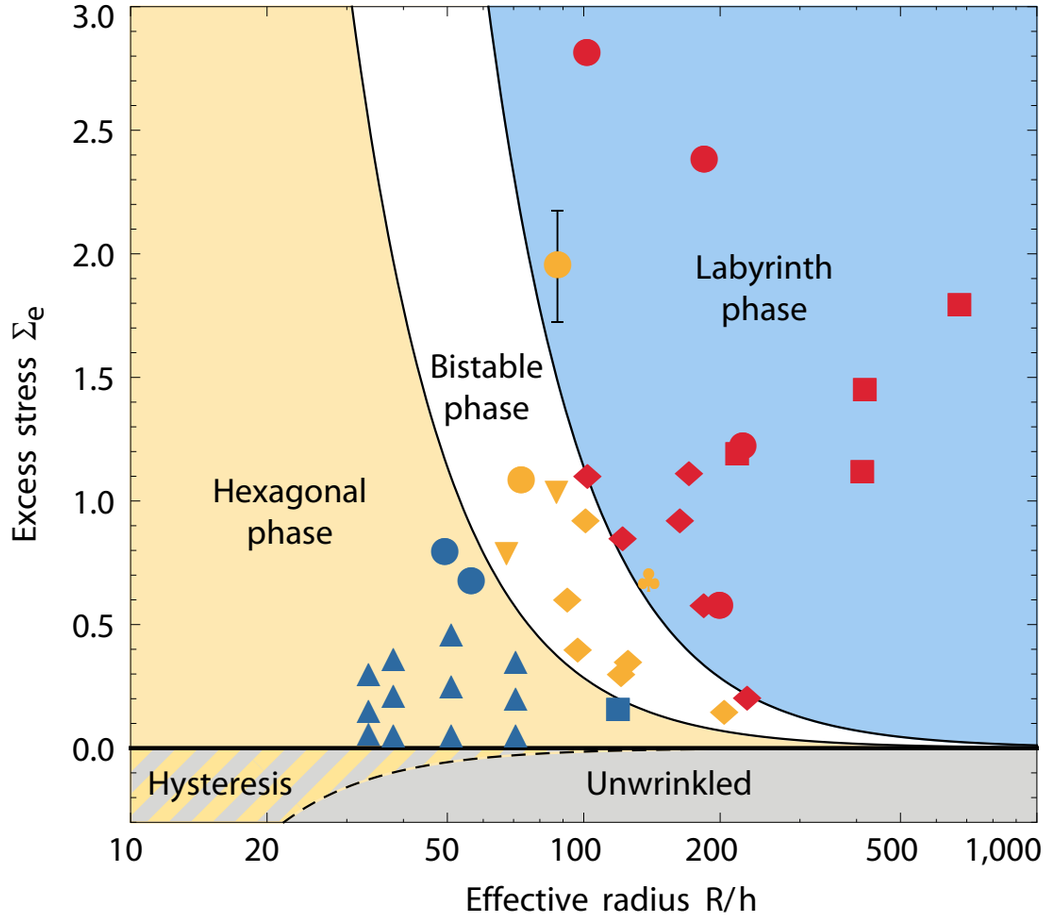
Nature Materials 2015 (joint work with Reis lab MIT MechE)

Theory correctly predicts

morphology



phase transitions



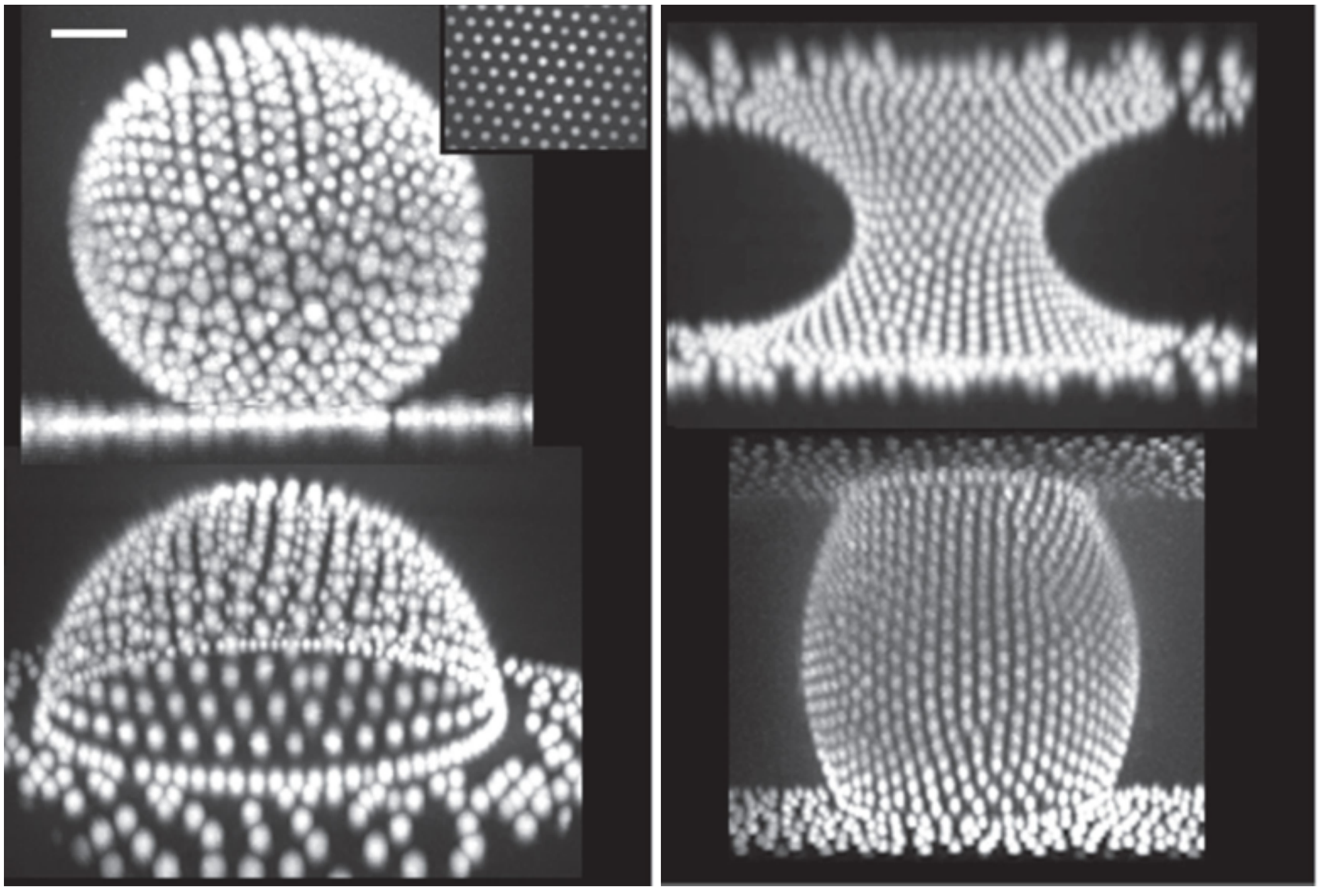
Nature Materials 2015 (joint work with Reis lab MIT MechE)

Arbitrarily curved surfaces

$$
\begin{aligned}
\mu\partial_t u = \; & \gamma_0\triangle u - \gamma_2\triangle^2 u - au - bu^2 - cu^3 + \\
& \frac{h}{2}\left\{(\nu-1)\left[b^{\alpha\beta}\nabla_\alpha u\nabla_\beta u + 2u\nabla_\beta\left(b^{\alpha\beta}\nabla_\alpha u\right)\right] + \right. \\
& \left. 2\nu\left[\mathcal{H}(\nabla u)^2 - 2\nabla\cdot(\mathcal{H}u\nabla u)\right]\right\} + \\
& \frac{h}{2}\left[(1-\nu)u\nabla_\beta\left(uc^{\alpha\beta}\nabla_\alpha u\right) - \nu\mathcal{R}u(\nabla u)^2 + \right. \\
& \left. \nu\nabla\cdot(\mathcal{R}u^2\nabla u)\right]
\end{aligned}
$$

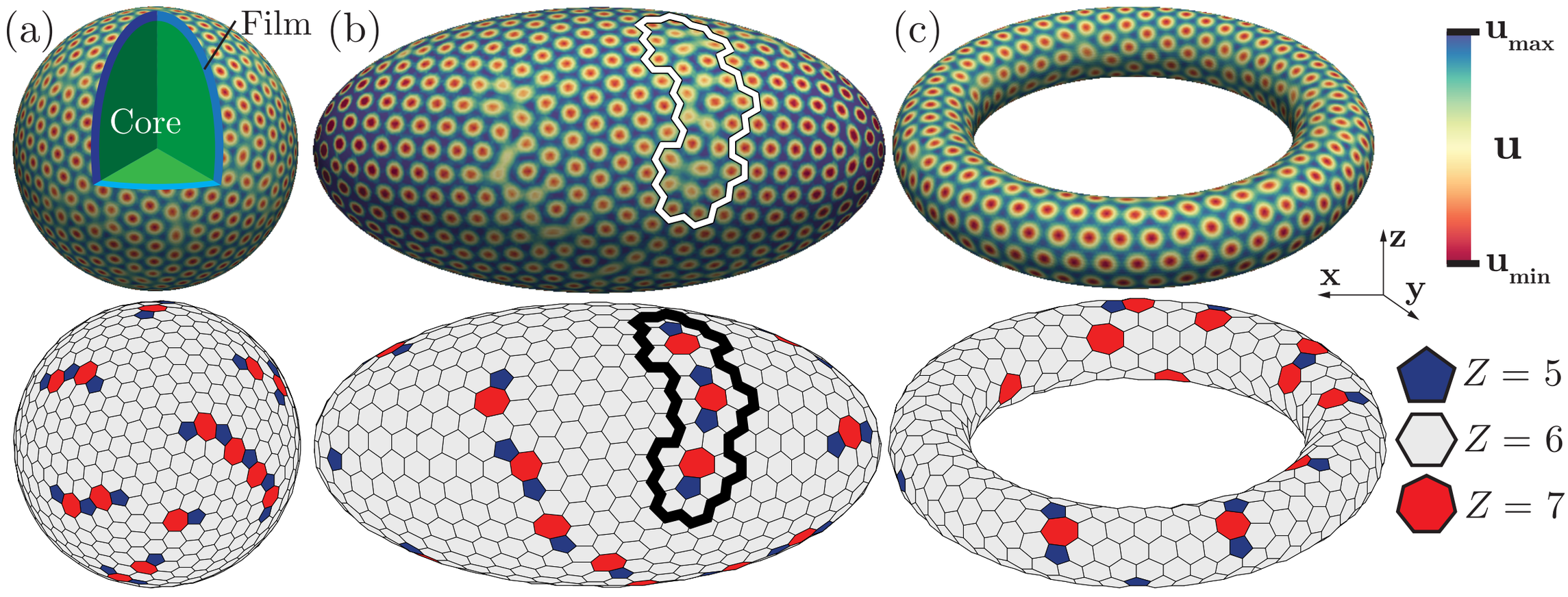


Outlook: Surface crystallography



Irvine et al (2010) Nature

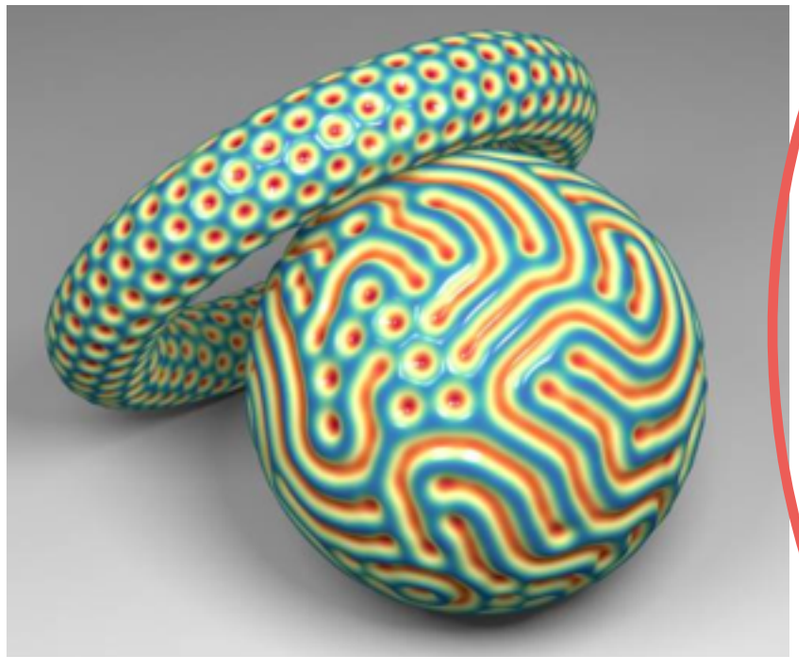
Outlook: Surface crystallography



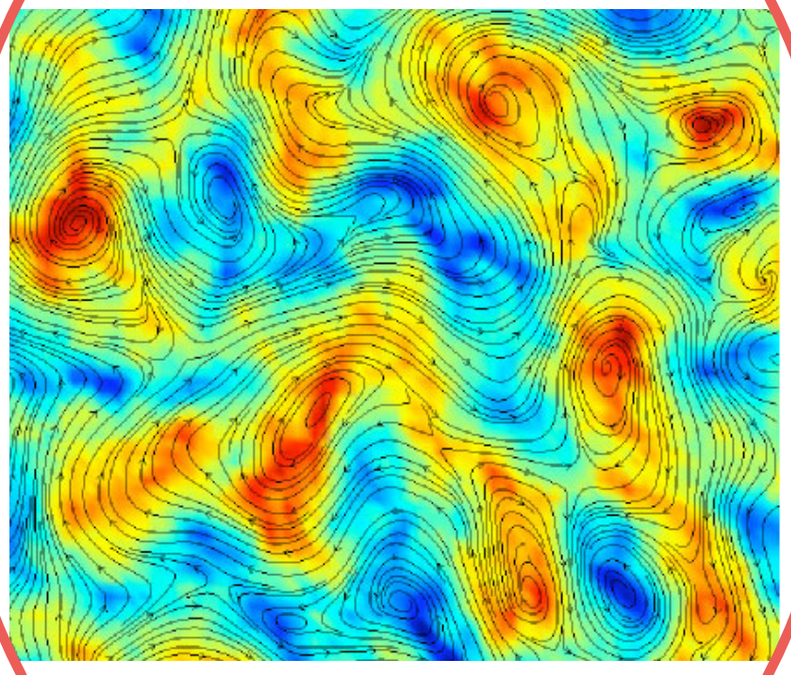
PRL 2016 (joint work with Reis lab MIT MechE)

Can we describe complex patterns through generic (minimal) PDE models ?

