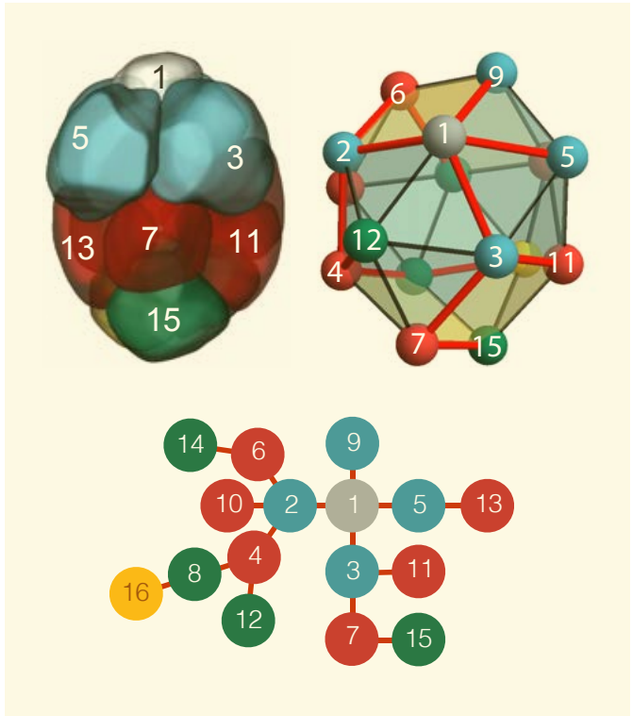
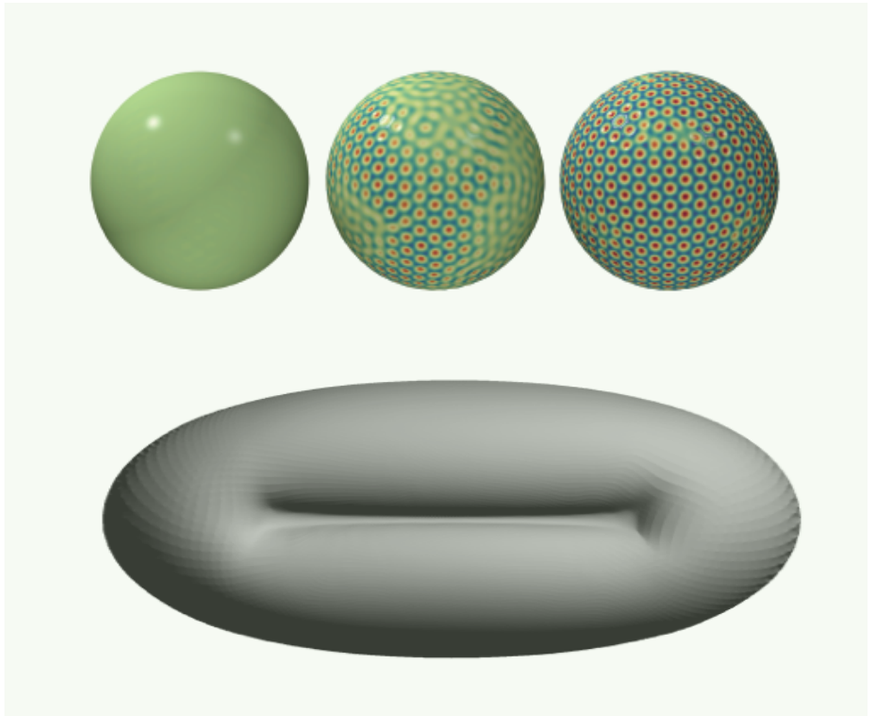
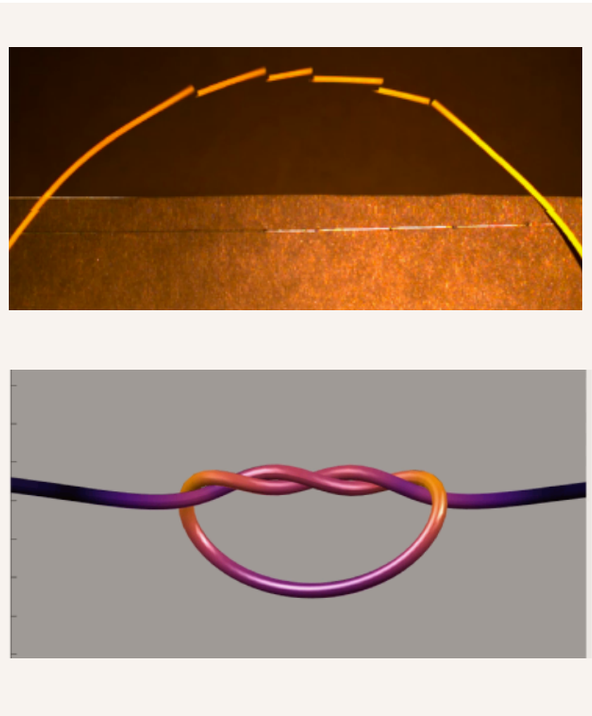
How cells pack, tissue folds, and spaghetti break

Jörn Dunkel





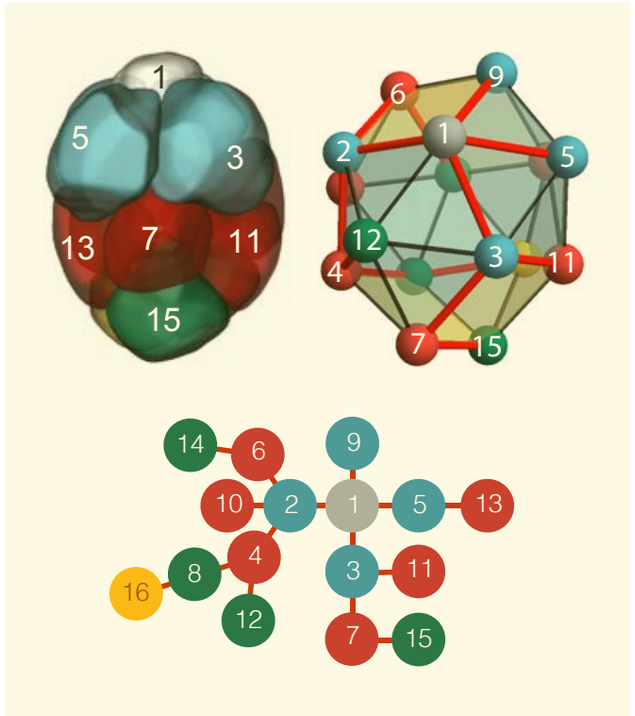


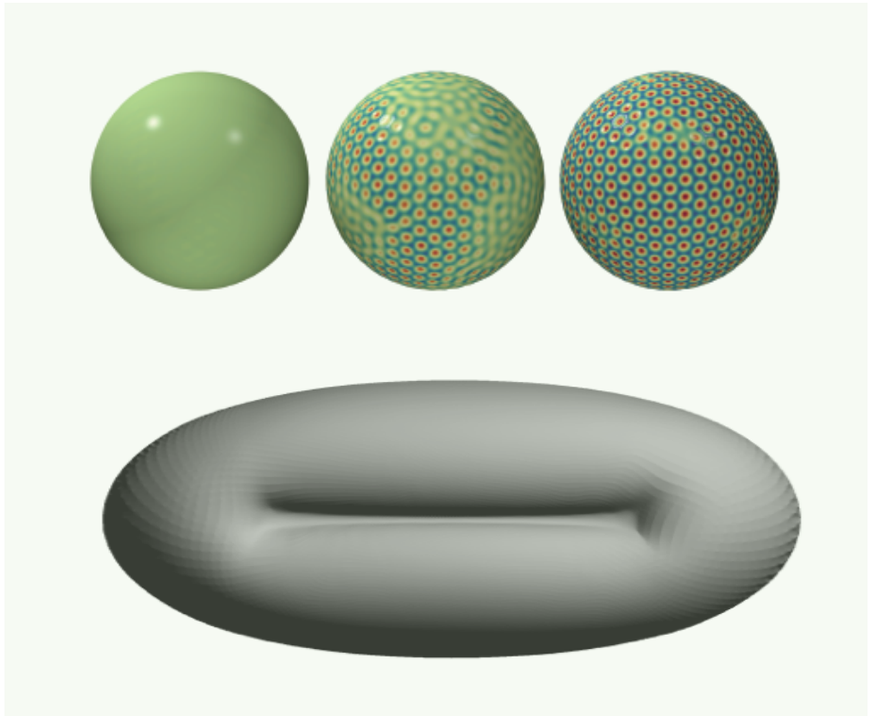
MIT
MATHEMATICS

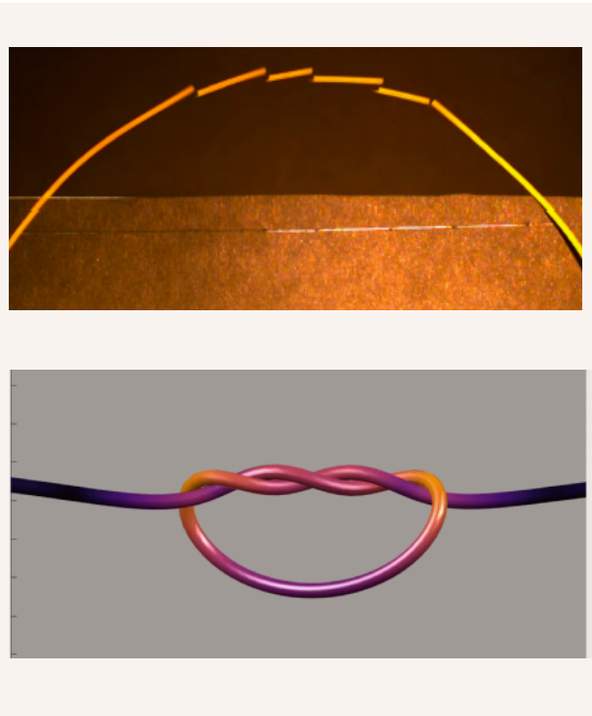
dunkel@math.mit.edu

How cells pack, tissue folds, and spaghetti break

Jörn Dunkel







MIT
MATHEMATICS

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Many thanks to

our group

MIT MATHEMATICS

Matthias Kolle

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Elastic fibers

Nikta Fakhri

Physics of Living Systems



Active surface waves



Pedro Reis

MITMECHE

Surface wrinkling

Alex Shalek

Chemistry



scRNA-seq protocol optimization



Adam Martin

MIT DEPARTMENT OF BIOLOGY

Fly morphogenesis

In Song Kim

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International trade data

Martin Zwierlein

rle RESEARCH LABORATORY OF ELECTRONICS AT MIT

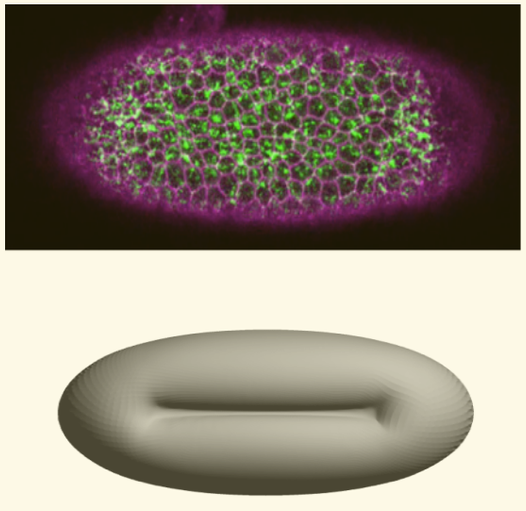


Quantum hydrodynamics

Started @ MIT daycare

Development 2017

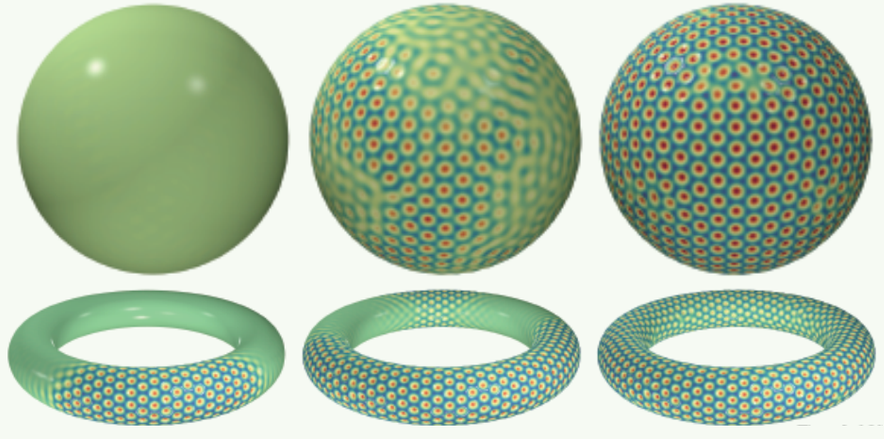
Morphogenesis



Martin lab, MIT Biology

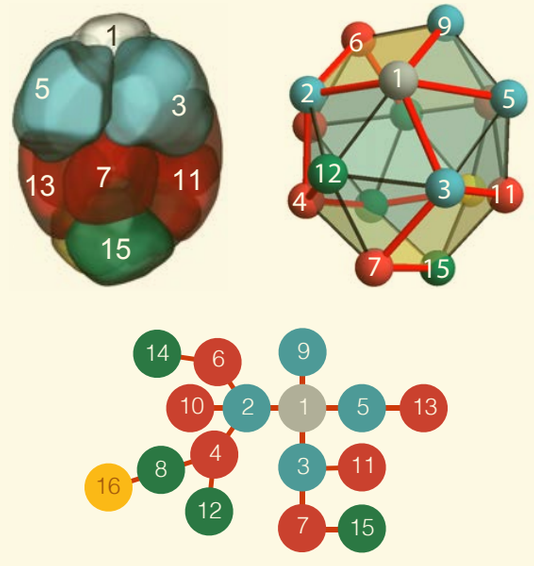
Nature Materials 2015 & PRL 2016

Pattern formation in soft materials



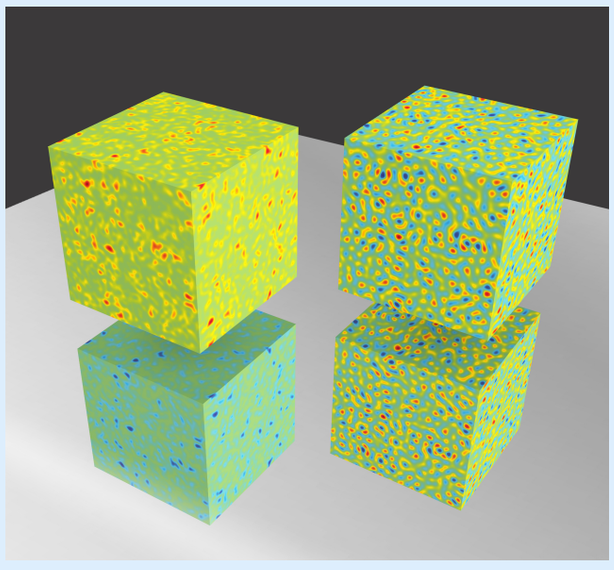
Reis lab, MIT MechE

Oogenesis



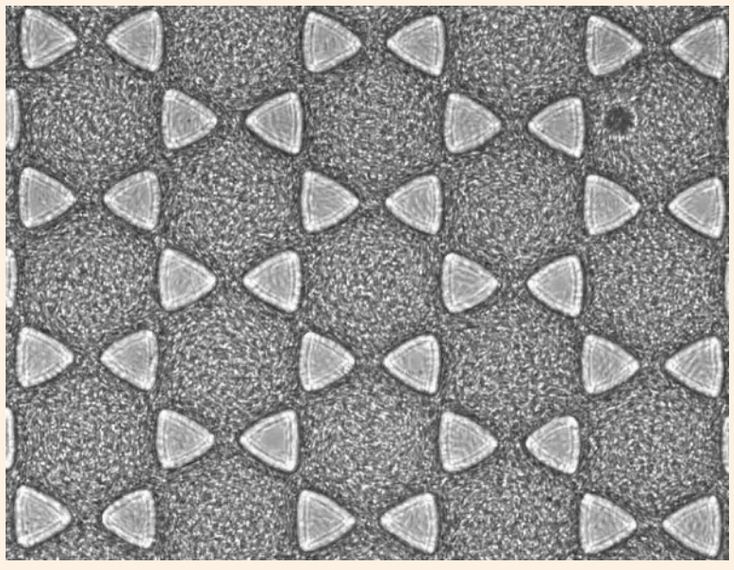
Shvartsman lab, Princeton

Active turbulence



PNAS 2017

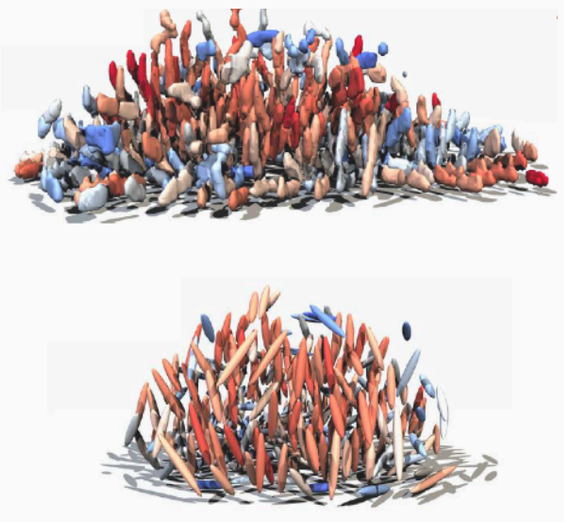
Geometric control of microbial suspensions



Goldstein lab, Cambridge

Nature Physics 2016 & Nature Comm 2017

Bacterial biofilm architecture

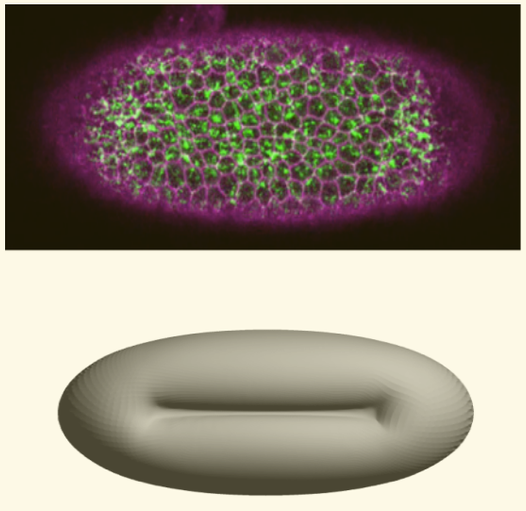


Drescher lab, MPI Marburg

PNAS 2016

Development 2017

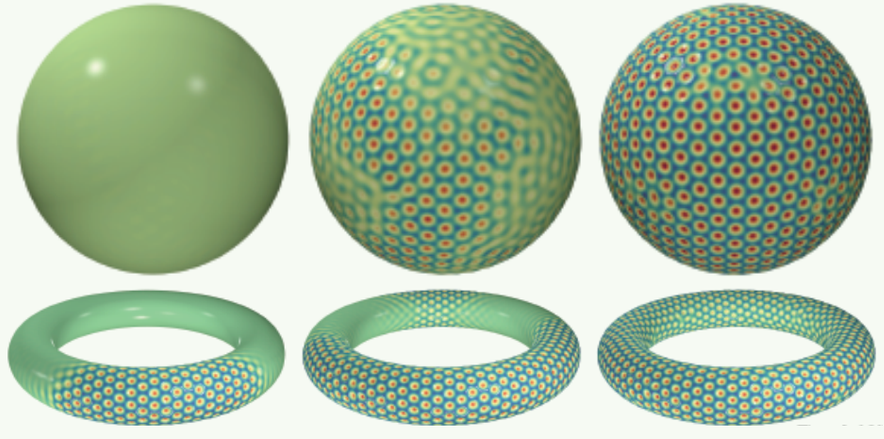
Morphogenesis



Martin lab, MIT Biology

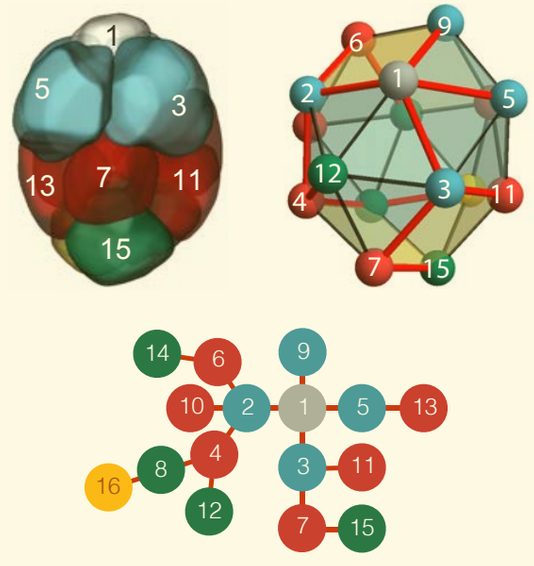
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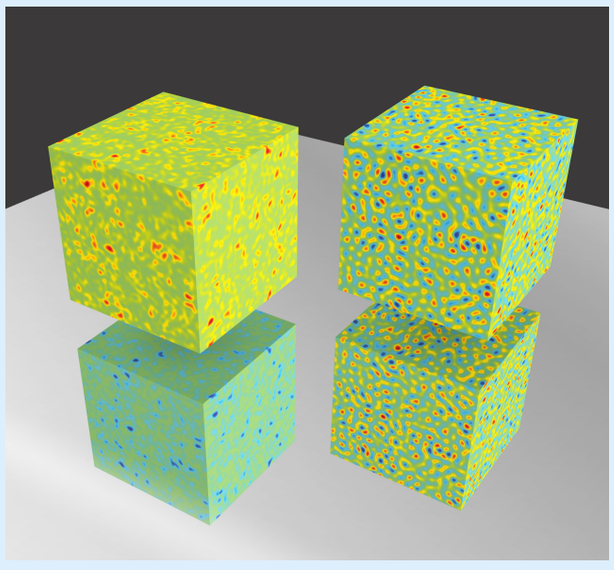
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Oogenesis



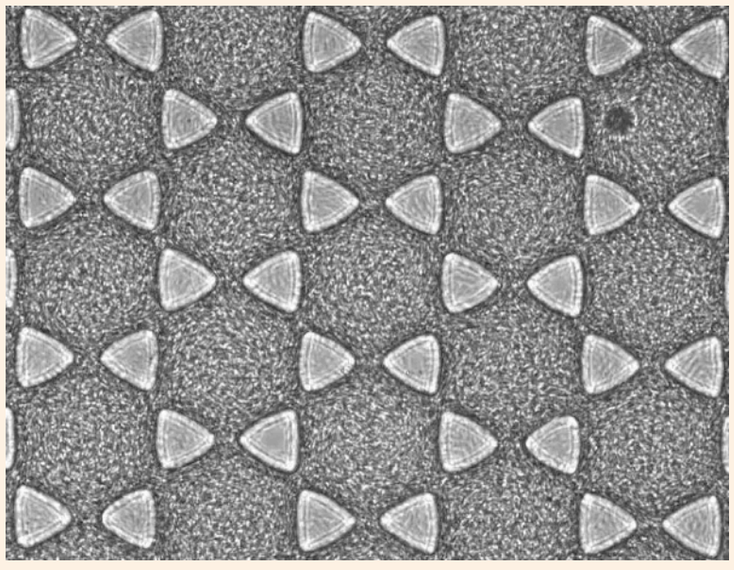
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PNAS 2017

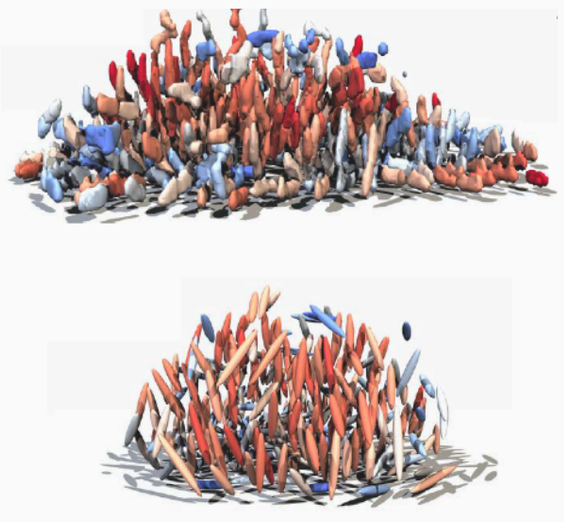
Geometric control of microbial suspensions



Goldstein lab, Cambridge

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Bacterial biofilm architecture



Drescher lab, MPI Marburg

PNAS 2016

What we do typically looks like this

$$
\begin{aligned}
\mu\partial_t u = \; & \gamma_0 \triangle u - \gamma_2 \triangle^2 u - au - bu^2 - cu^3 + \\
& \frac{h}{2}\left\{(\nu-1)\left[b^{\alpha\beta}\nabla_\alpha u \nabla_\beta u + 2u\nabla_\beta\left(b^{\alpha\beta}\nabla_\alpha u\right)\right] + \right. \\
& \left. 2\nu\left[\mathcal{H}(\nabla u)^2 - 2\nabla\cdot(\mathcal{H}u\nabla u)\right]\right\} + \\
& \frac{h}{2}\left[(1-\nu)u\nabla_\beta\left(uc^{\alpha\beta}\nabla_\alpha u\right) - \nu\mathcal{R}u(\nabla u)^2 + \right. \\
& \left. \nu\nabla\cdot(\mathcal{R}u^2\nabla u)\right]
\end{aligned}
$$



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& \left. \nu\nabla\cdot(\mathcal{R}u^2\nabla u)\right]
\end{aligned}
$$



or this

The change in the action is then given by:

$$
\begin{aligned}
\delta S &= \int_0^T dt\,(\delta\mathcal{T} - \delta\mathcal{E}) \\
&= \int_0^T dt \int_0^L ds\Big[(\rho A\ddot{\boldsymbol{x}}\cdot\delta\boldsymbol{x} - \mathbf{F}\cdot\delta\boldsymbol{x}') + (L_i\delta\omega_i - M_i\delta\kappa_i + \mathbf{F}\cdot\delta\boldsymbol{\phi}\times\mathbf{d_3}) + (\boldsymbol{x}' - \mathbf{d_3})\cdot\delta\mathbf{F}\Big] \\
&= \int_0^T dt \int_0^L ds\left[(\mathbf{F}' - \rho A\ddot{\boldsymbol{x}})\cdot\delta\boldsymbol{x} + \left(\mathbf{L}\cdot\delta\dot{\boldsymbol{\phi}} - \mathbf{M}\cdot\delta\boldsymbol{\phi}' + \mathbf{d_3}\times\mathbf{F}\cdot\delta\boldsymbol{\phi}\right) + (\boldsymbol{x}' - \mathbf{d_3})\cdot\delta\mathbf{F}\right] \\
&= \int_0^T dt \int_0^L ds\left[(\mathbf{F}' - \rho A\ddot{\boldsymbol{x}})\cdot\delta\boldsymbol{x} + \left(-\dot{\mathbf{L}} + \mathbf{M}' + \mathbf{d_3}\times\mathbf{F}\right)\cdot\delta\boldsymbol{\phi} + (\boldsymbol{x}' - \mathbf{d_3})\cdot\delta\mathbf{F}\right]
\end{aligned}
$$