scalar fields



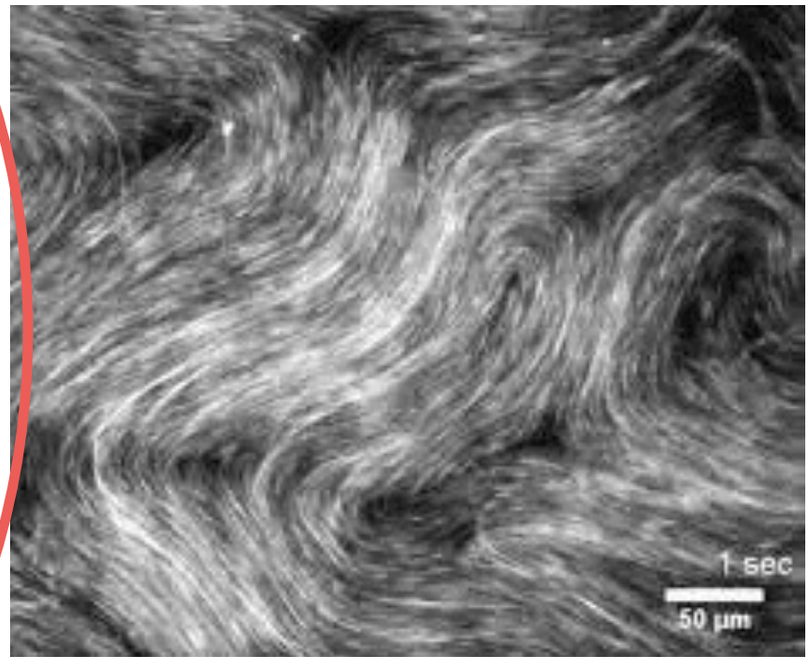
curved elastic materials



vector fields

bacterial suspensions

matrix fields

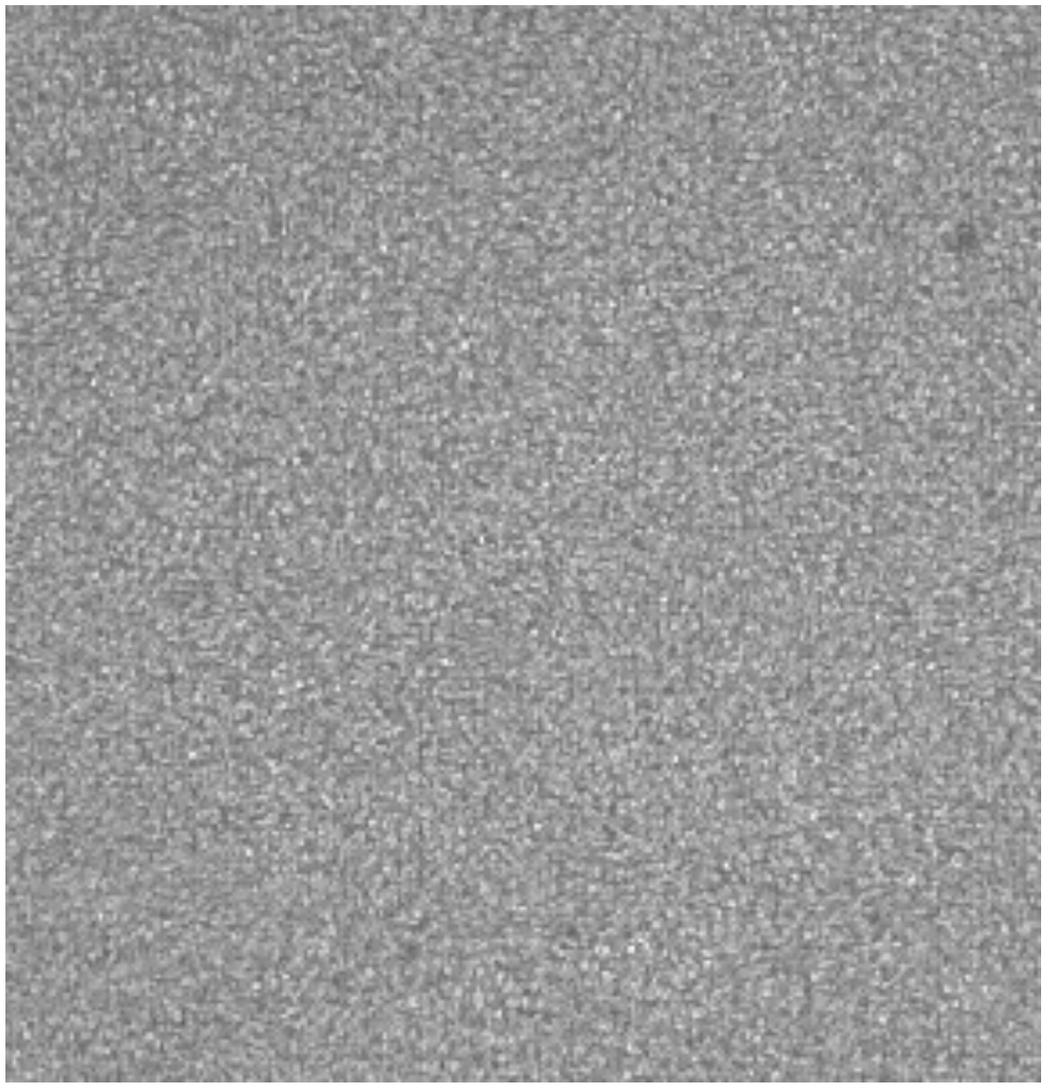


ATP driven active nematics (Dogic lab)

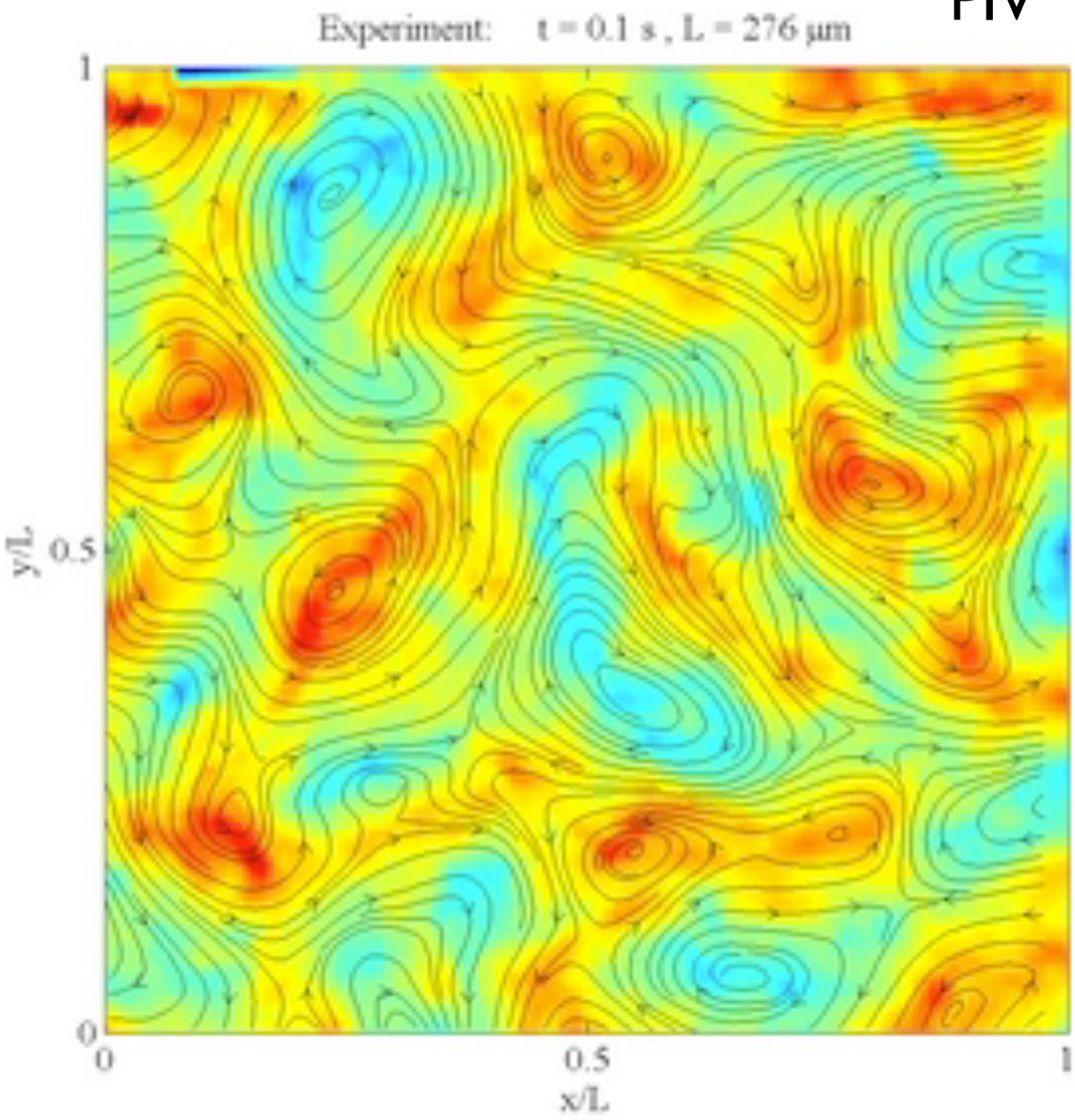
Bacterial ‘turbulence’

Bacillus subtilis

bright-field



PIV



PNAS 2012
PRL 2013

Minimal continuum theory for bacterial velocity field

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

nematic stresses

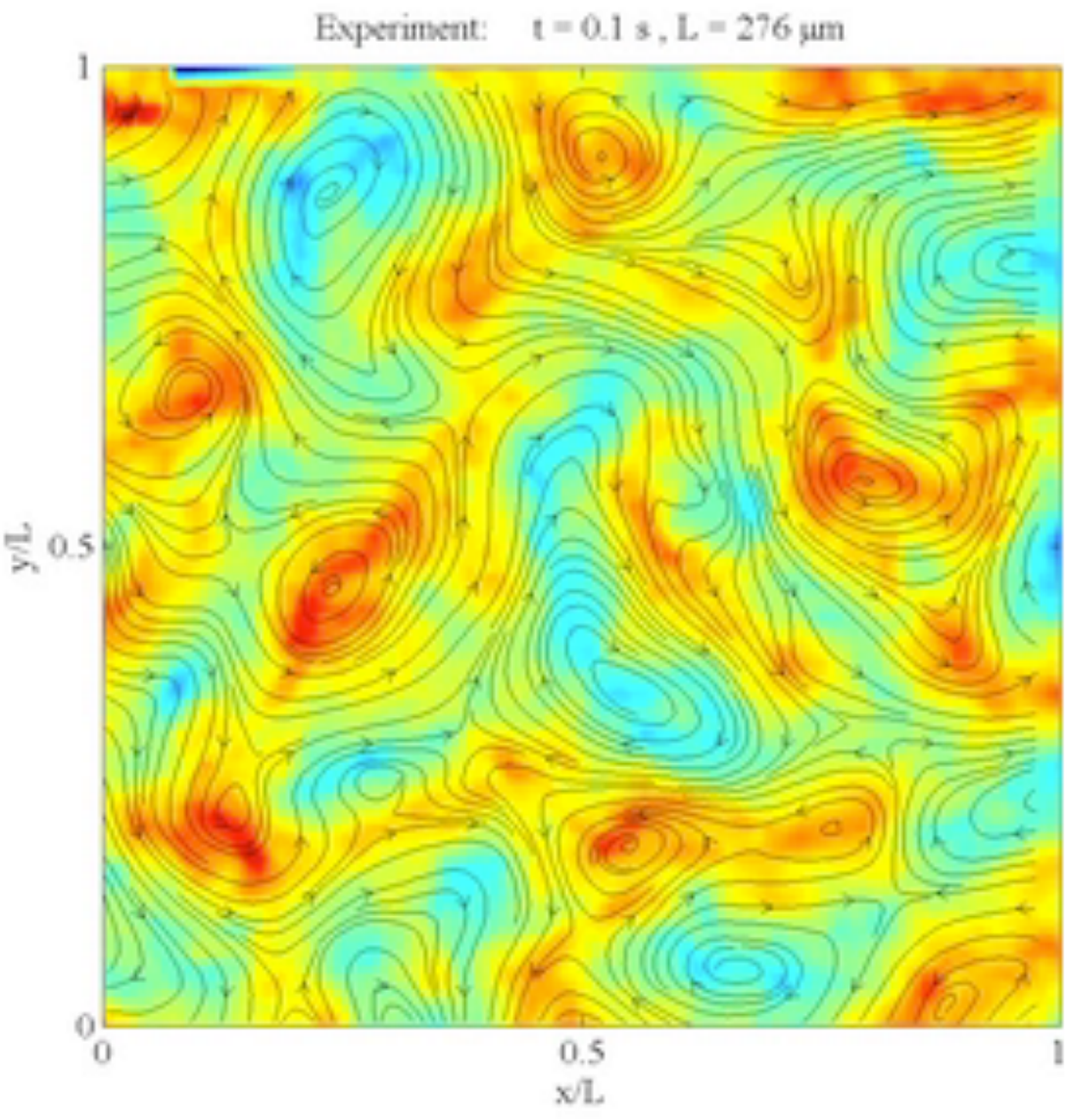
$\alpha < 0$
polar alignment

$$(\partial_t + \lambda_0 \boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla(p + \lambda_1 \boldsymbol{v}^2) - (\beta \boldsymbol{v}^2 + \alpha)\boldsymbol{v} + \\ + \Gamma_0 \nabla^2 \boldsymbol{v} - \Gamma_2 (\nabla^2)^2 \boldsymbol{v}$$