Setting $\delta S = 0$ yields the constraint $\boldsymbol{x}' = \mathbf{d_3}$ and the Kirchhoff equations:

$$
\mathbf{F}' = \rho A\ddot{\boldsymbol{x}}
$$

$$
\mathbf{M}' + \mathbf{d_3}\times\mathbf{F} = \dot{\mathbf{L}}
$$

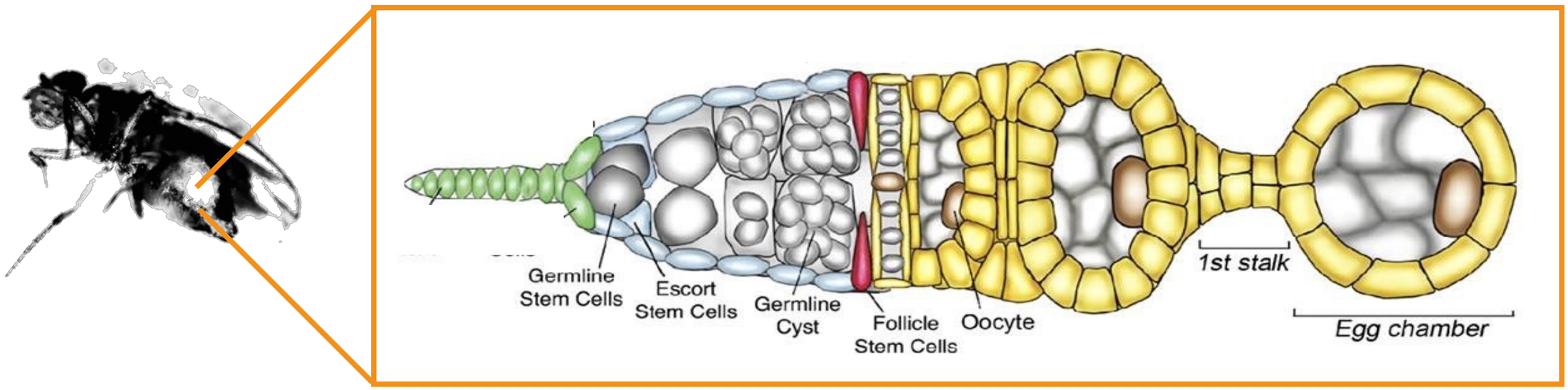




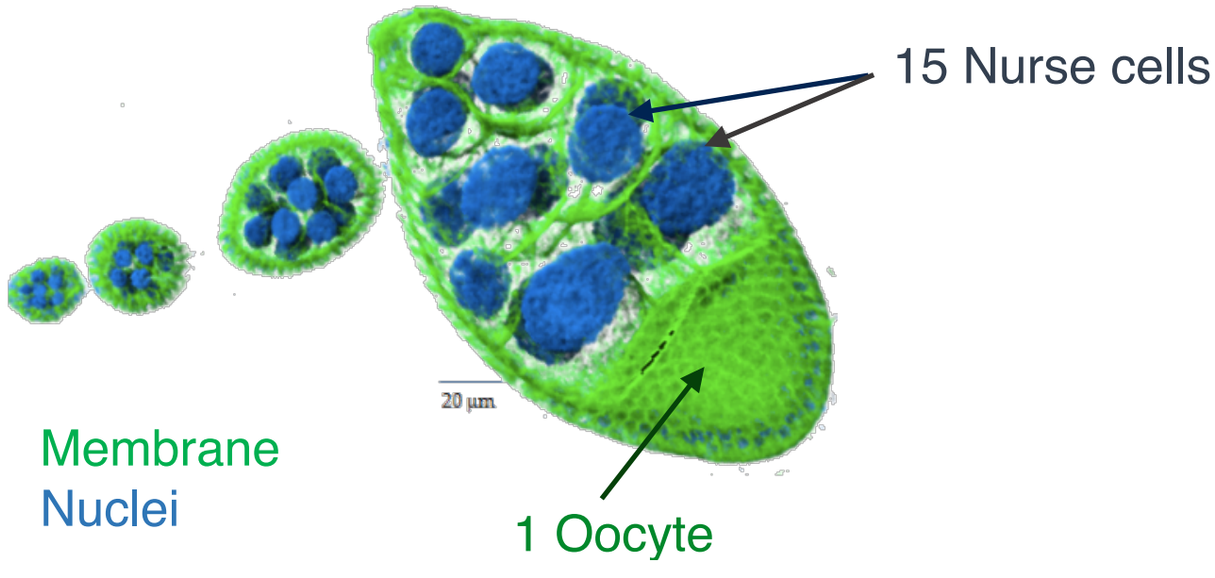
How do cell-lineage trees pack ?

In collaboration with Stas Shvartsman's lab (Princeton)





Hartmann et al, J. Cell. Biol. (2010)

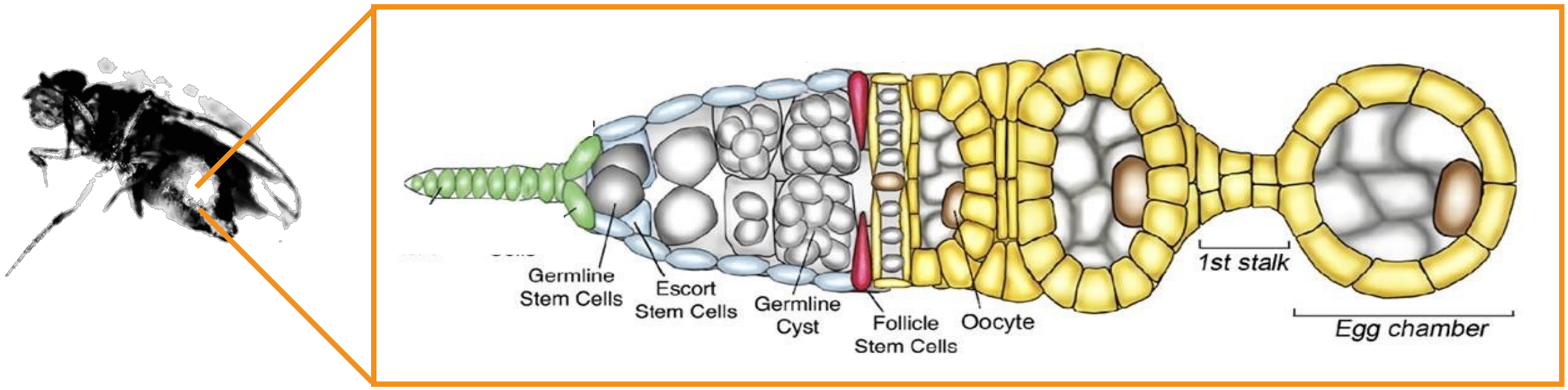




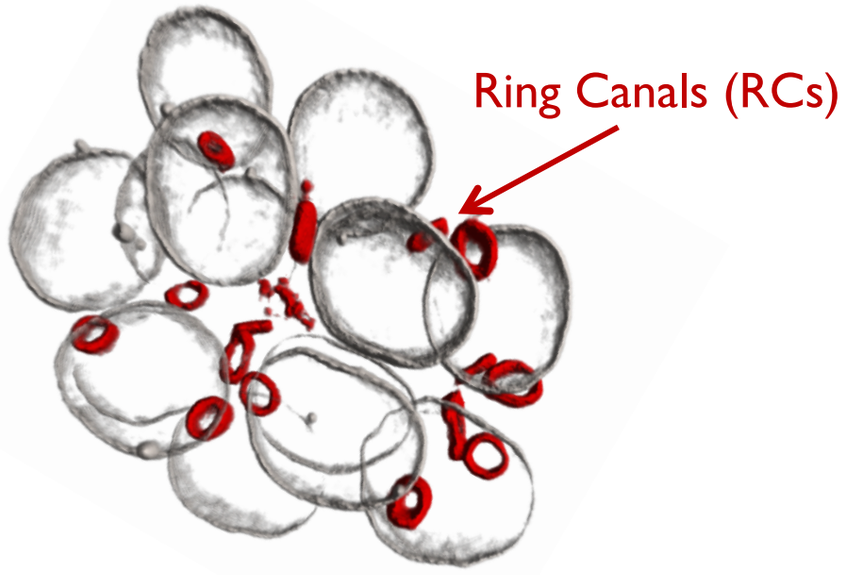
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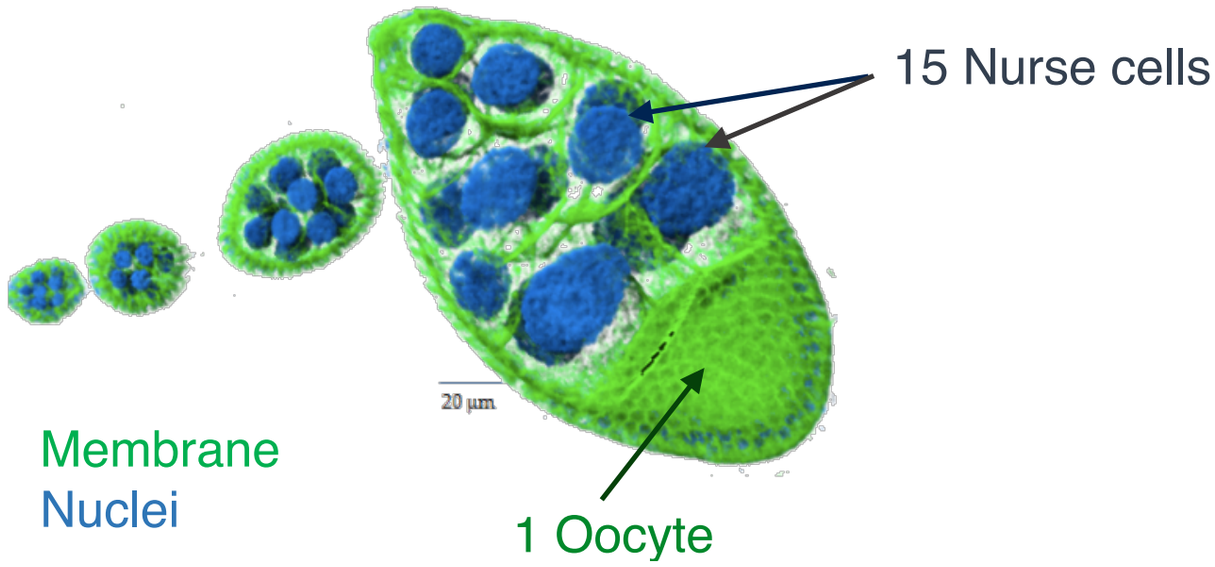
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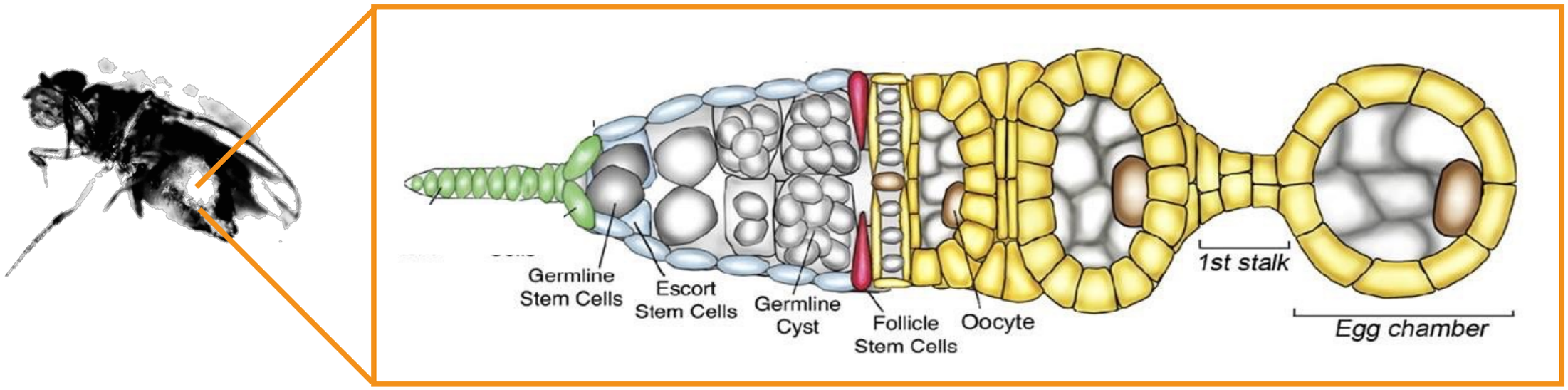


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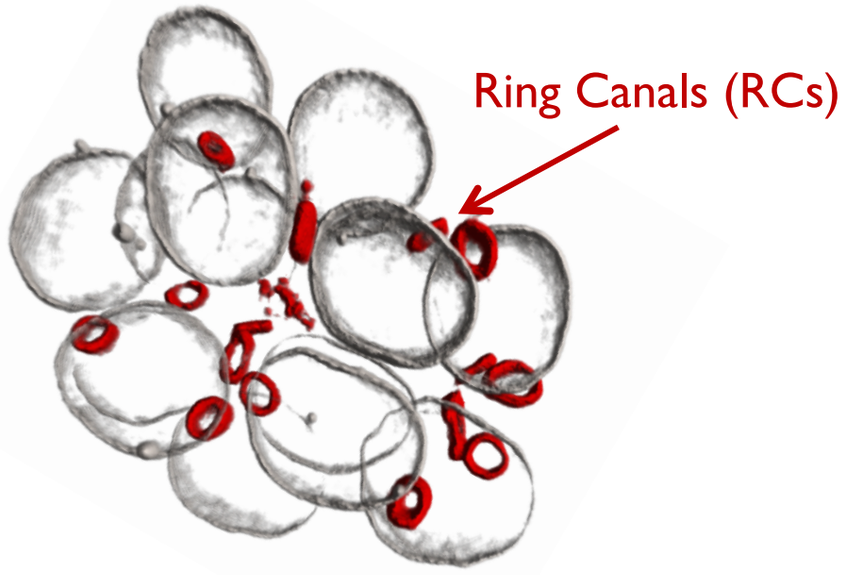
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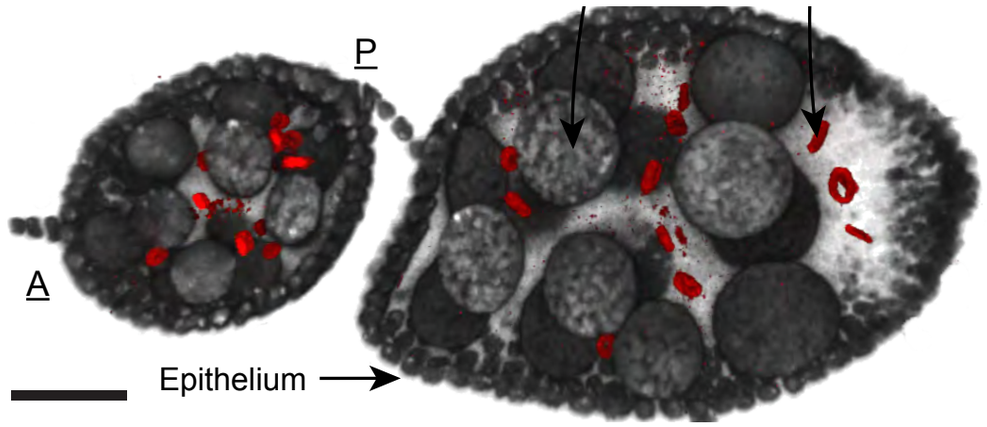


Incomplete cell division in early oogenesis/embryogenesis of many organisms leads to generic constrained tree-packing problem



Hartmann et al, J. Cell. Biol. (2010)

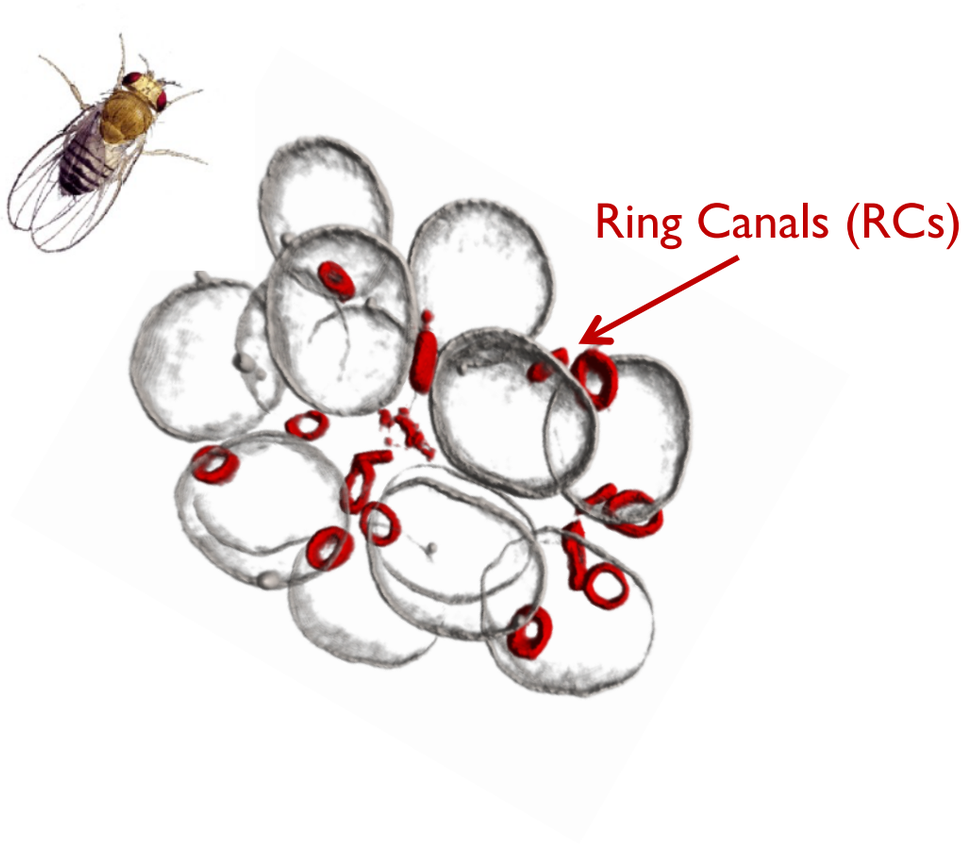


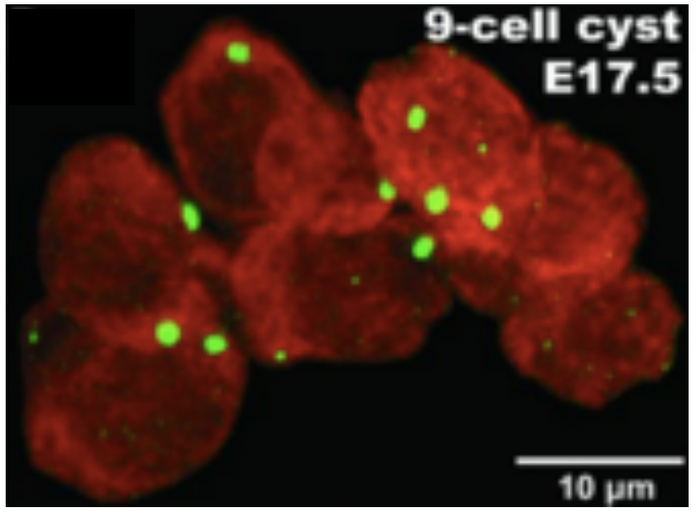


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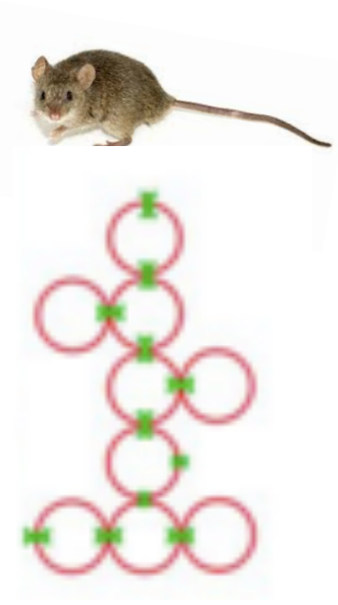
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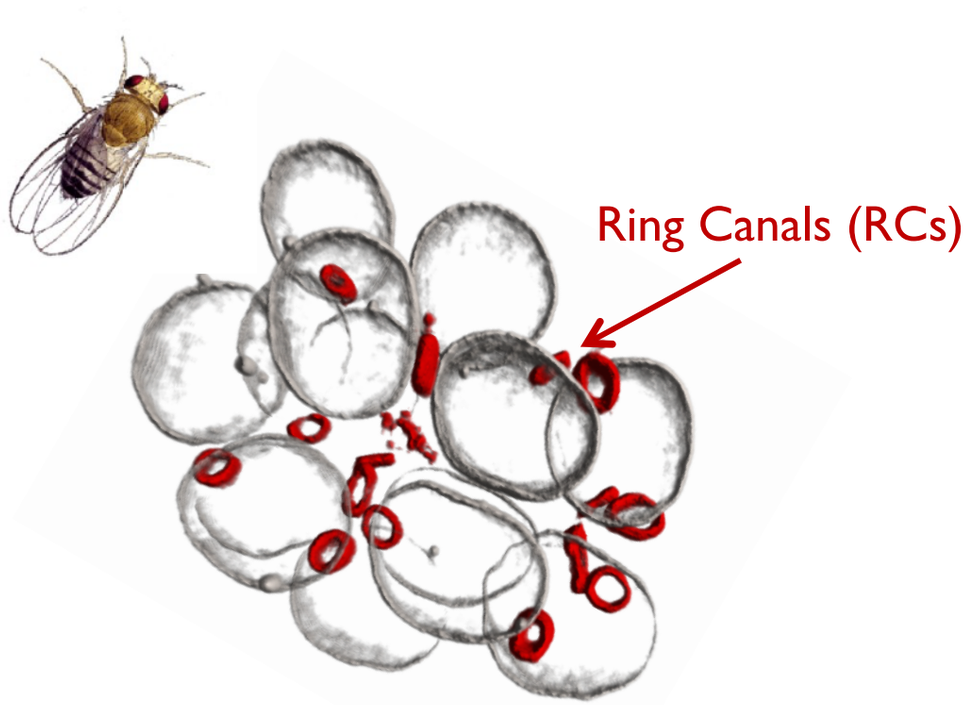
Lei & Spradling, Science (2016)



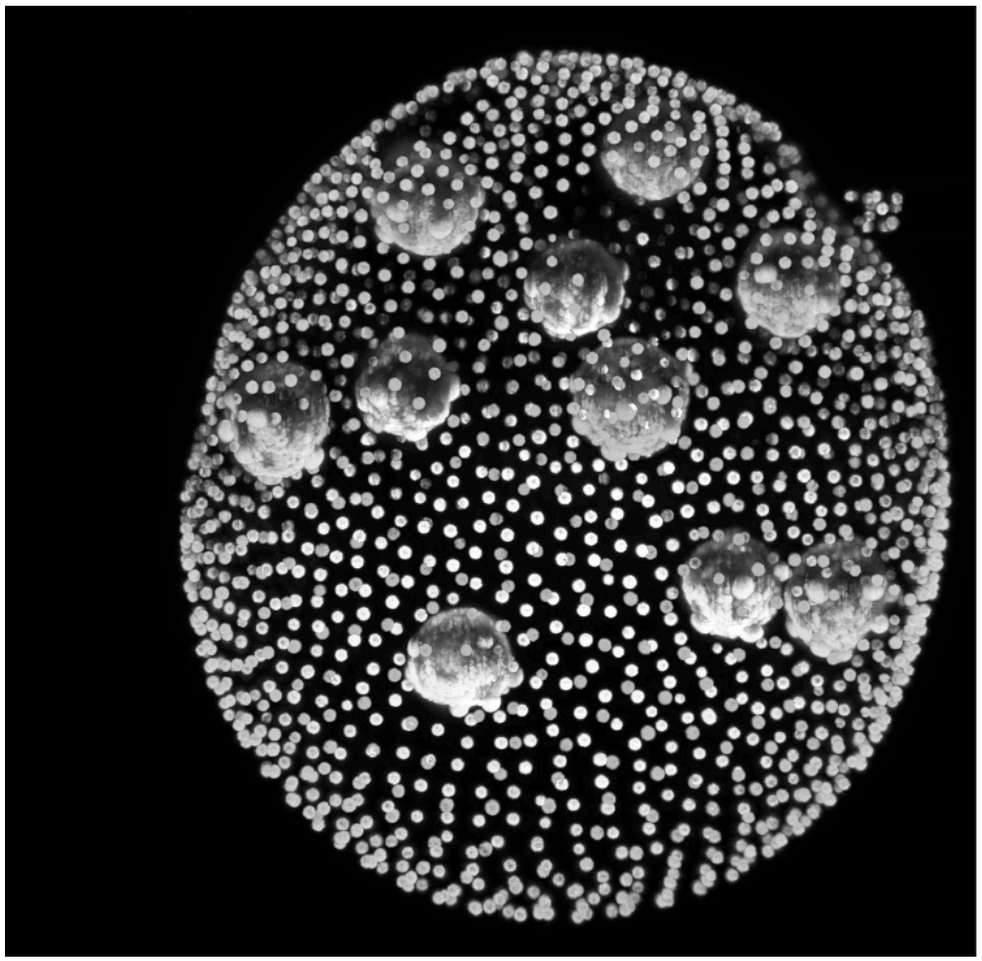
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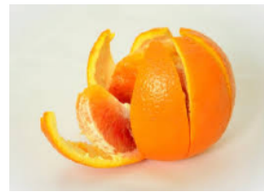
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Goldstein lab, Cambridge