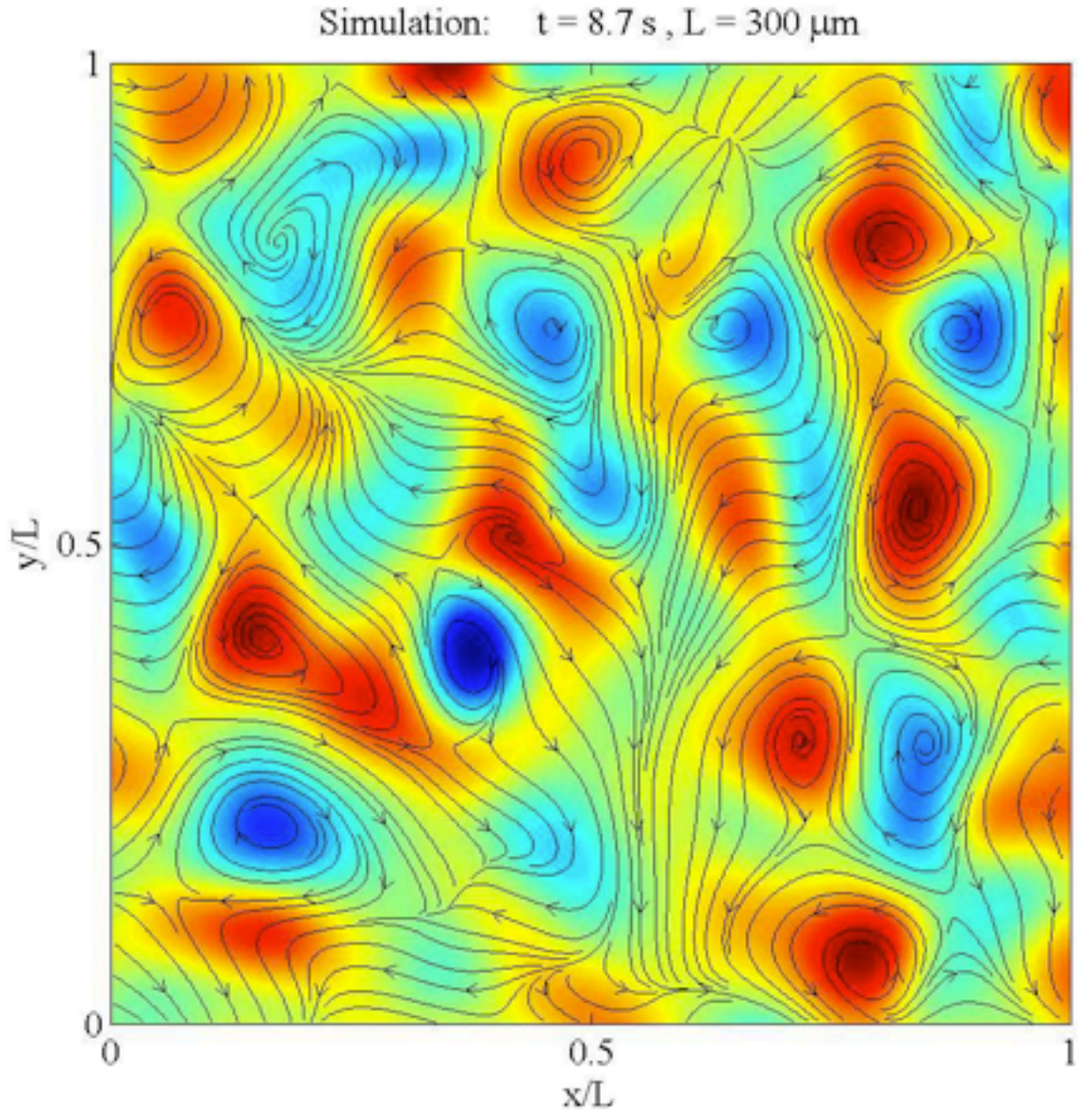
vortices

PNAS 2012
PRL 2013

experiment *vs.* theory



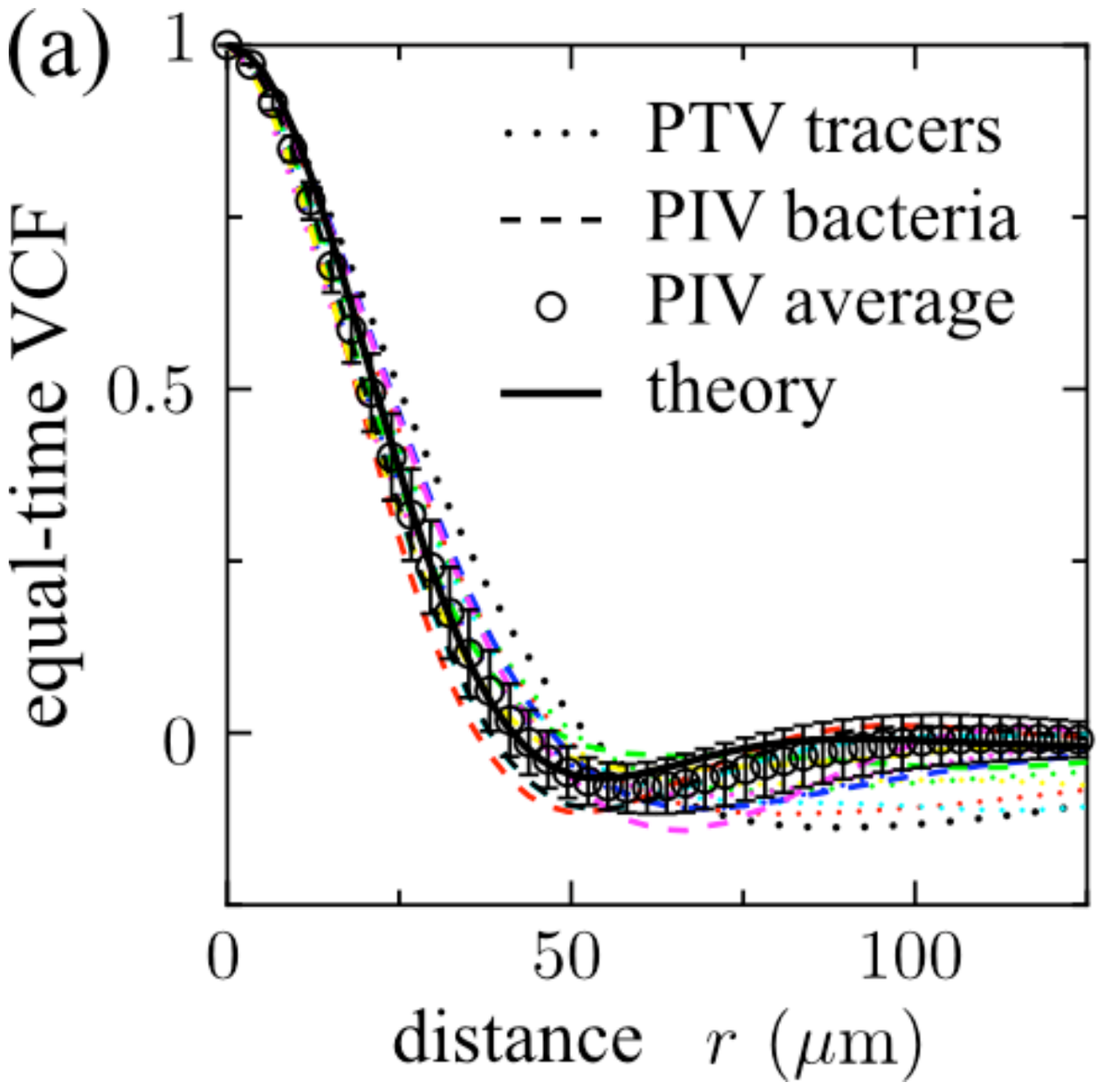
quasi-2D slice



2D slice
from 3D simulation

Dunkel et al PRL 2013

Velocity correlations



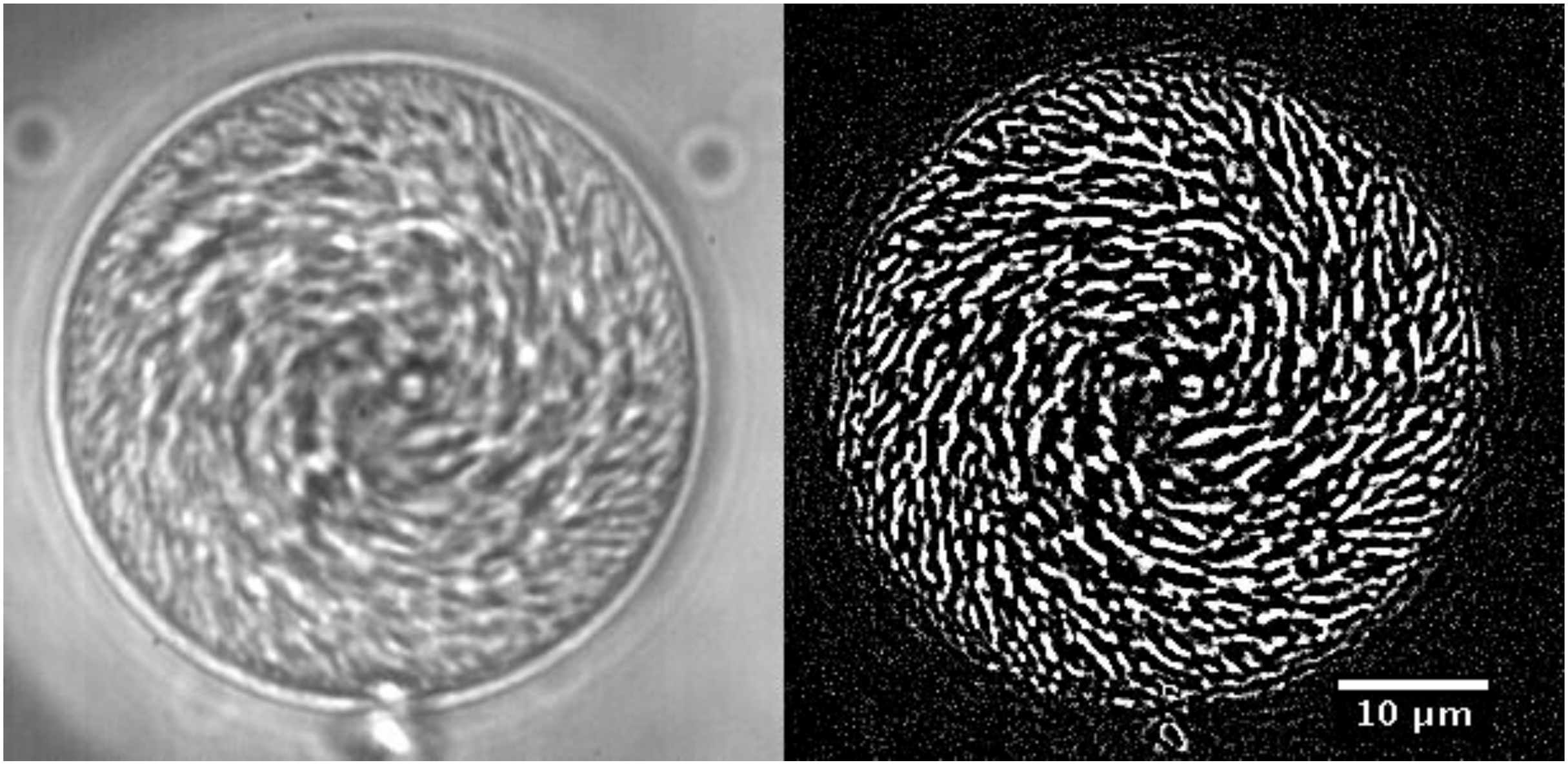
Vortex diameter ~ 70μm

PRL 2013

Stable bacterial spiral vortex



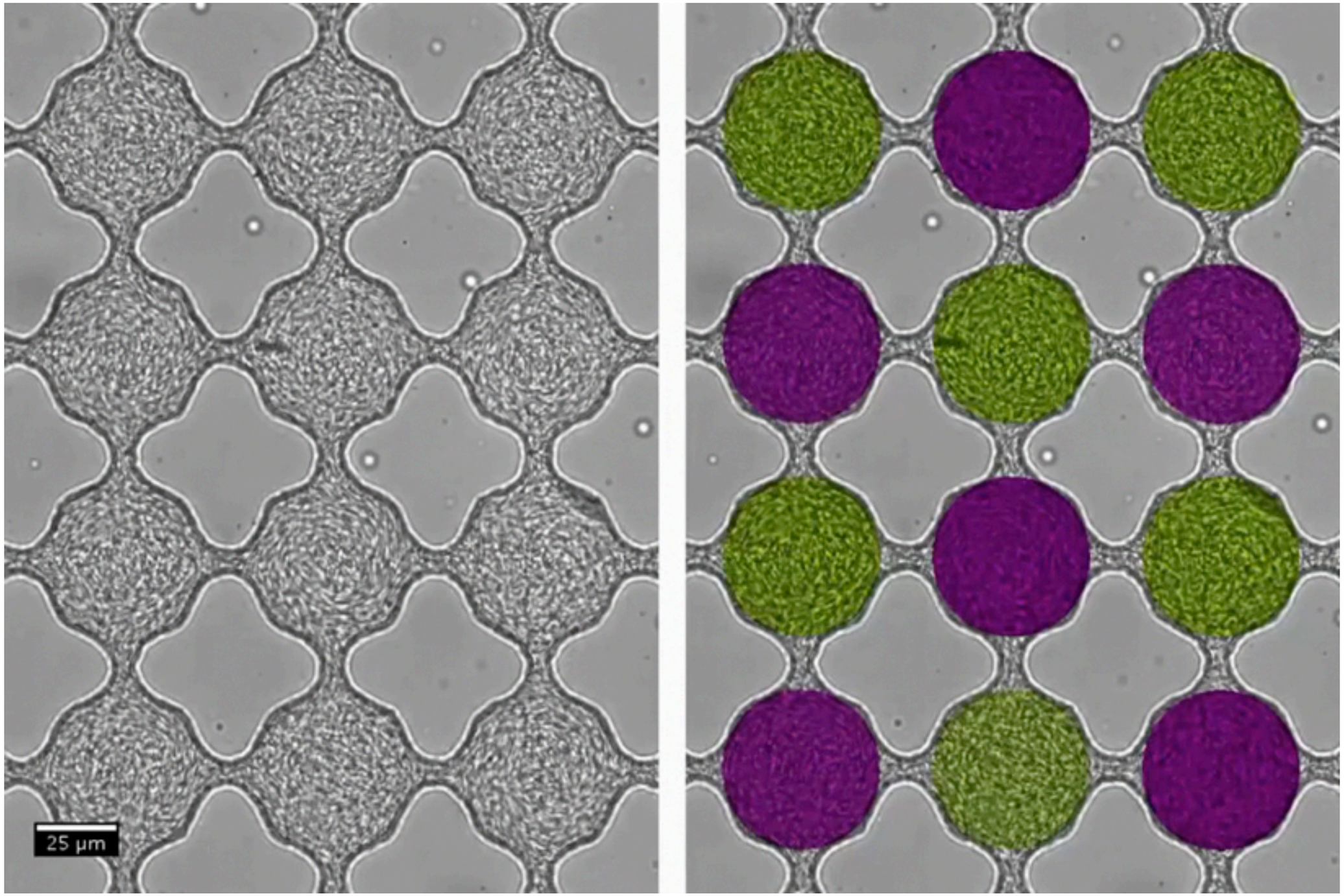
Vortex life time ~ minutes



Wioland et al (2013) PRL
Lushi et al (2014) PNAS

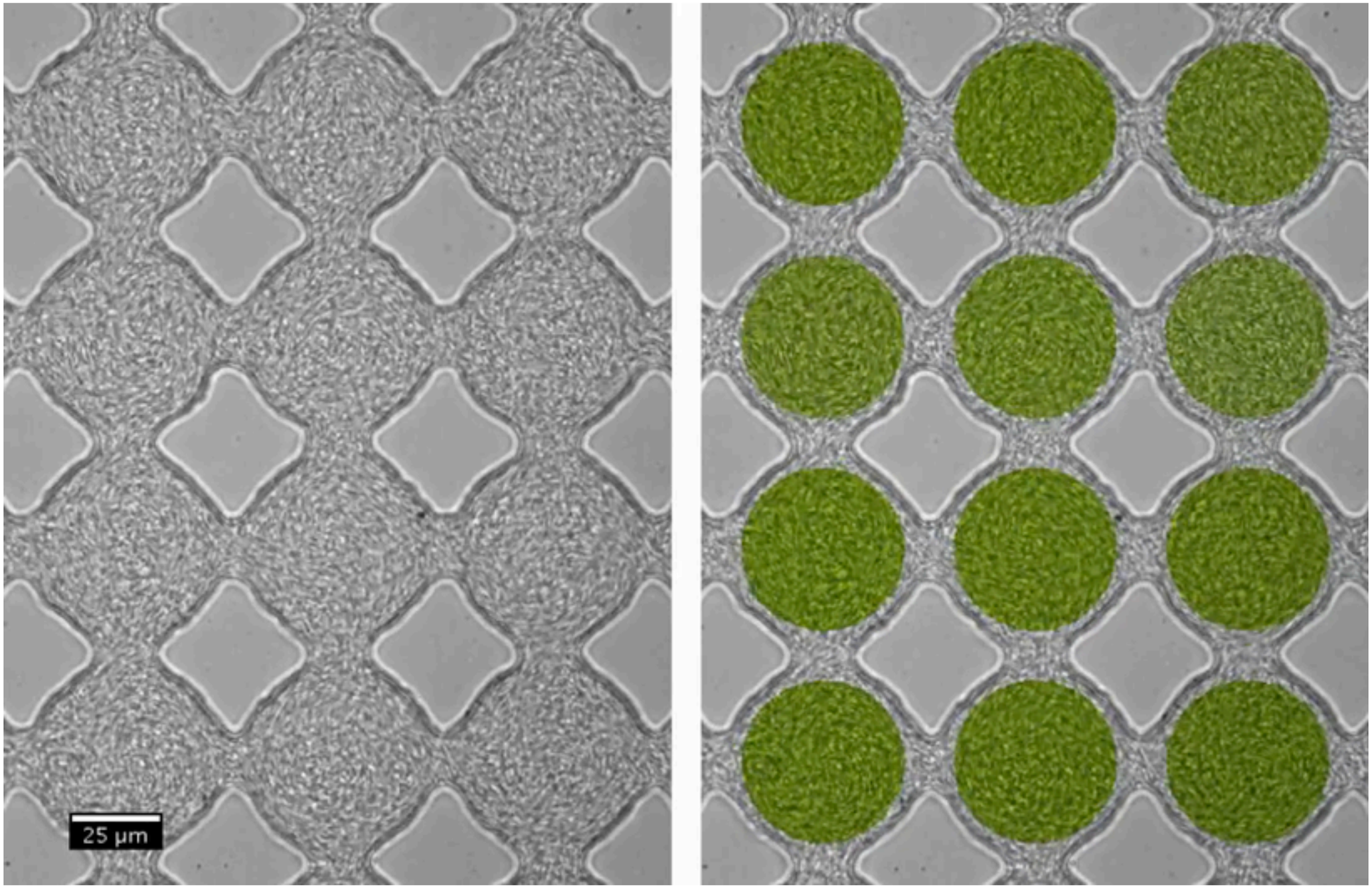
Edge currents !