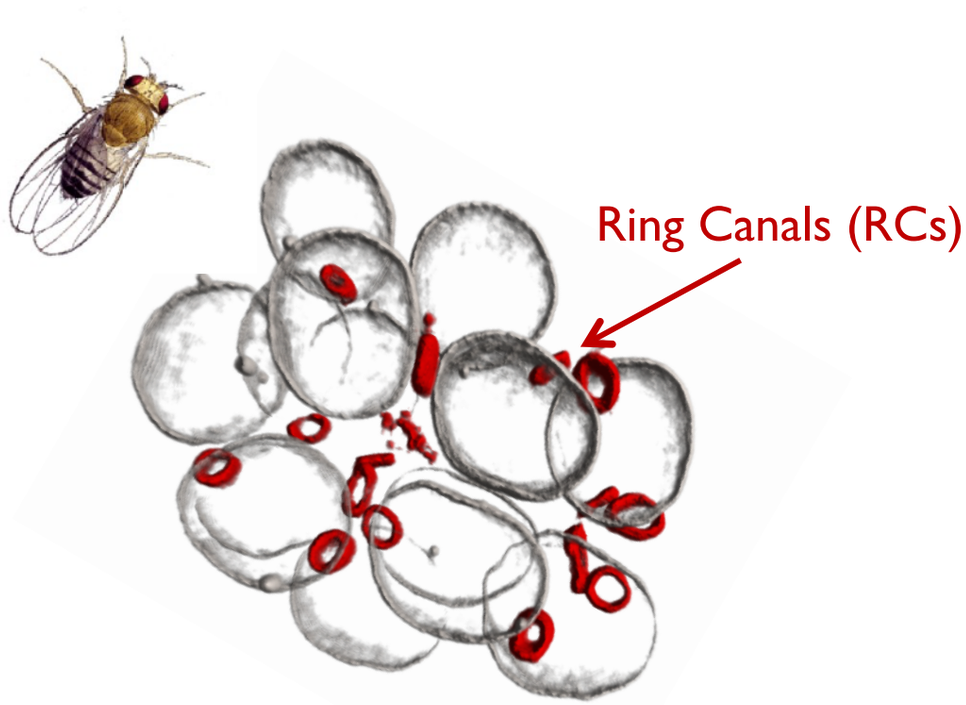




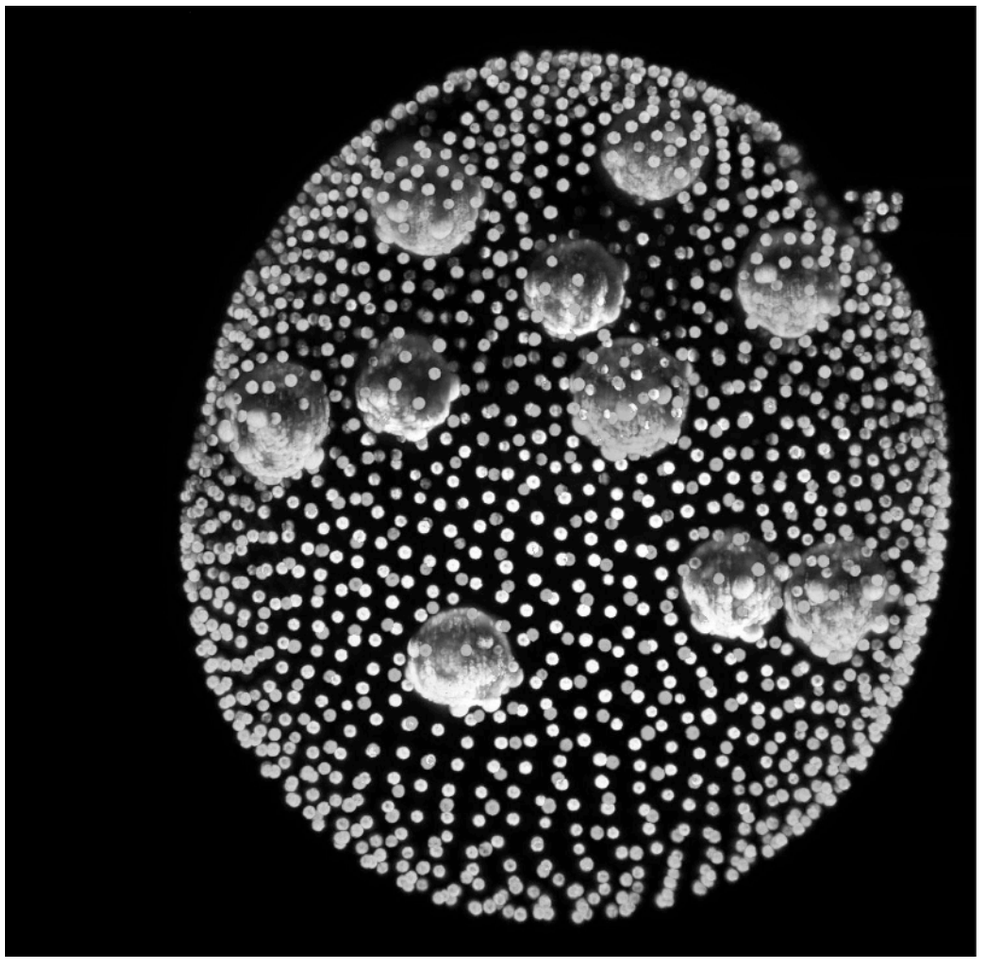
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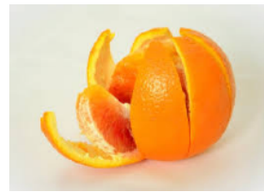
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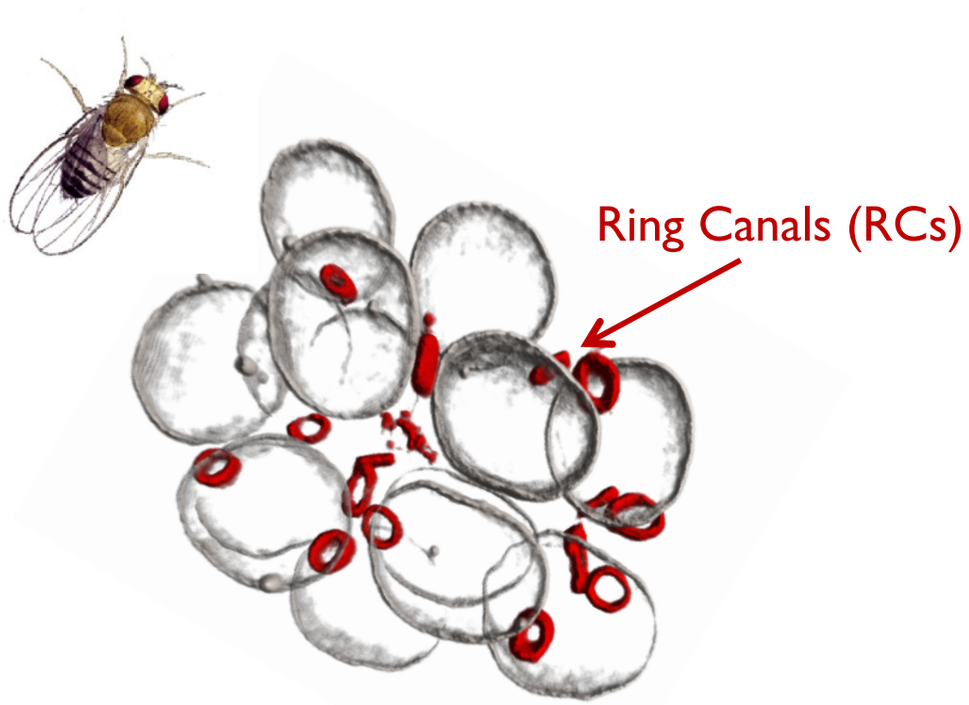




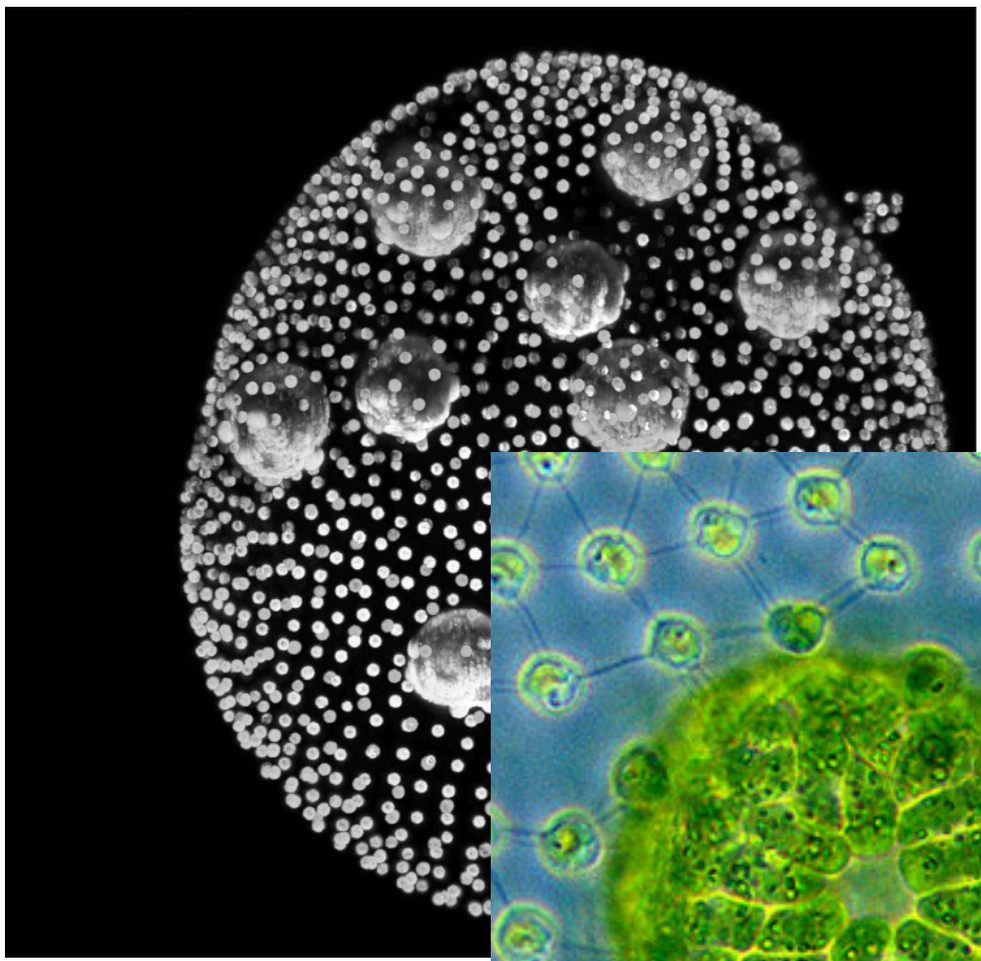
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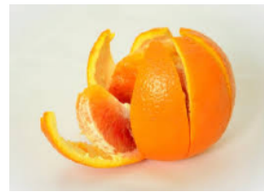
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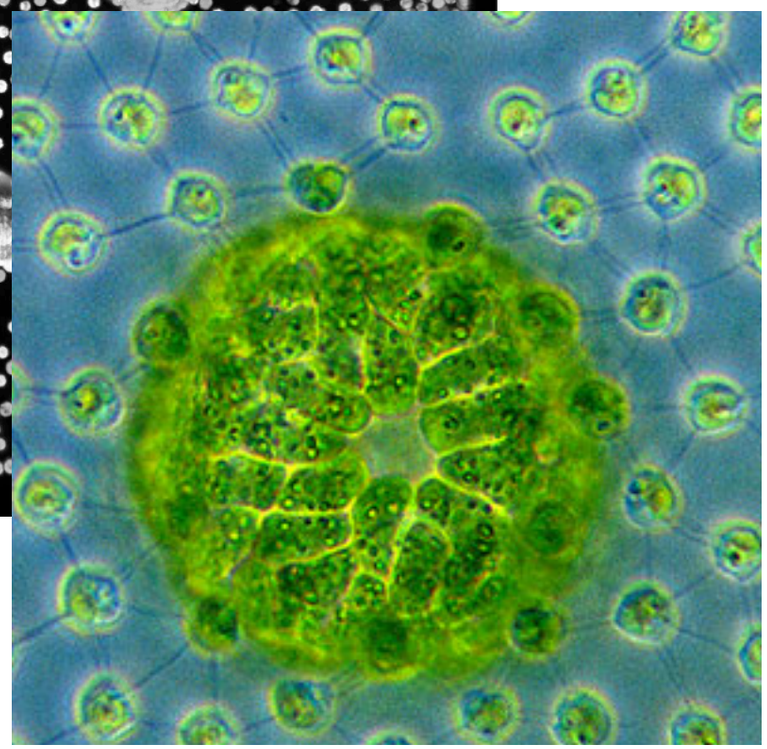
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Goldstein lab, Cambridge