Anti-ferromagnetic order

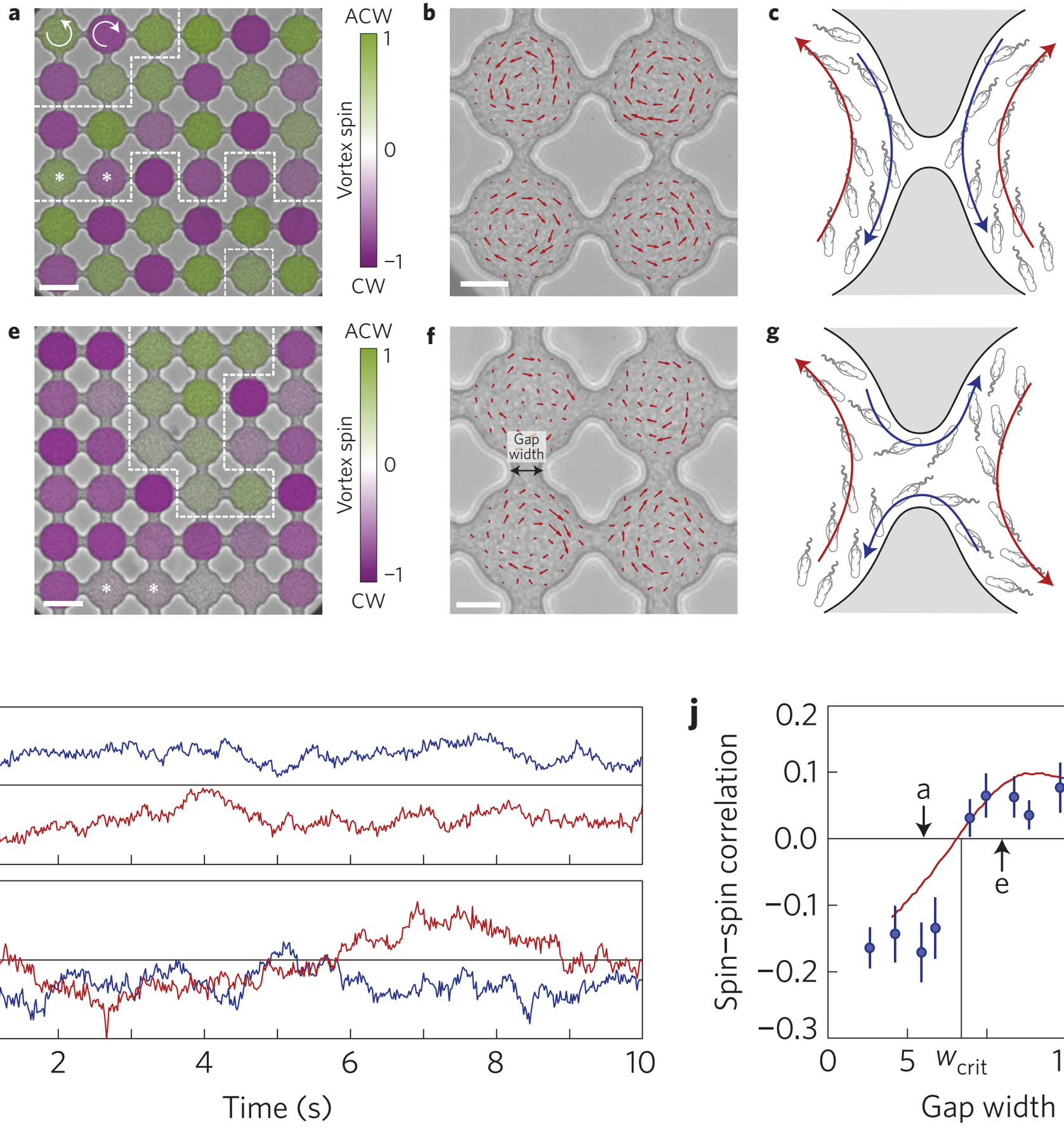


Nature Physics 2016

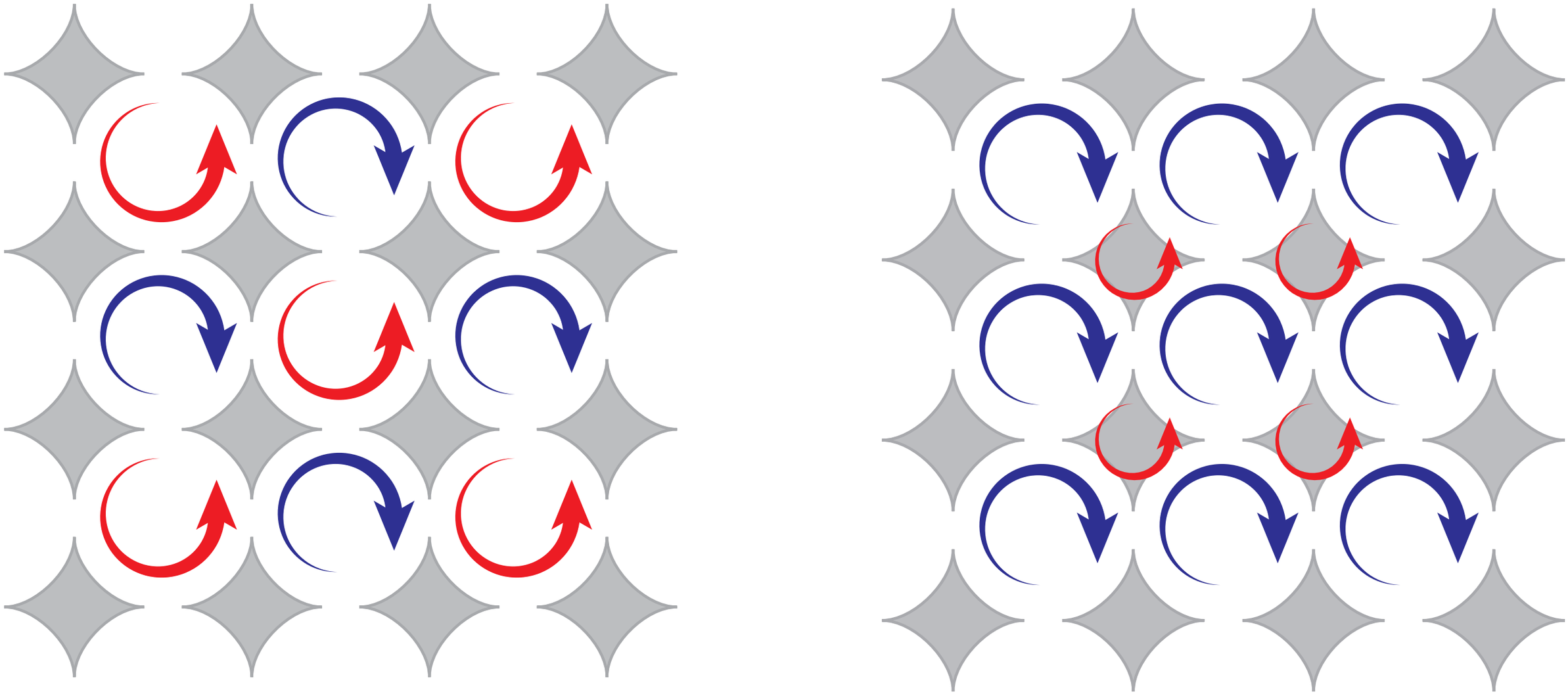
Ferromagnetic order



Nature Physics 2016


a
ACW
1
Vortex spin
0
−1
CW
b
c
e
ACW
1
Vortex spin
0
−1
CW
f
Gap
width
g
i
Vortex spin
1
0
−1
1
0
−1
0
2
4
6
8
10
Time (s)
j
Spin–spin correlation
0.2
0.1
0.0
−0.1
−0.2
−0.3
a
e
0
5
w_{crit}
15
20
25
Gap width (μm)

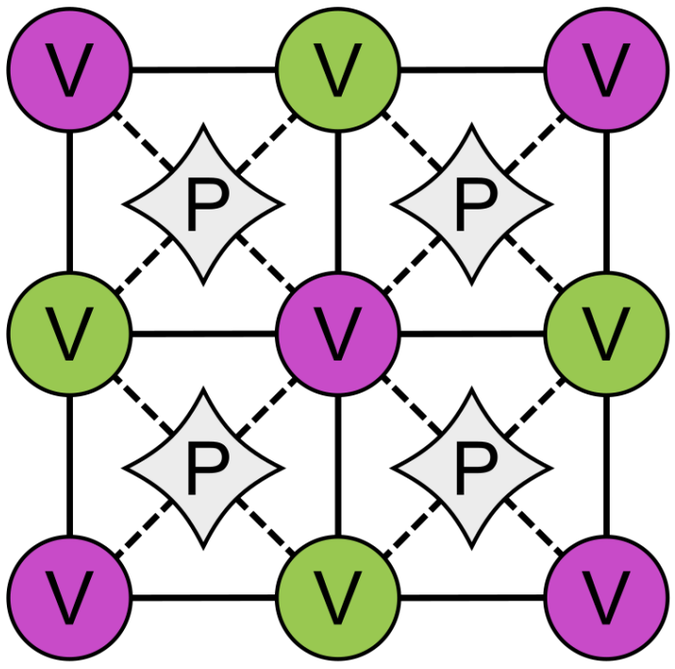
anti-ferro *vs.* ferro

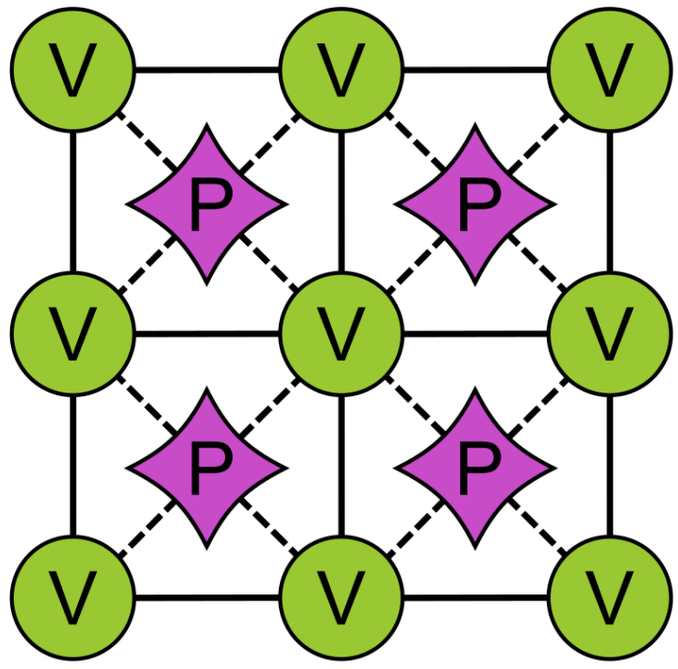


can be completely characterized in terms
of a generalized Ising model

Wioland et al (2015)

anti-ferro *vs.* ferro





$$H(\mathbf{V},\mathbf{P}) = -J_{\mathrm{v}} \sum_{V_i \sim V_j} V_i V_j - J_{\mathrm{p}} \sum_{V_i \sim P_j} V_i P_j \quad + \sum_{V_i} \left(\frac{1}{2} a_v V_i^2 + \frac{1}{4} b_v V_i^4 \right) + \sum_{P_j} \frac{1}{2} a_{\mathrm{p}} P_j^2$$

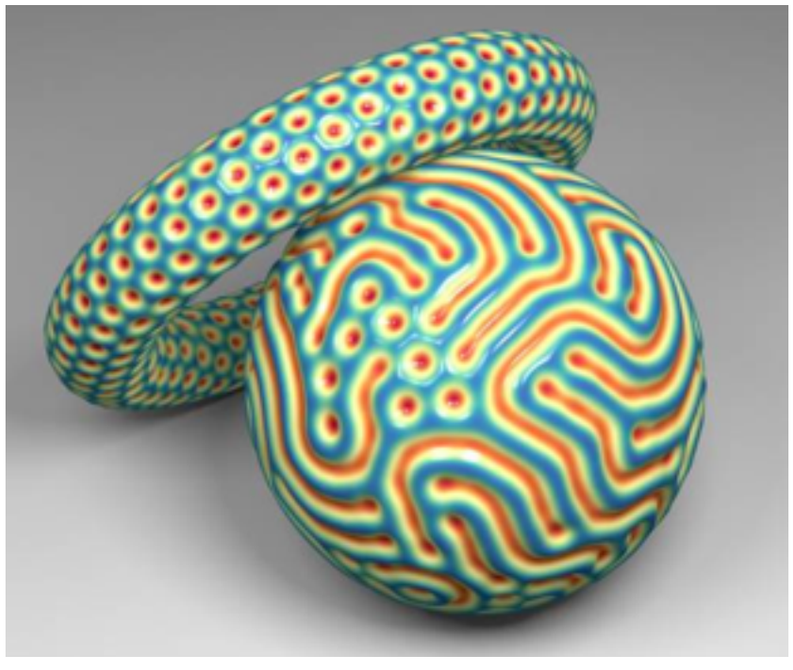
$$\mathrm{d}\mathbf{V} = -(\partial H / \partial \mathbf{V})\,\mathrm{d}t + \sqrt{2T_{\mathrm{v}}}\,\mathrm{d}\mathbf{W}_{\mathrm{v}}$$

$$\mathrm{d}\mathbf{P} = -(\partial H / \partial \mathbf{P})\,\mathrm{d}t + \sqrt{2T_{\mathrm{p}}}\,\mathrm{d}\mathbf{W}_{\mathrm{p}}$$

dunkel@mit.edu

Can we describe complex patterns through generic (minimal) PDE models ?

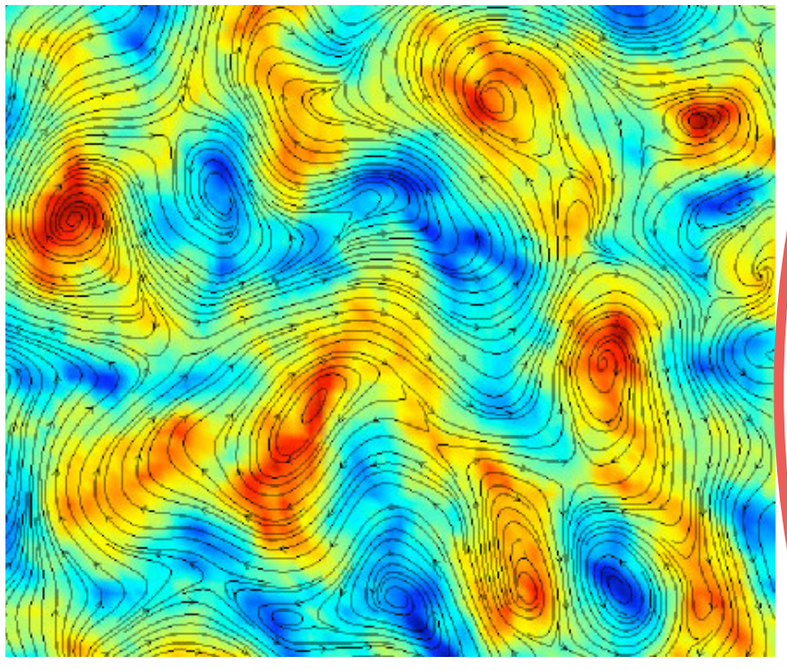
scalar fields



curved elastic materials

✓

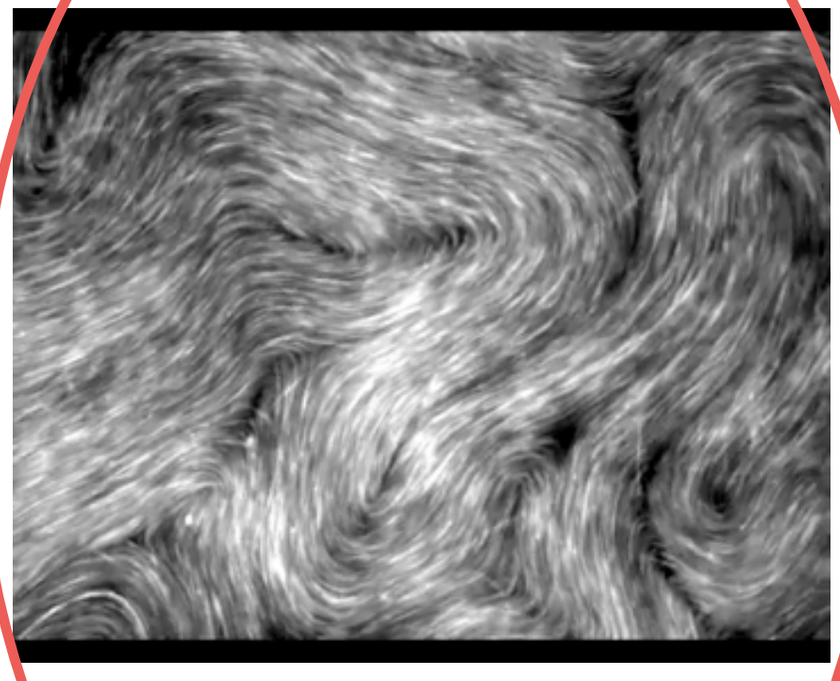
vector fields



bacterial suspensions

✓

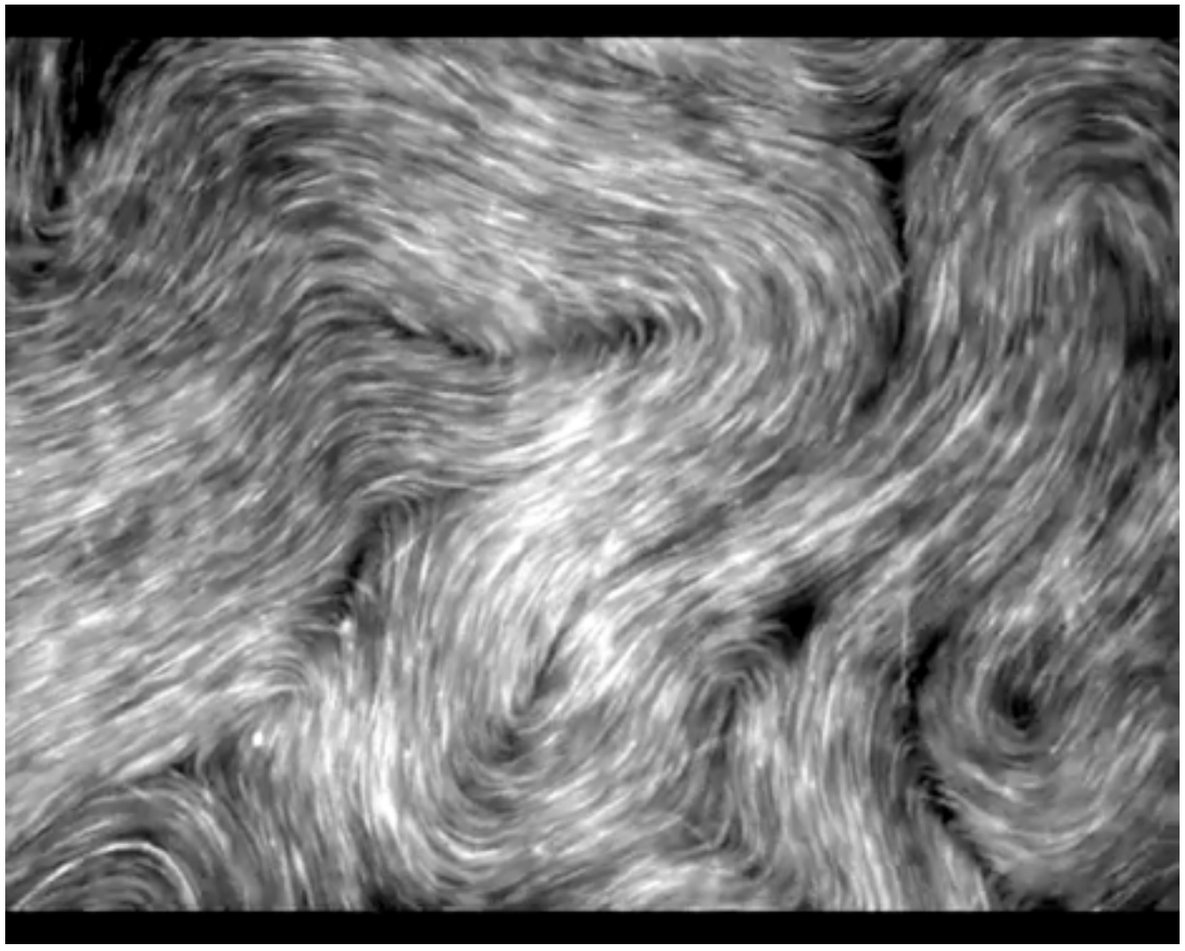
matrix fields

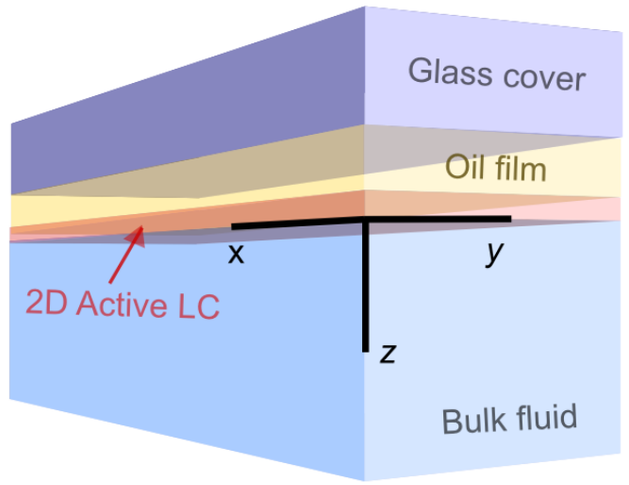


ATP driven active nematics (Dogic lab)

Multi-field theories: Marchetti, Yeomans, Baskaran, Giomi, Chate, …

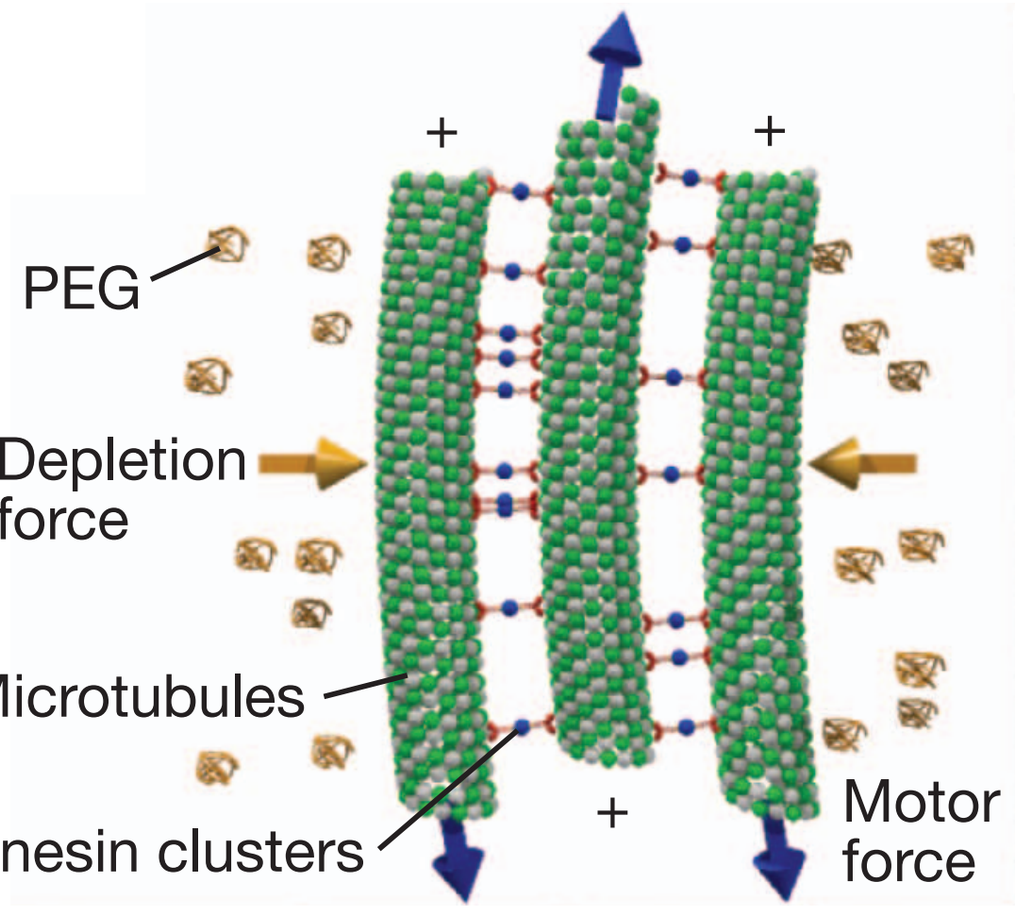
Active LCs





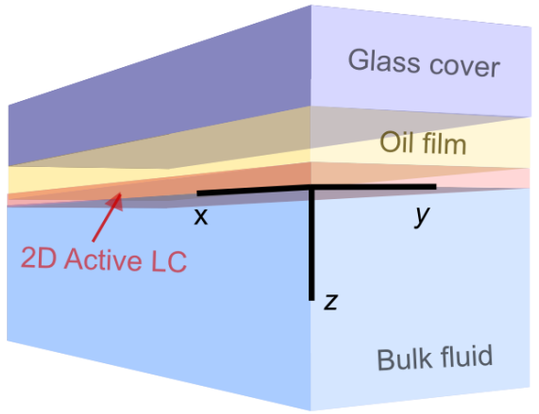
$$Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix} \qquad \mathrm{Tr}\, Q = 0$$