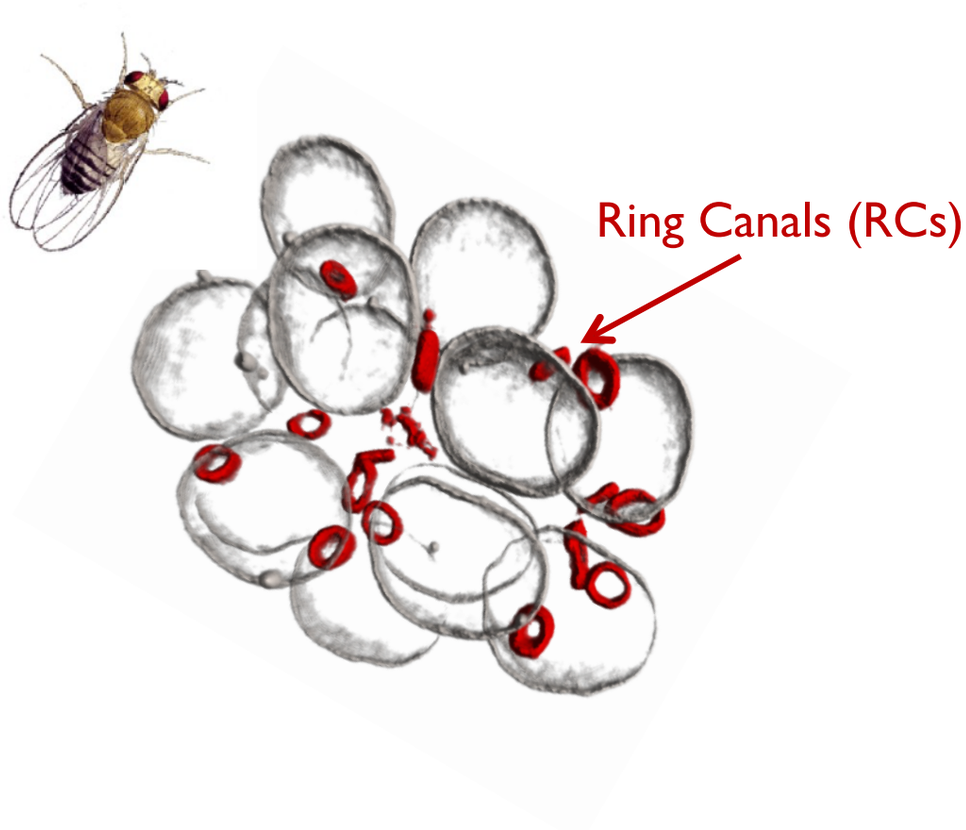


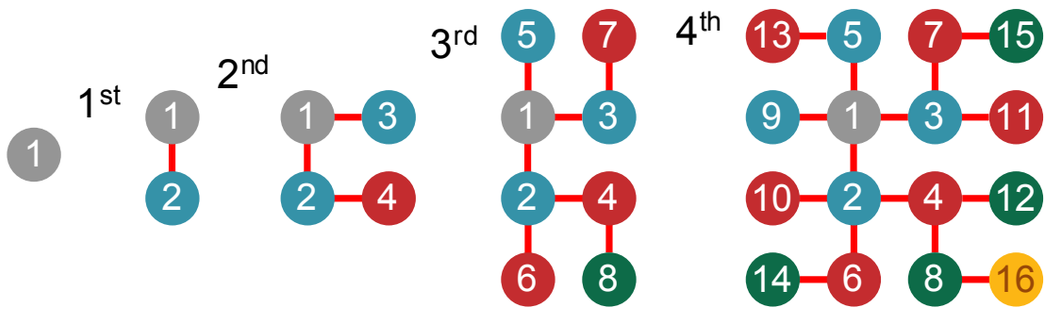
Wim van Egmond

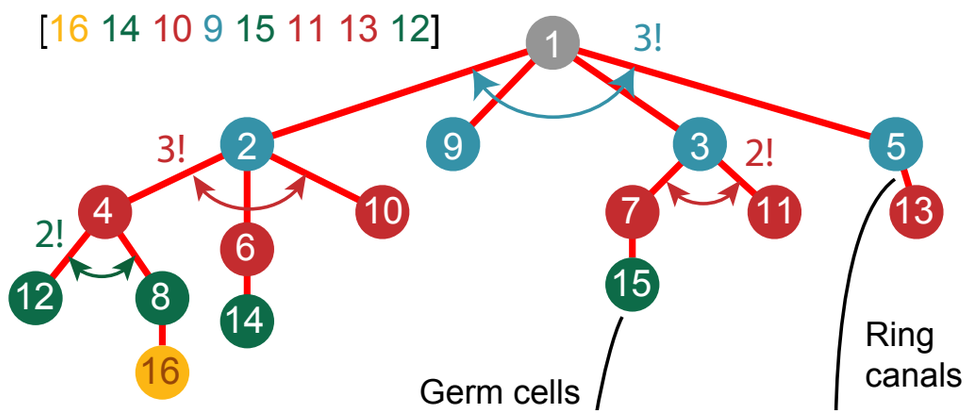
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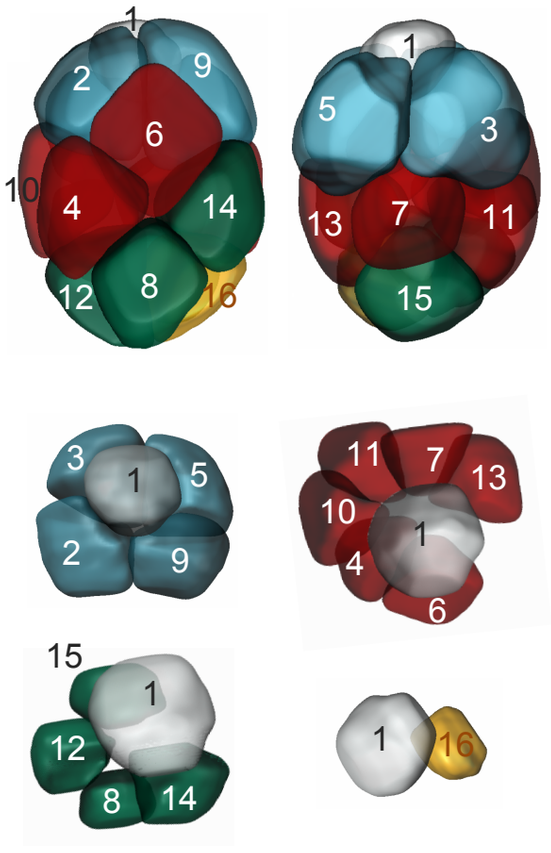


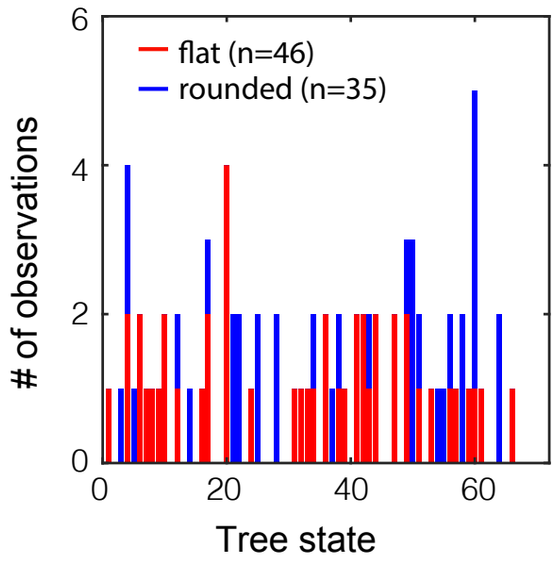


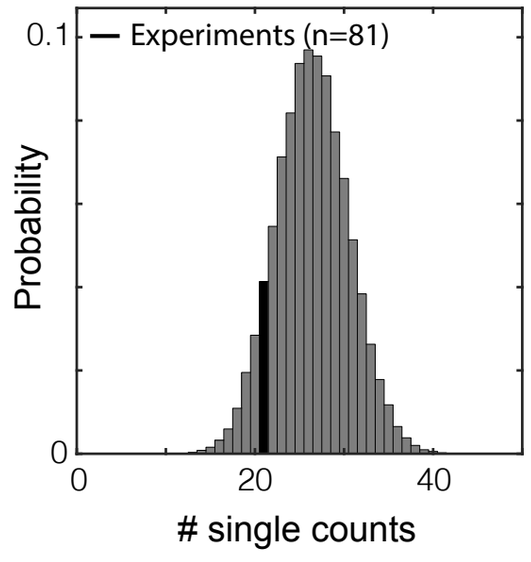


2 x 72 tree configurations

Jasmin Imran Alsous *et al*, under review (2017)



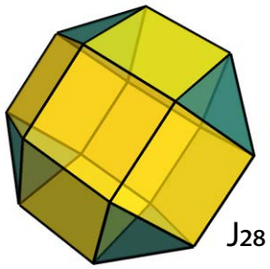




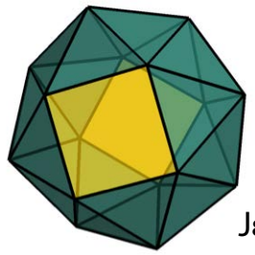
data suggest non-uniform distribution over tree topologies …

4 Johnson solids
with 16 vertices

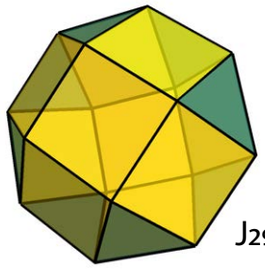
favored by adjacency data



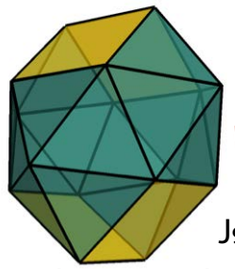
J28
Square Orthobicupola



J85
Snub Square Antiprism

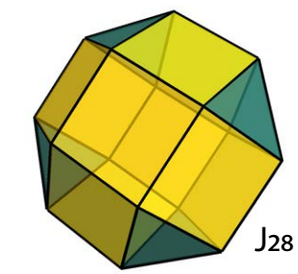


J29
Square Gyrobicupola

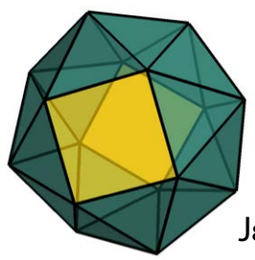


J90
Disphenocingulum

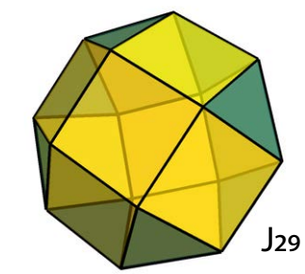
4 Johnson solids
with 16 vertices



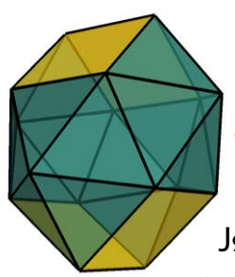
Square Orthobicupola



Snub Square Antiprism

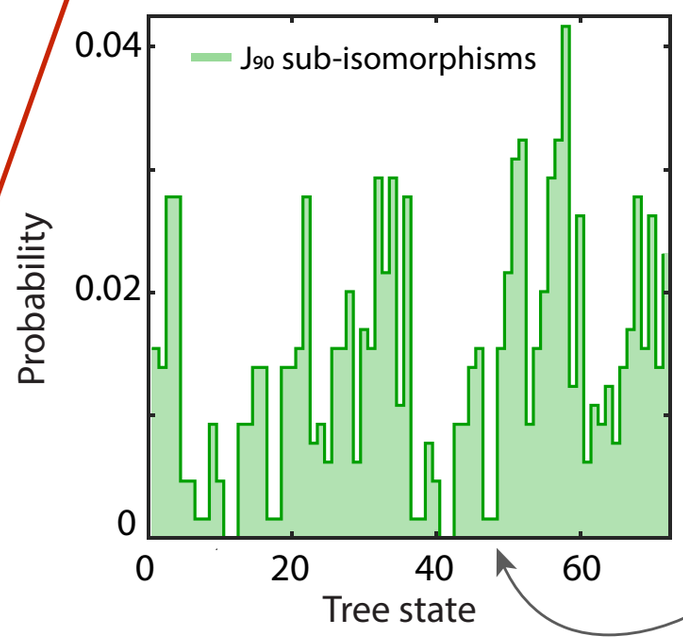


Square Gyrobicupola

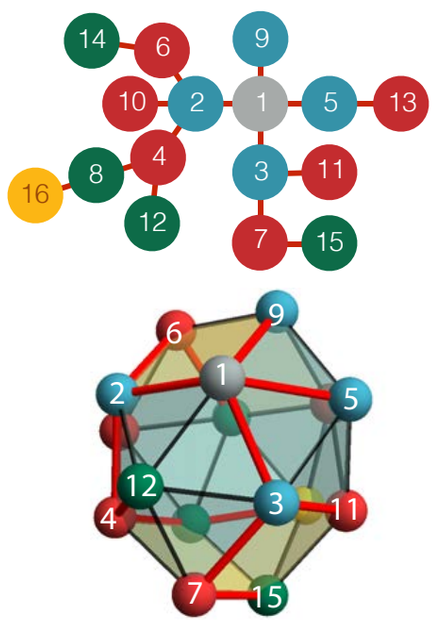


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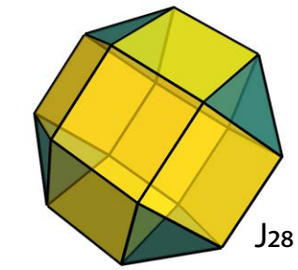
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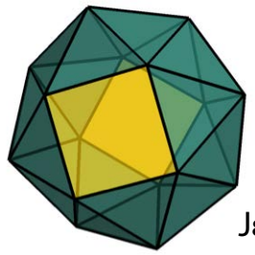
State 48 (1 isomorphism)
[16, 12, 15, 11, 13, 9, 14, 10]