$$\Delta = \sqrt{\lambda^2 + \mu^2} \qquad \Lambda^{\pm} = \pm\Delta$$



Dogic Lab, Nature 2012, Nature Materials 2015

Active LCs

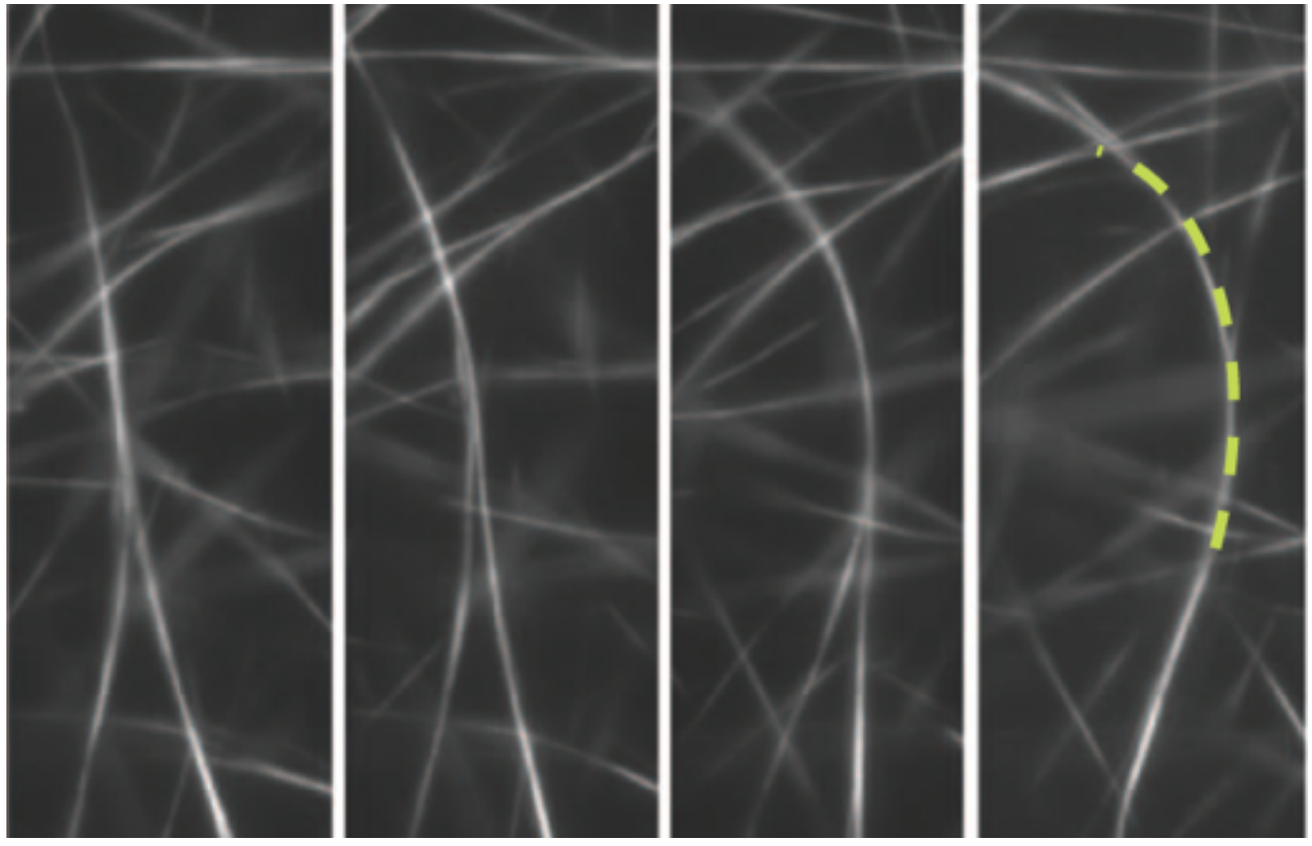


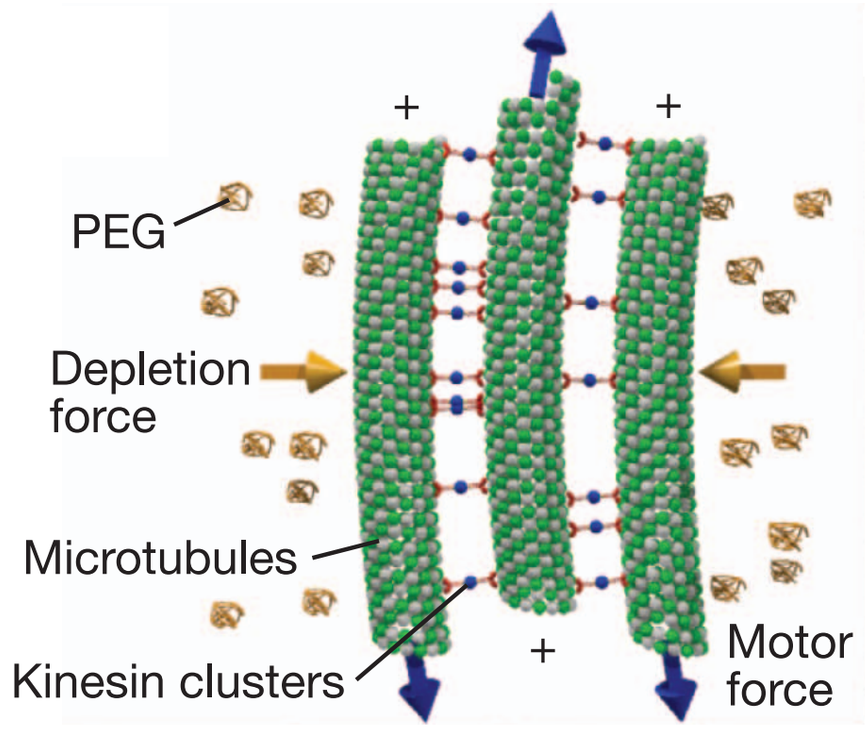


Anand Oza

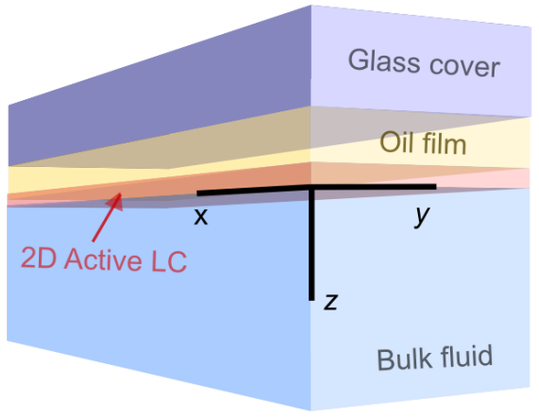
$$\partial_t Q + \nabla \cdot (\boldsymbol{v} Q) = -\frac{\delta \mathcal{F}}{\delta Q}$$

$$F = \mathrm{Tr}\left\{ -\frac{a}{2}Q^2 + \frac{b}{4}Q^4 - \frac{\gamma_2}{2}(\nabla Q)^2 + \frac{\gamma_4}{4}(\nabla\nabla Q)^2 \right\}$$





Active LCs





Anand Oza

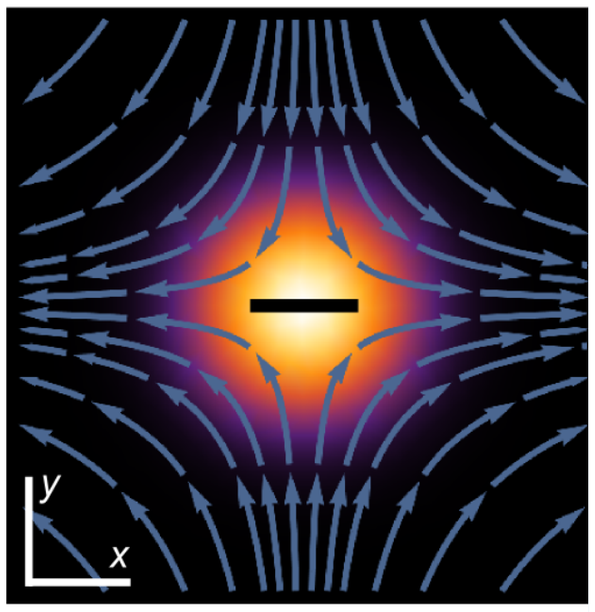
$$\partial_t Q + \boxed{\nabla \cdot (\boldsymbol{v} Q)} = -\frac{\delta \mathcal{F}}{\delta Q}$$

$$F = \mathrm{Tr}\left\{ -\frac{a}{2}Q^2 + \frac{b}{4}Q^4 - \frac{\gamma_2}{2}(\nabla Q)^2 + \frac{\gamma_4}{4}(\nabla\nabla Q)^2 \right\}$$

Non-incompressible overdamped HD

$$-\eta \nabla^2 \boldsymbol{v} + \nu \boldsymbol{v} = -\zeta \nabla \cdot Q$$

$$\boldsymbol{v} = -D \nabla \cdot Q \qquad\qquad D = \zeta/\nu$$



arXiv:1507.01055

Complex representation

Generalization of analogy between smectic LCs and Abrikosov vortex
(De Gennes 1972, Renn & Lubensky 1988, Pindak and co-workers 1990)

$$\partial_t Q - D\nabla\cdot[(\nabla\cdot Q)Q] = aQ - bQ^3 - \gamma_2\nabla^2 Q - \gamma_4(\nabla^2)^2 Q$$

$$Q = \begin{pmatrix} \lambda & \mu \\ \mu & -\lambda \end{pmatrix} \qquad \psi(t,z) = \lambda + i\mu \qquad \partial = \tfrac{1}{2}(\partial_x - i\partial_y)$$

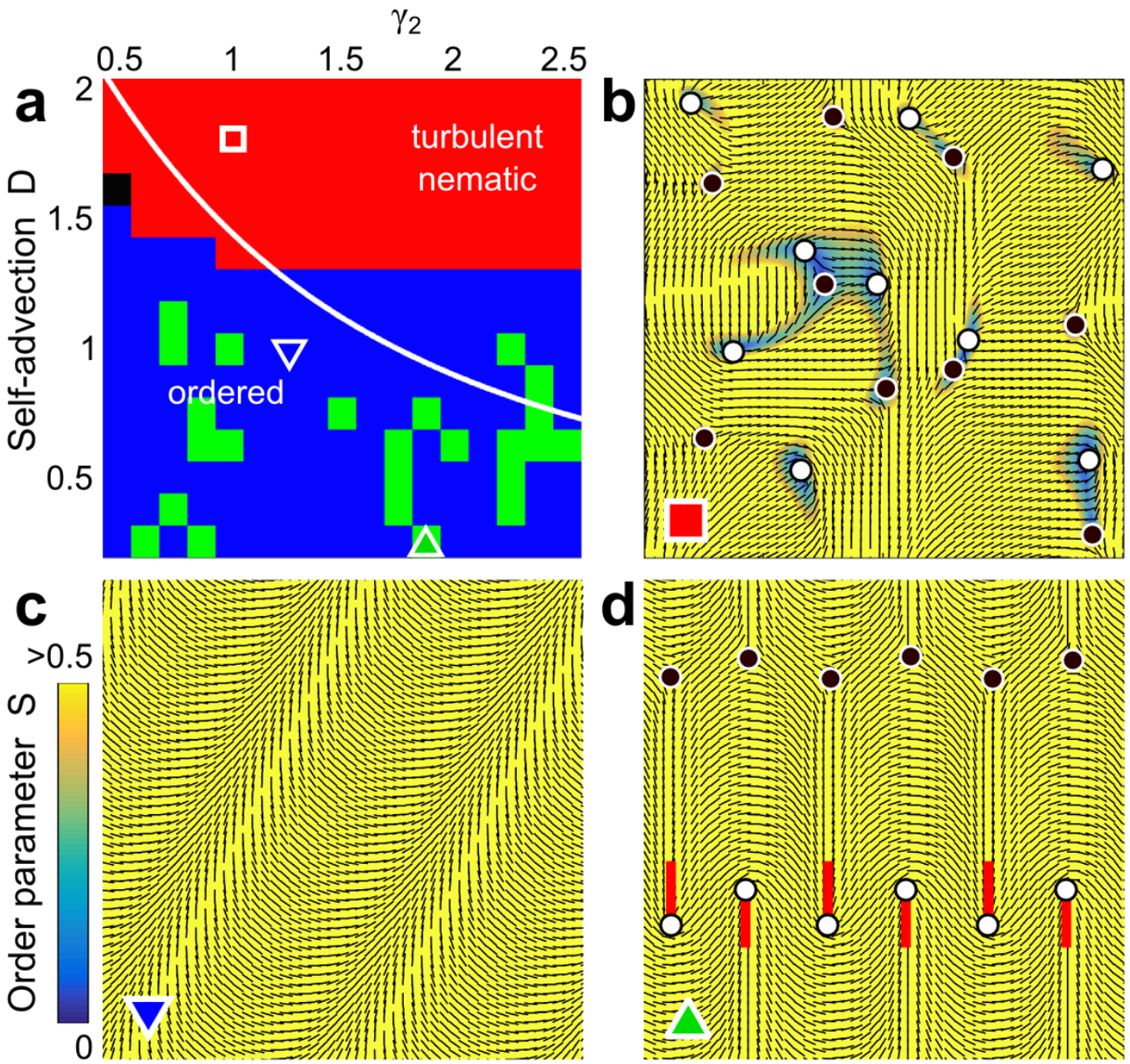
$$\partial_t\psi - 4D\,\Re\{(\partial^2\psi) + (\partial\psi)\partial\}\,\psi = \left(\frac{1}{4} - |\psi|^2\right)\psi - \gamma_2(4\bar\partial\partial)\psi - (4\bar\partial\partial)^2\psi$$

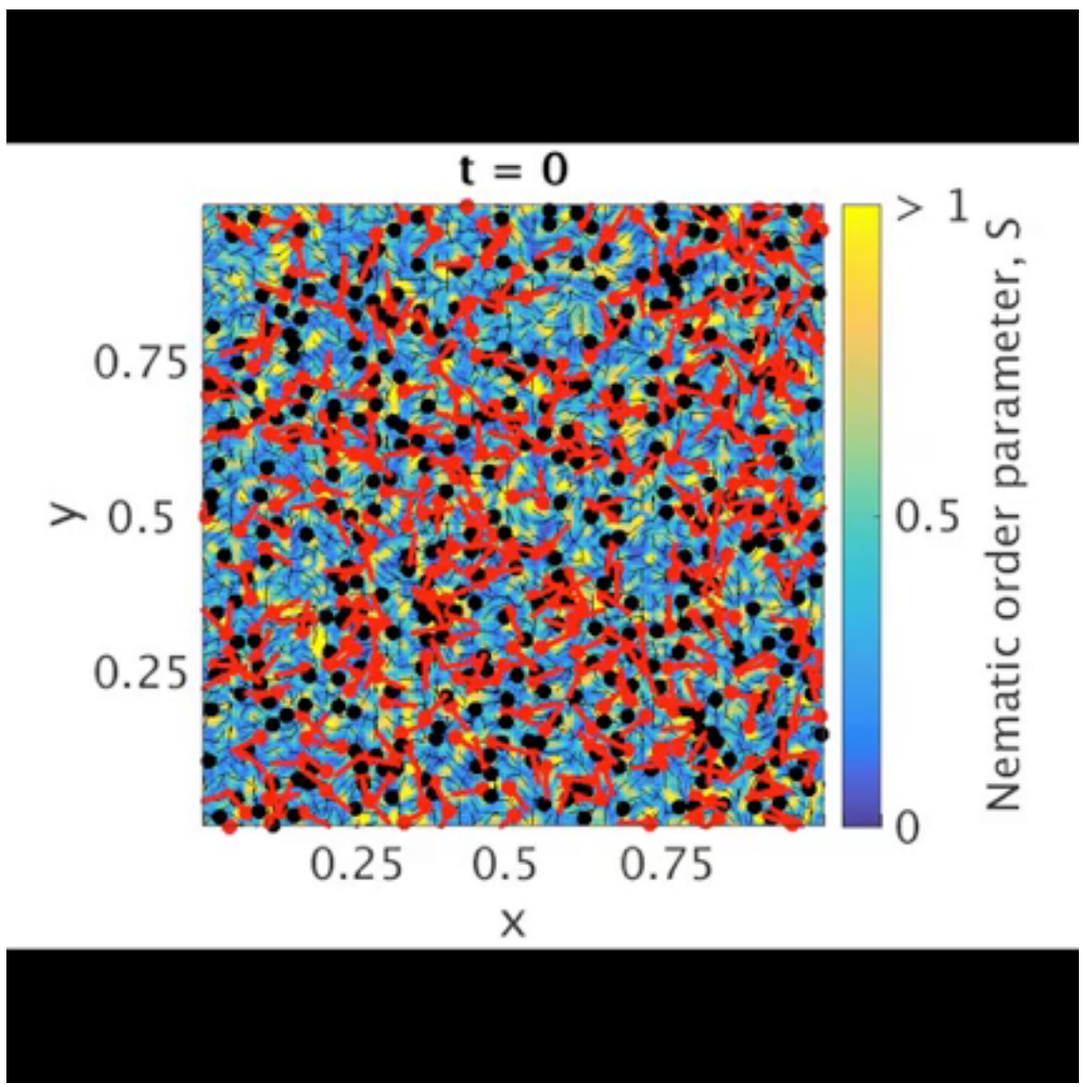
Generalized Gross-Pitaevski equation
with double-well dispersion

Lin et al Nature 2011 Cheuk et al PRL 2012 Parker et al Nature Phys 2013

Phases & long-range order

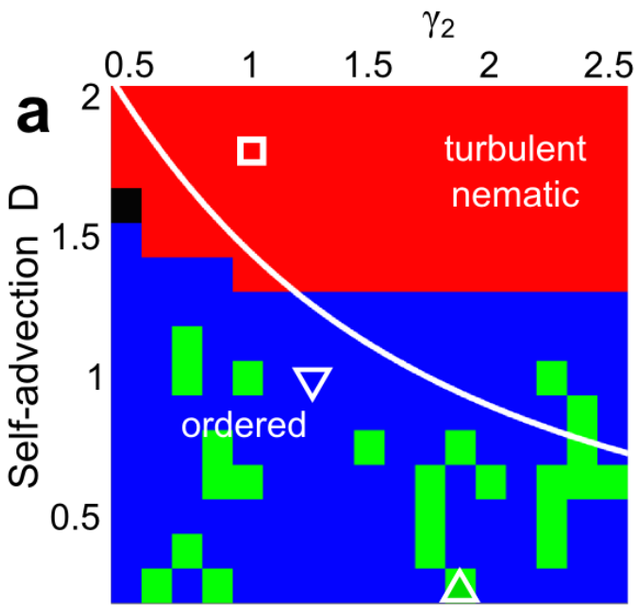
$$\partial_t \psi - 4D\,\Re\{(\partial^2\psi) + (\partial\psi)\partial\}\,\psi = \left(\frac{1}{4} - |\psi|^2\right)\psi - \gamma_2(4\bar{\partial}\partial)\psi - (4\bar{\partial}\partial)^2\psi$$