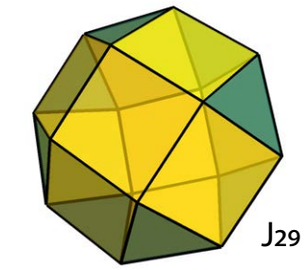
4 Johnson solids with 16 vertices



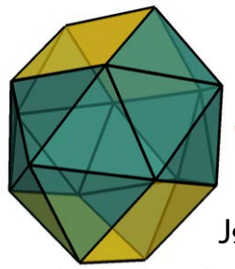
Square Orthobicupola



Snub Square Antiprism

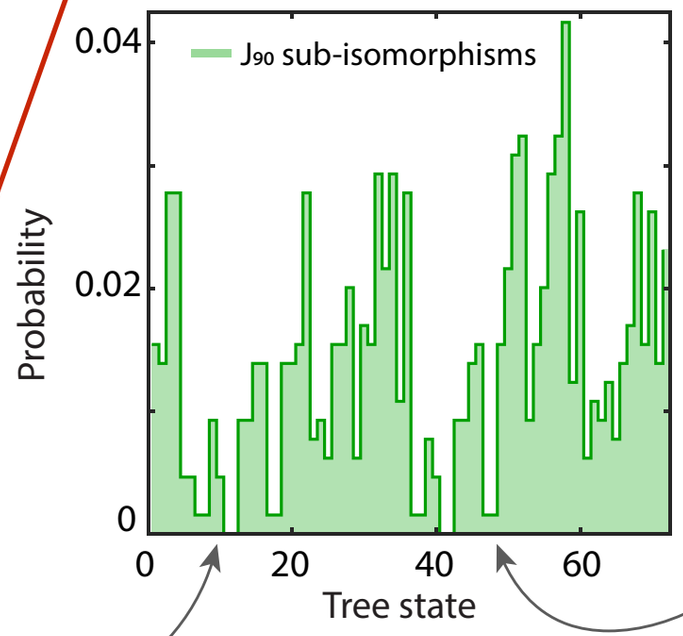


Square Gyrobicupola



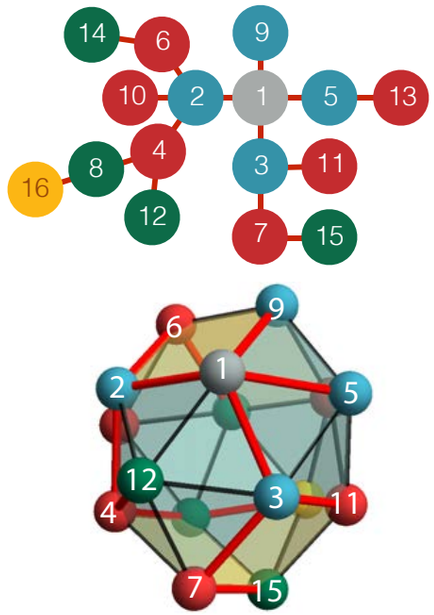
Disphenocingulum

favored by adjacency data



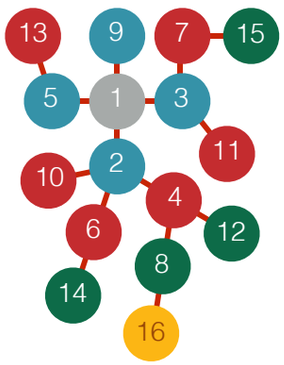
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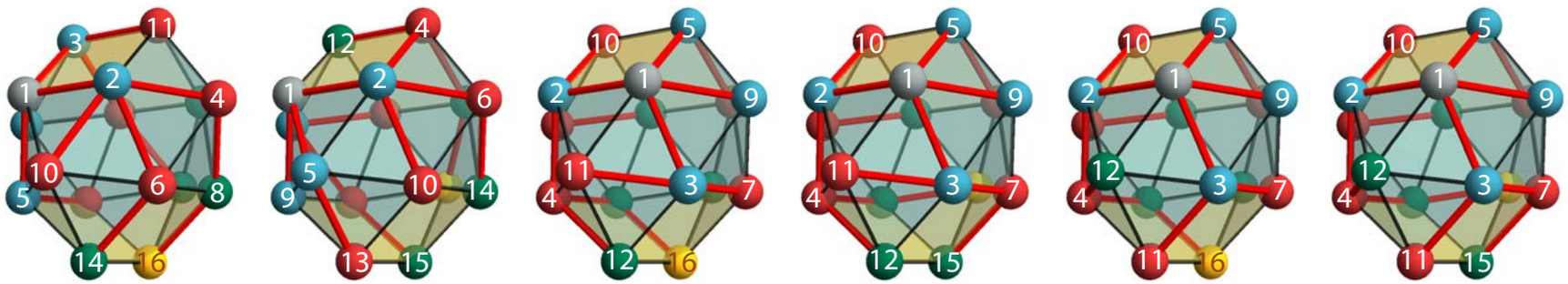
[16, 12, 15, 11, 13, 9, 14, 10]



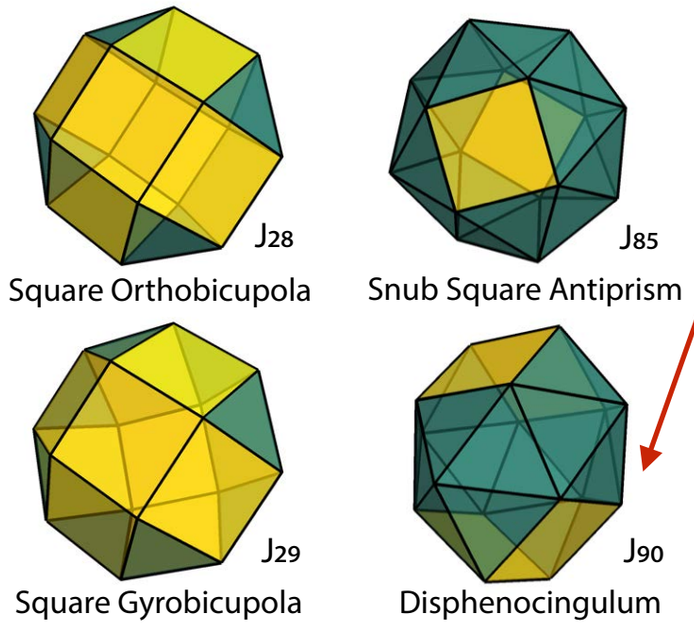
State 13 (6 isomorphisms)

[16, 12, 11, 15, 9, 13, 10, 14]

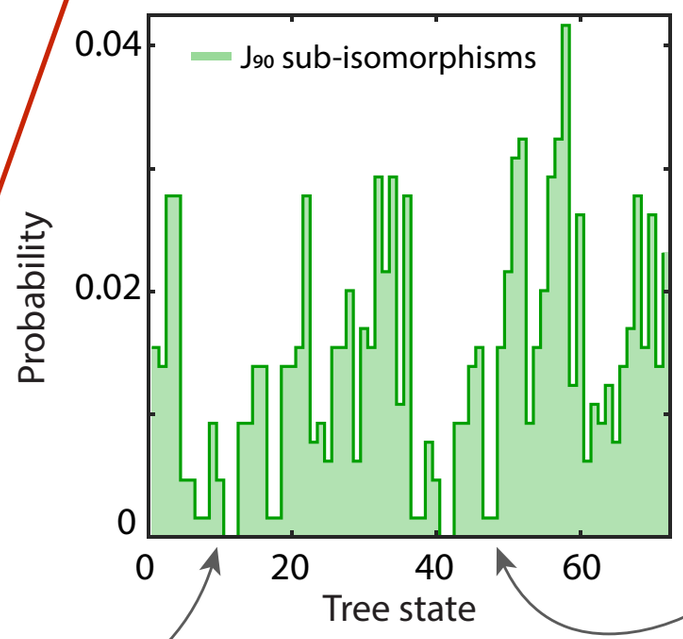




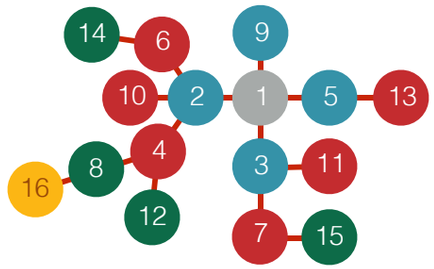
4 Johnson solids with 16 vertices

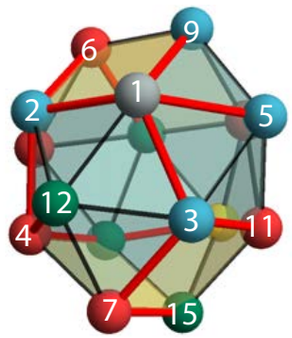


favored by adjacency data

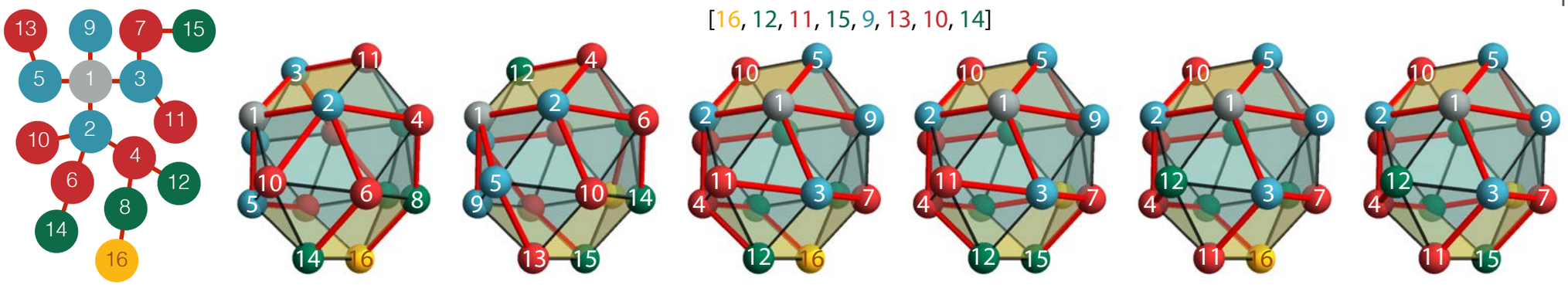


State 48 (1 isomorphism)
[16, 12, 15, 11, 13, 9, 14, 10]





State 13 (6 isomorphisms)
[16, 12, 11, 15, 9, 13, 10, 14]



non-uniform distribution expected based on counting arguments

Part II:

How do tissues fold during embryogenesis ?