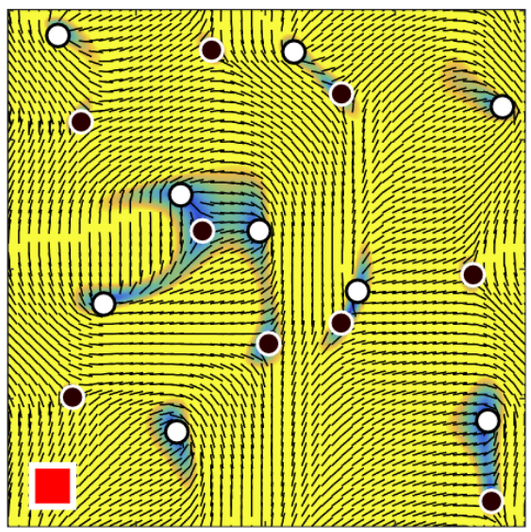


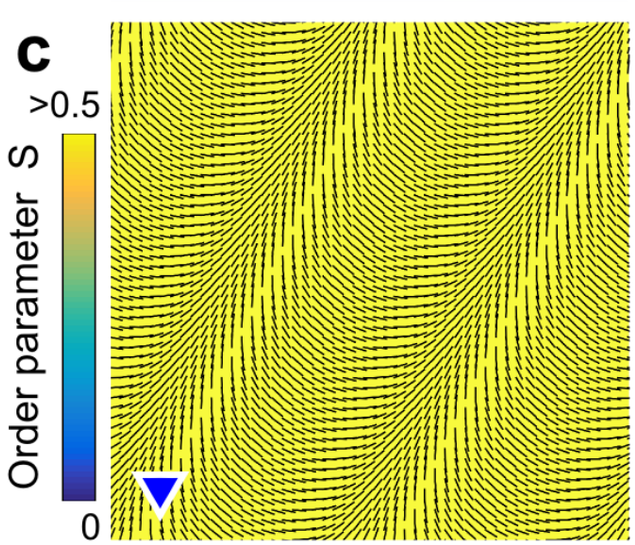


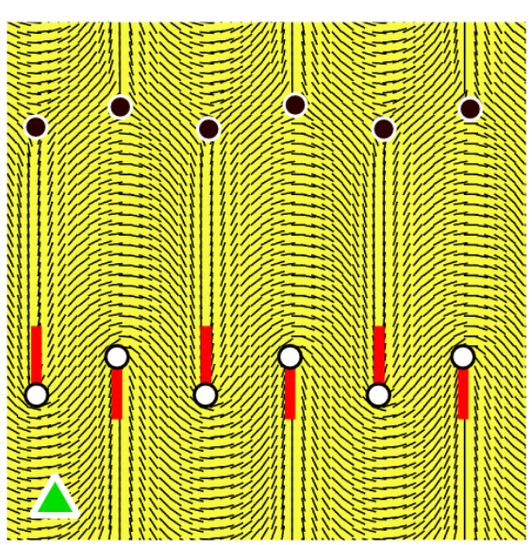
Phases & long-range order

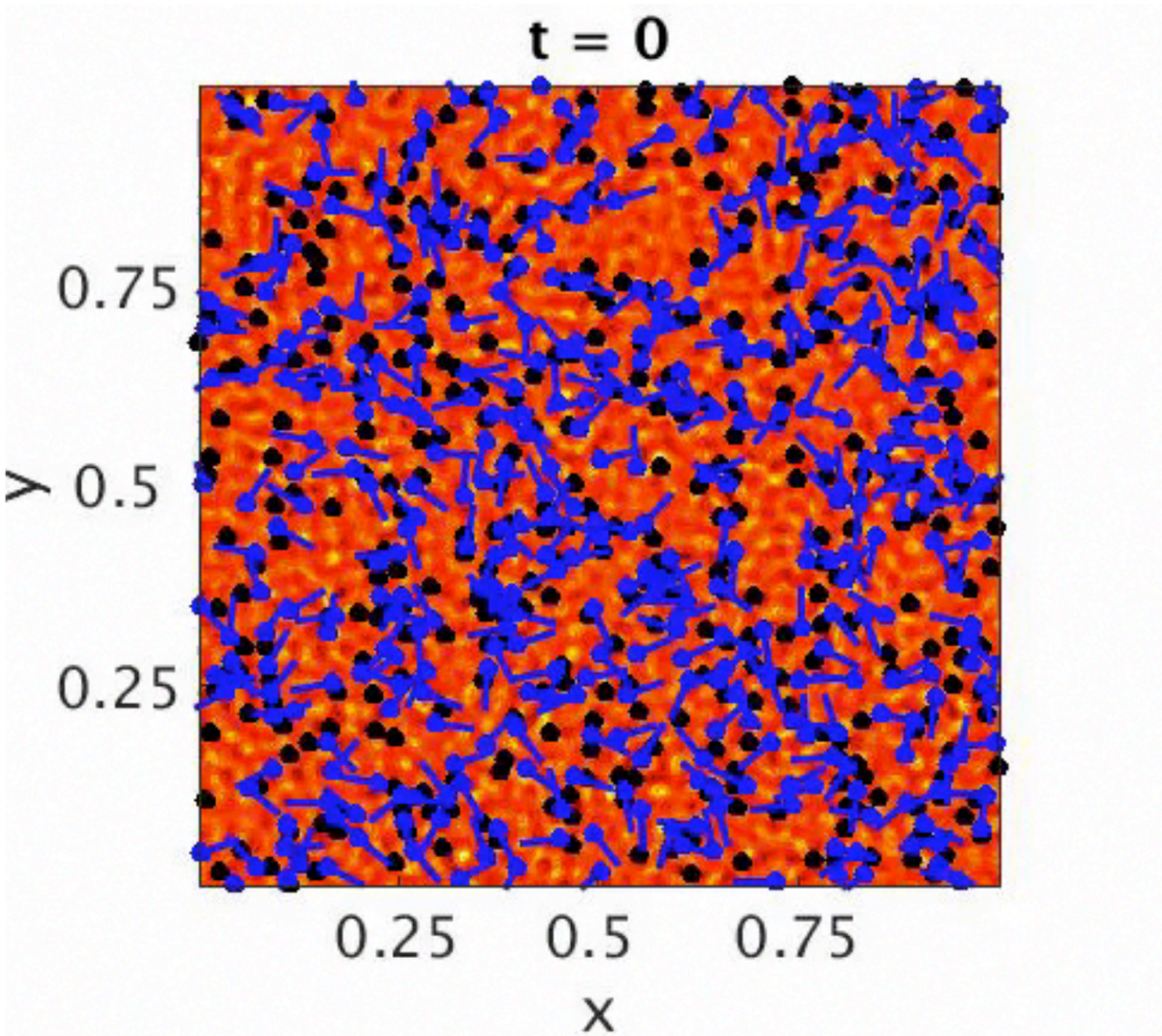
$$\partial_t \psi - 4D\,\Re\{(\partial^2\psi) + (\partial\psi)\partial\}\,\psi = \left(\frac{1}{4} - |\psi|^2\right)\psi - \gamma_2(4\bar{\partial}\partial)\psi - (4\bar{\partial}\partial)^2\psi$$



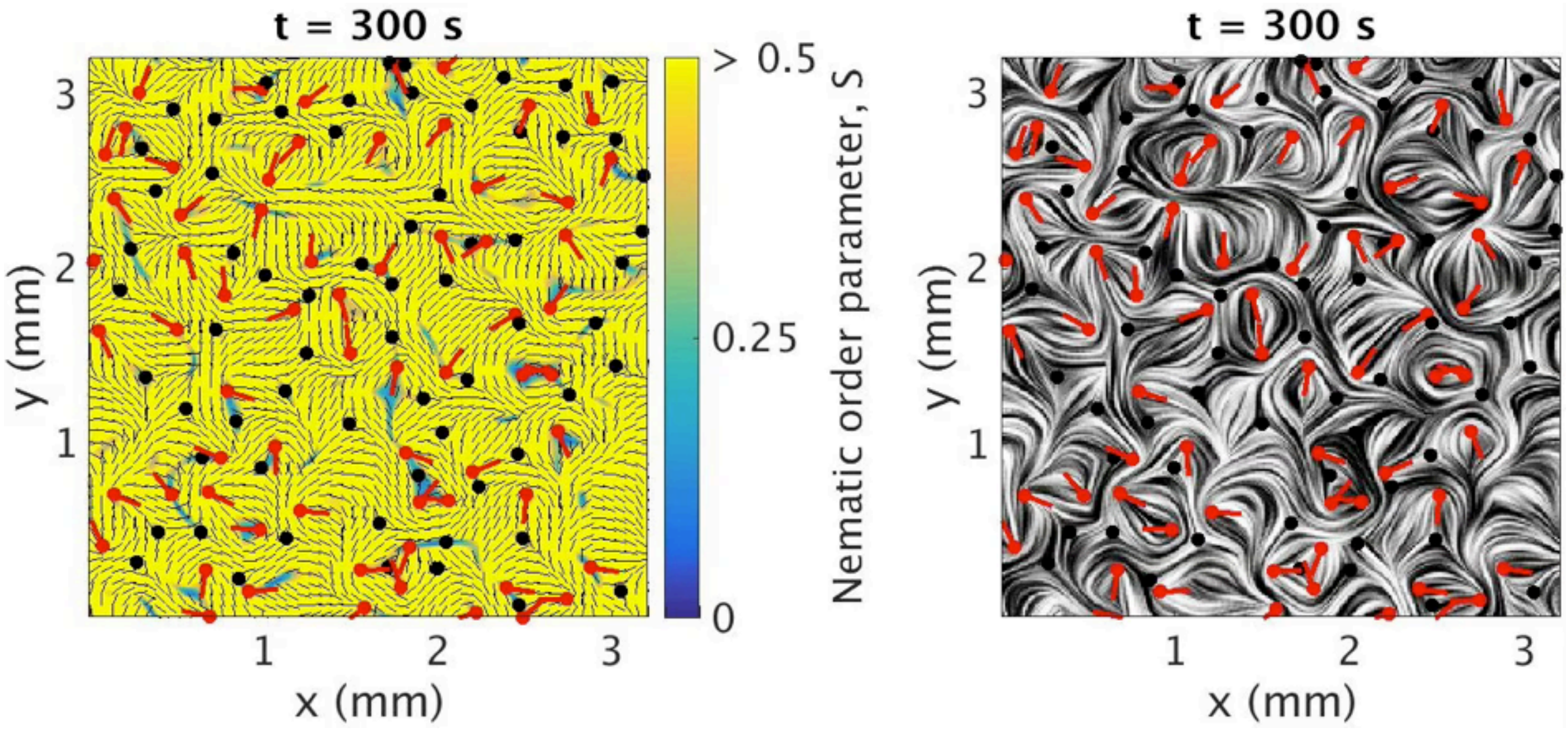








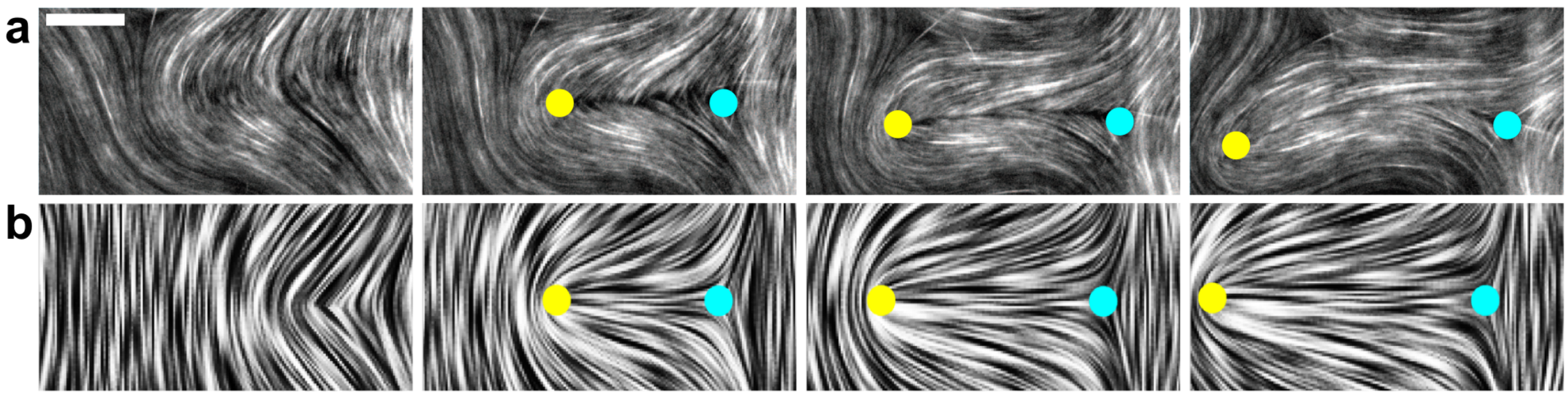
Chaotic phase



arXiv:1507.01055

Pair defect dynamics

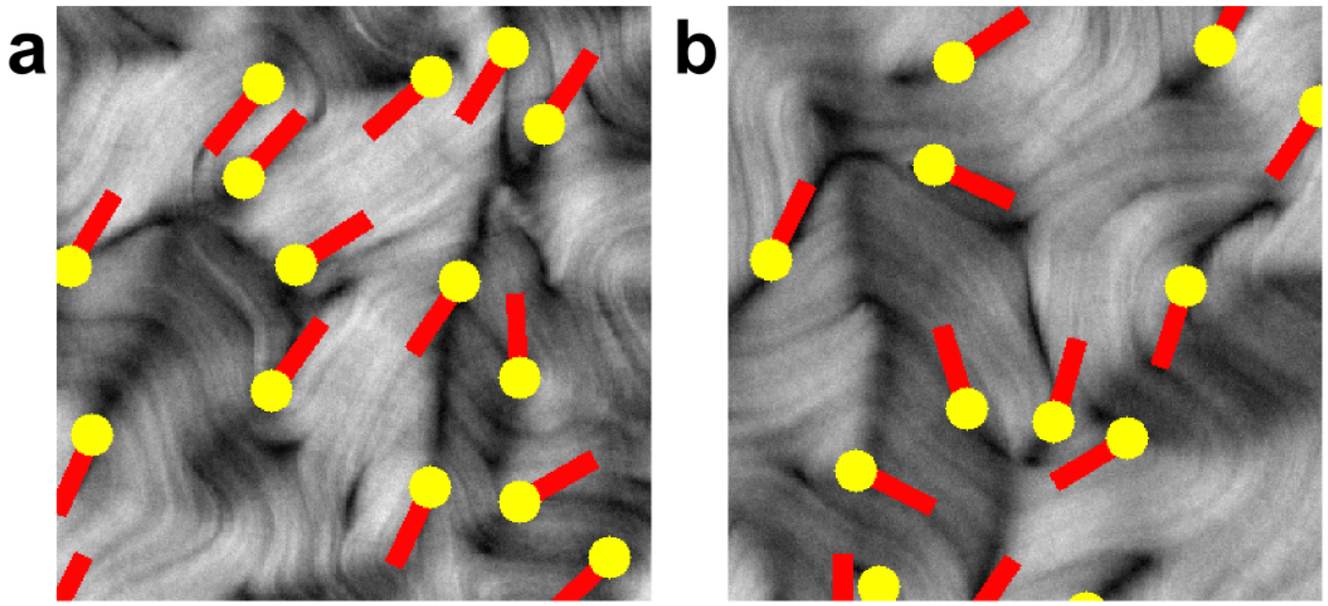
$$\partial_t \psi - 4D\,\Re\{(\partial^2\psi) + (\partial\psi)\partial\}\,\psi = \left(\frac{1}{4} - |\psi|^2\right)\psi - \gamma_2(4\bar{\partial}\partial)\psi - (4\bar{\partial}\partial)^2\psi$$

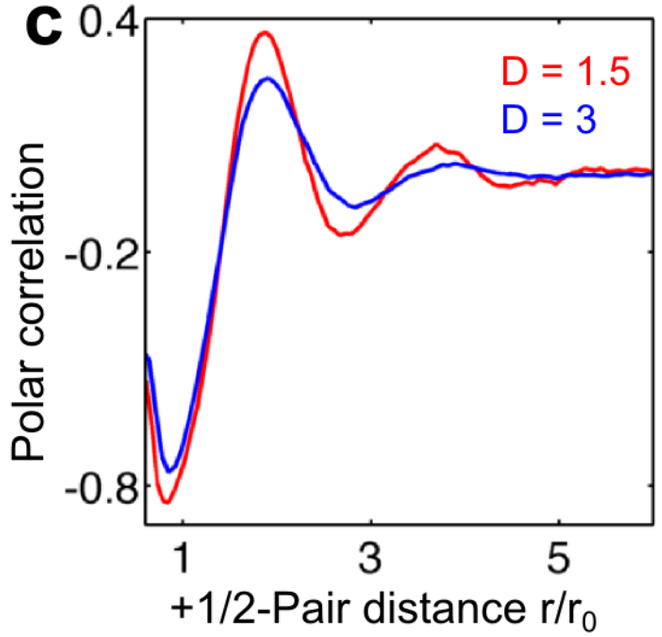


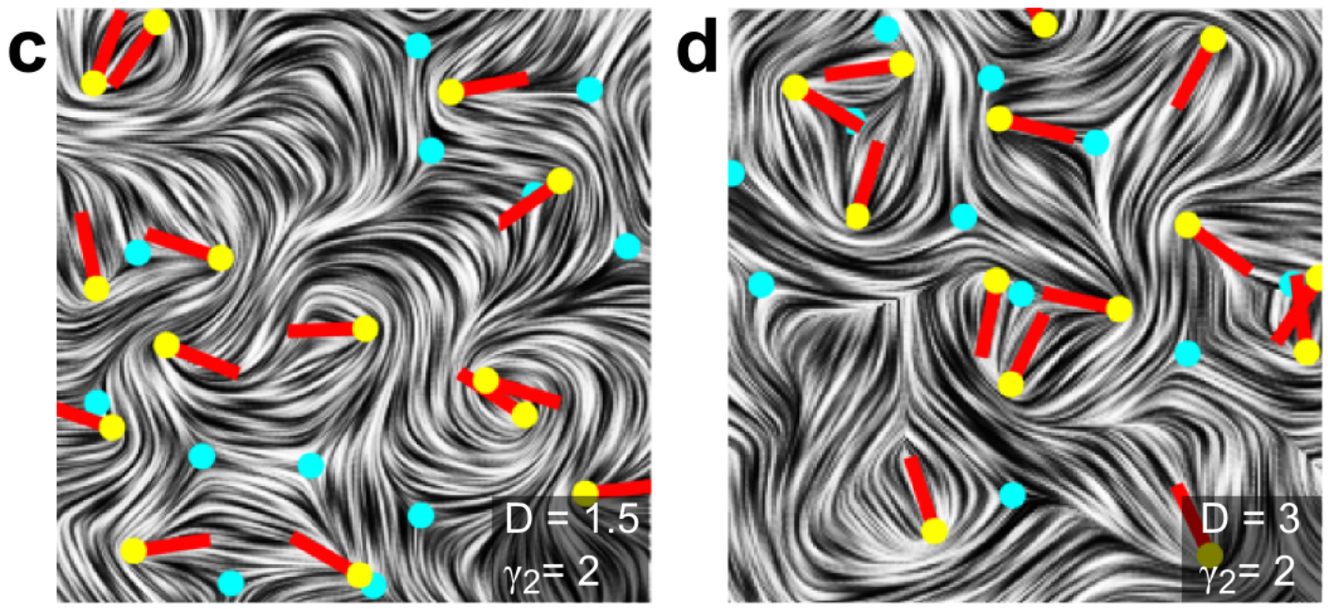
Similar to Giomi et al PRL 2013

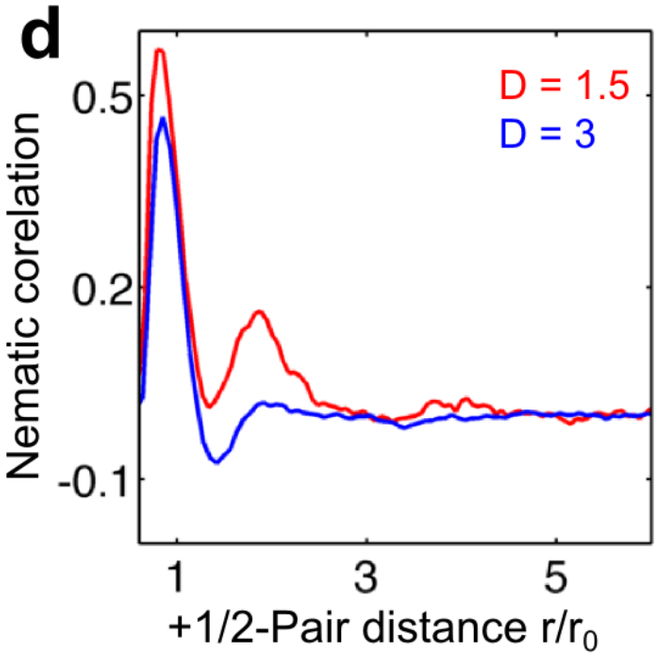
Anti-polar ordering

$$D = \zeta/\nu$$



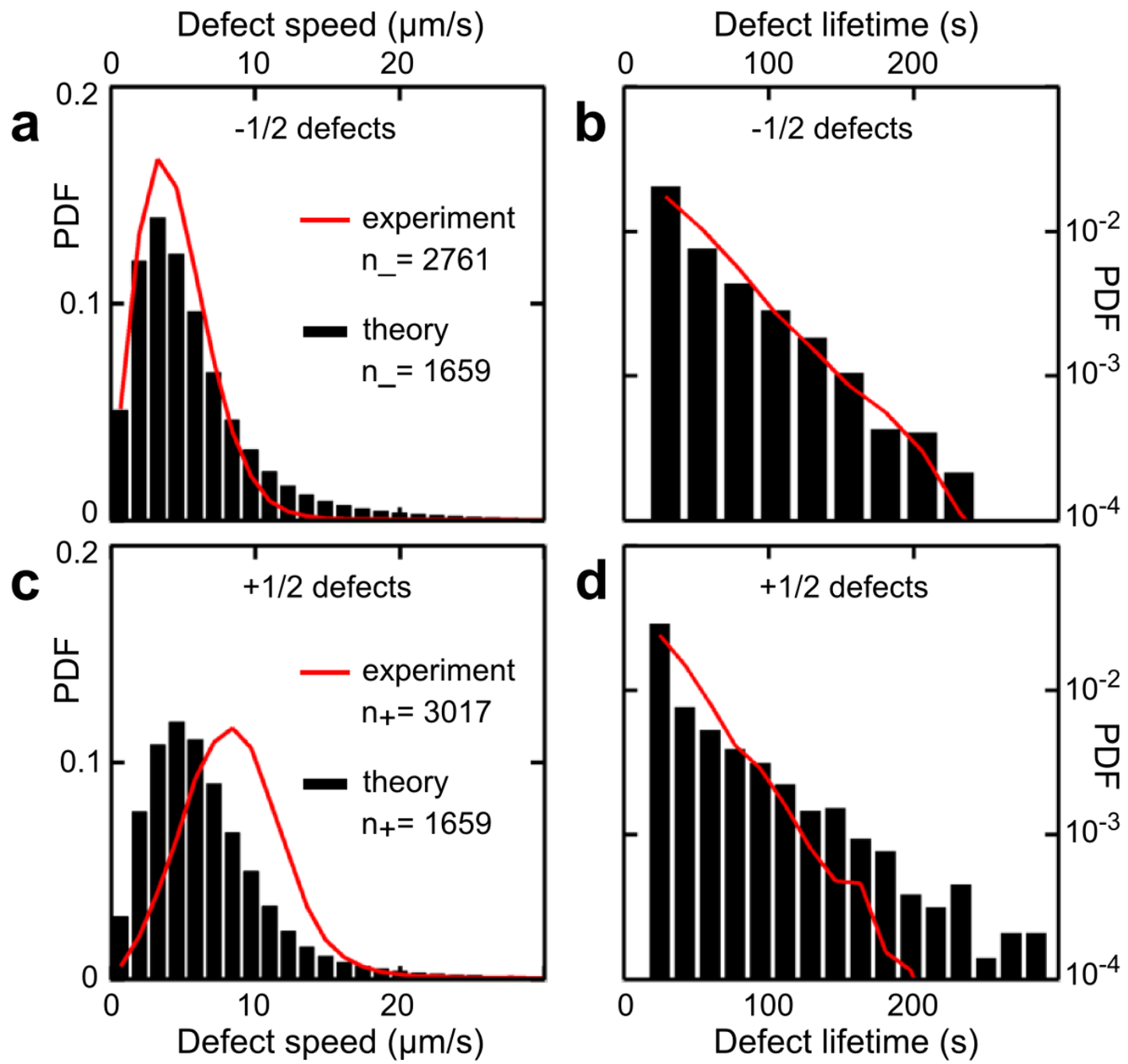






Experimental data kindly provided by Zvonimir Dogic and Steve DeCamp

Experiment vs. theory

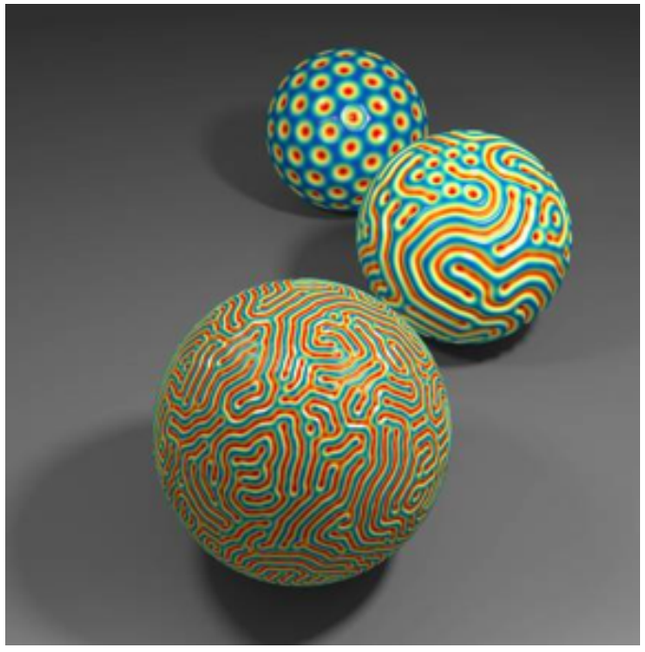


Experimental data kindly provided by Zvonimir Dogic and Steve DeCamp

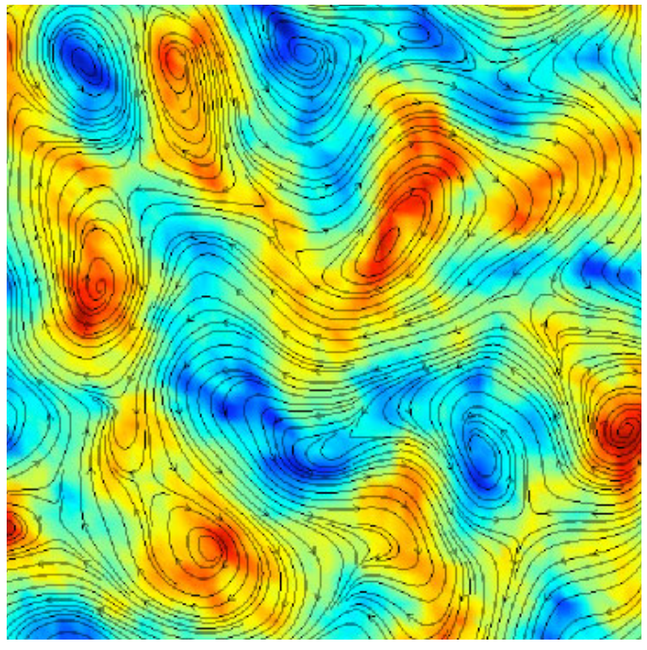
Summary

- ‘universal’ approach to pattern formation seems to describe 3 different experimental systems (scalar, vector, matrix order parameters) quite well

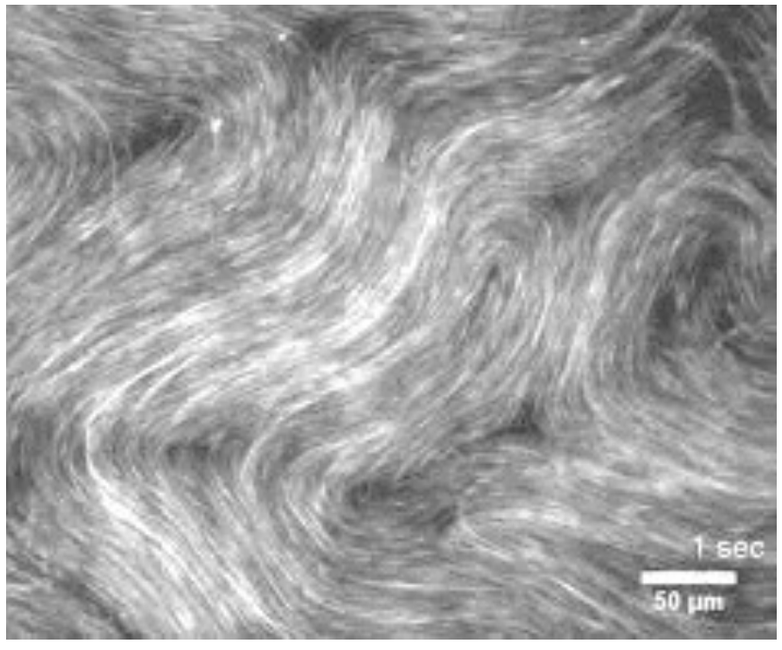
- Interesting analogies between active LCs and strongly interacting quantum systems



Nature Materials 2015



PRL 2013



Dogic Lab
Nature 2012
Nature Materials 2015

arXiv:1507.01055

Thanks to

Norbert Stoop (MIT)
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Sebastian Heidenreich (PTB Berlin)
Markus Bär (PTB Berlin)

Rik Wensink (CNRS Paris)
Hartmut Löwen (Düsseldorf)







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