Start with ‘smart’ golf balls





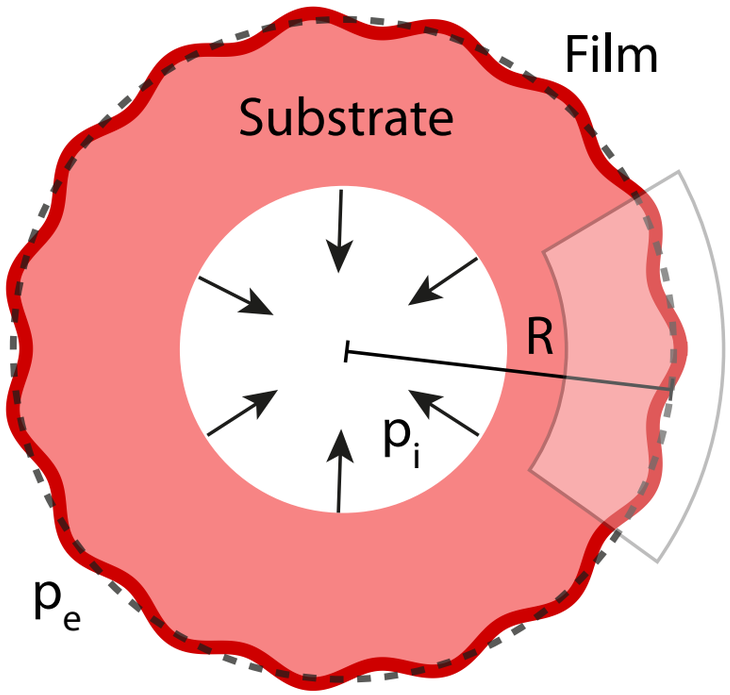




Start with ‘smart’ golf balls









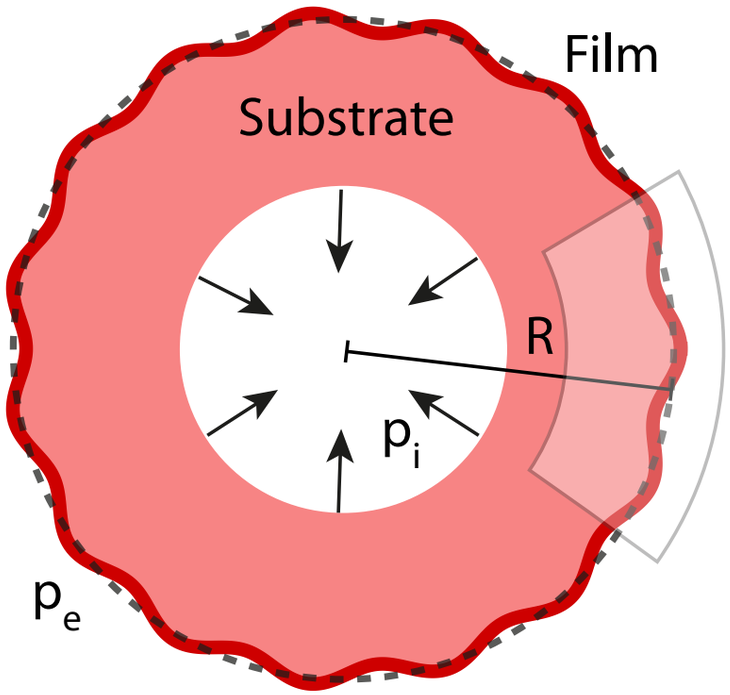




Start with ‘smart’ golf balls





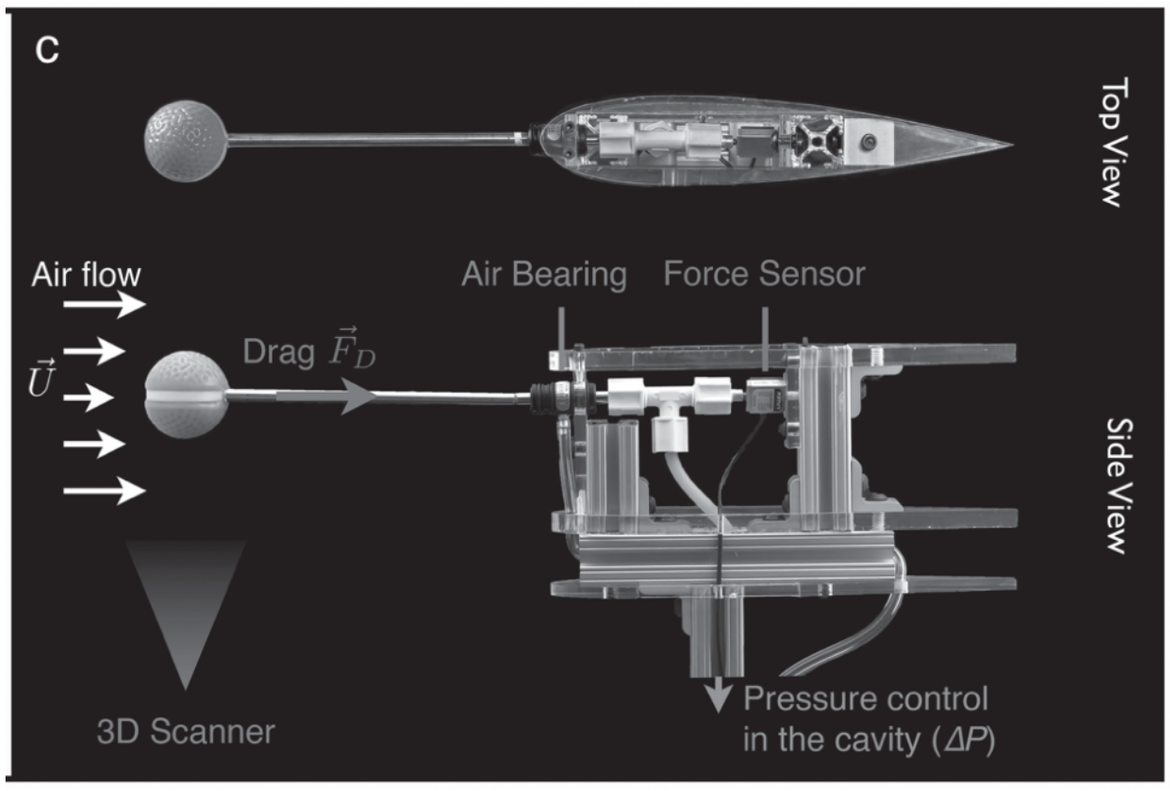


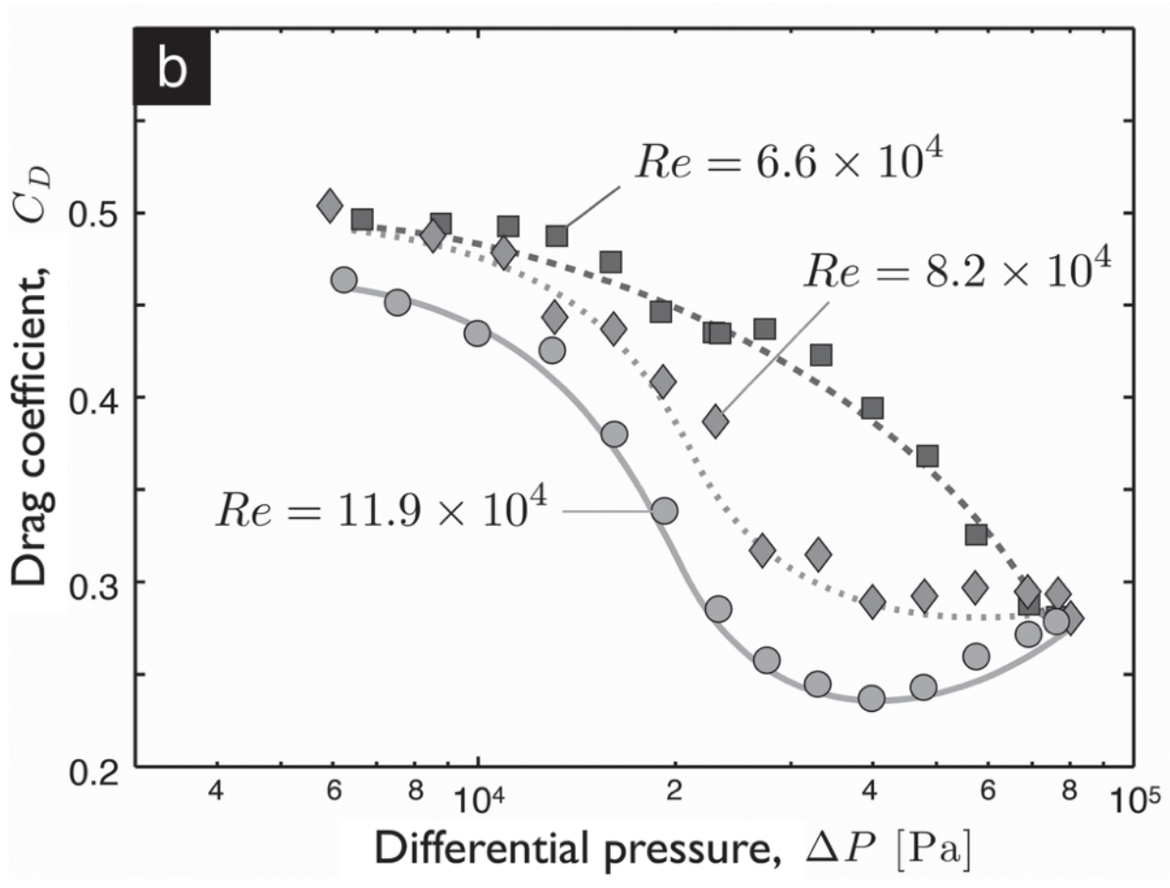




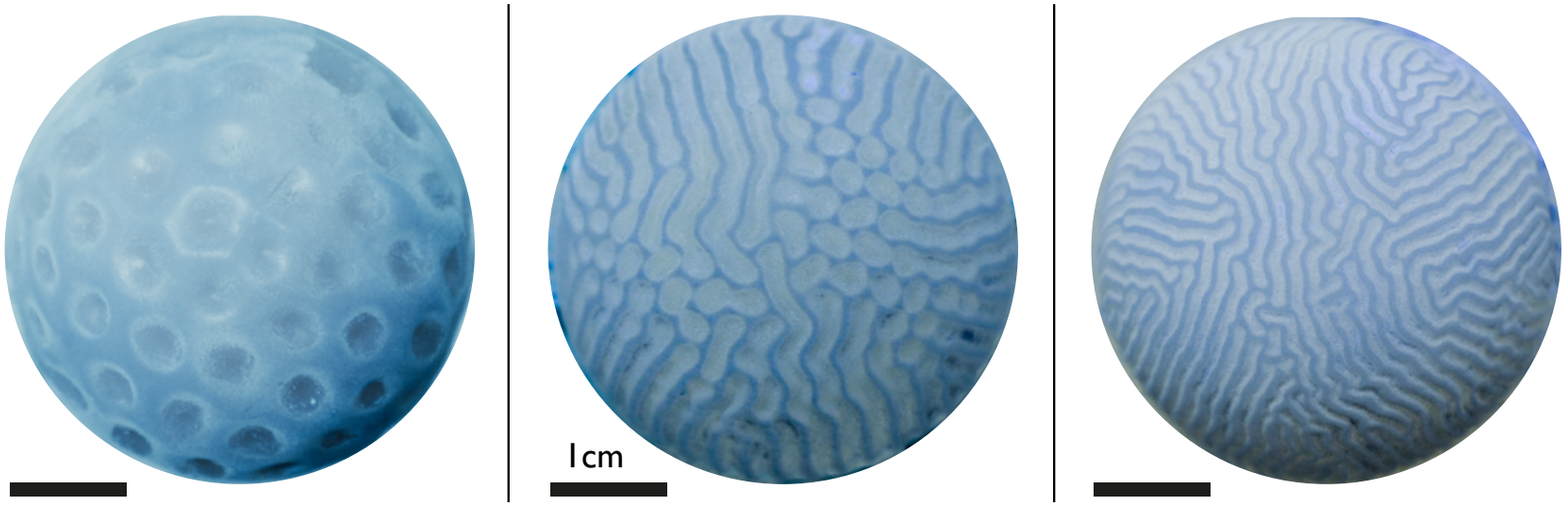


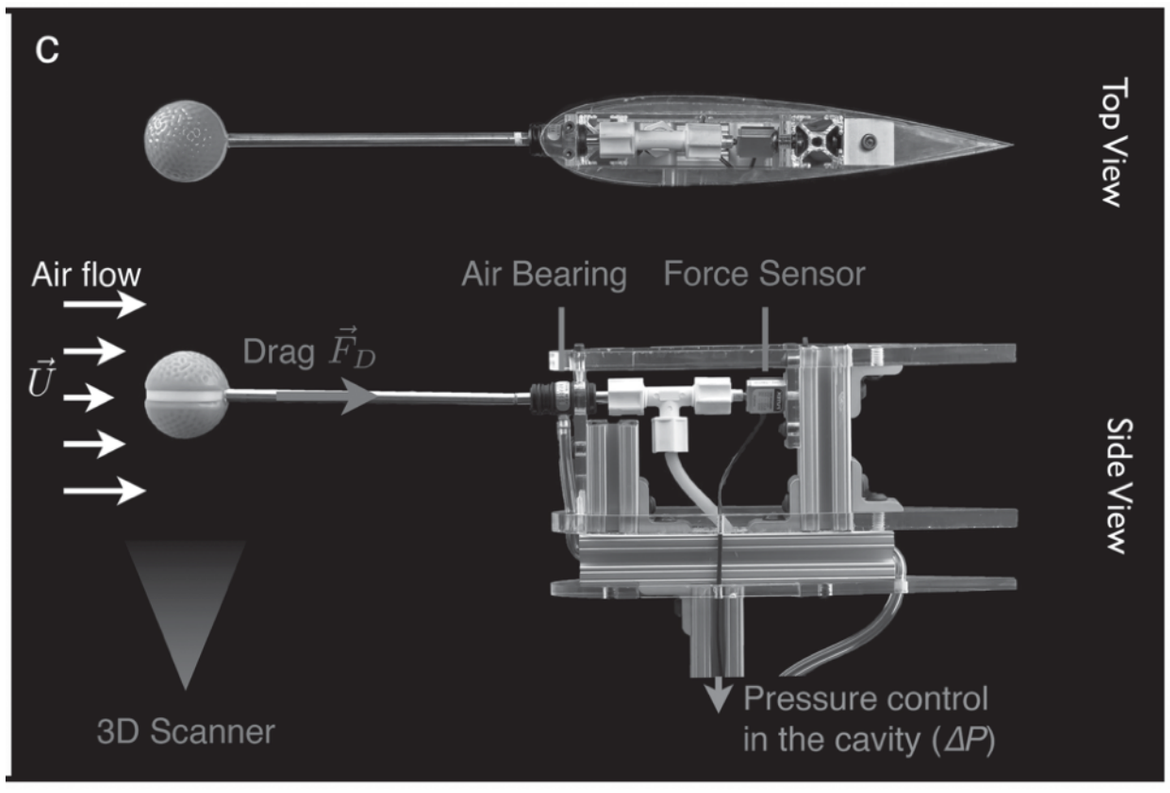


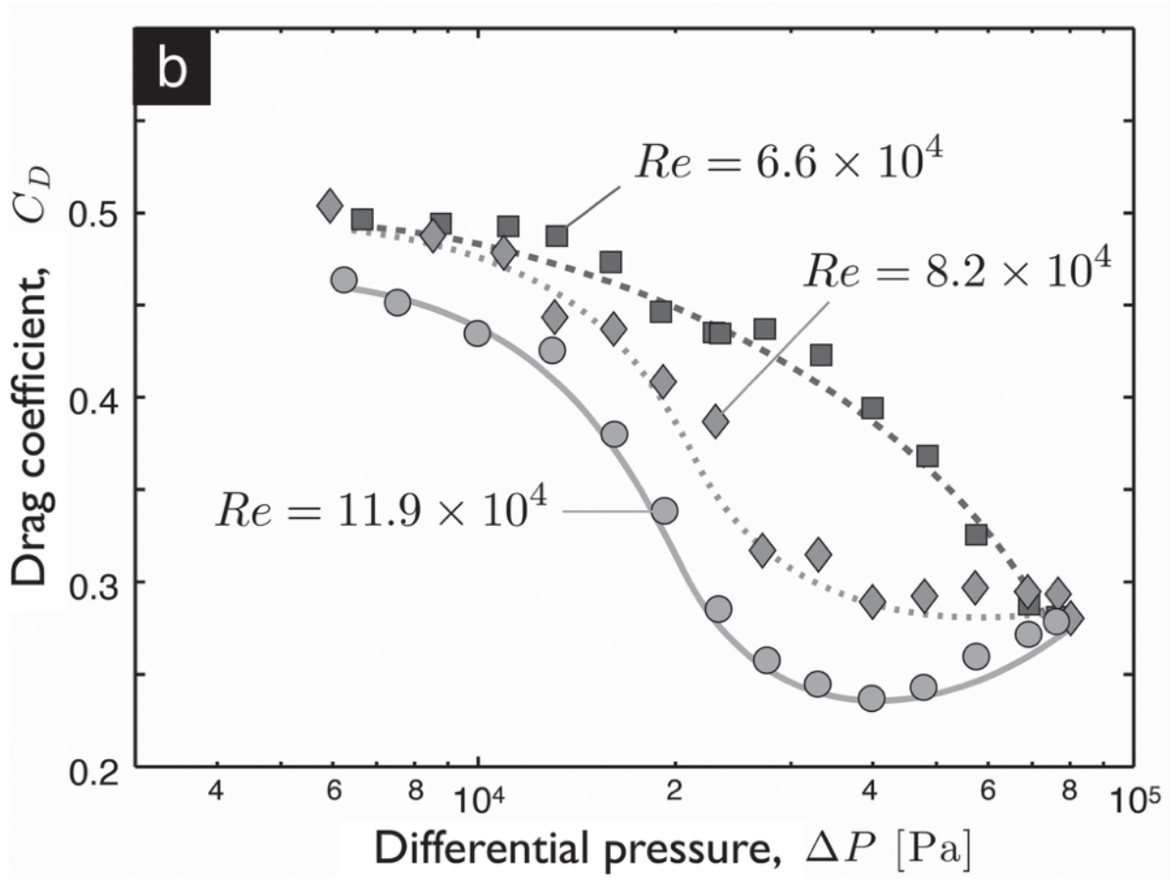




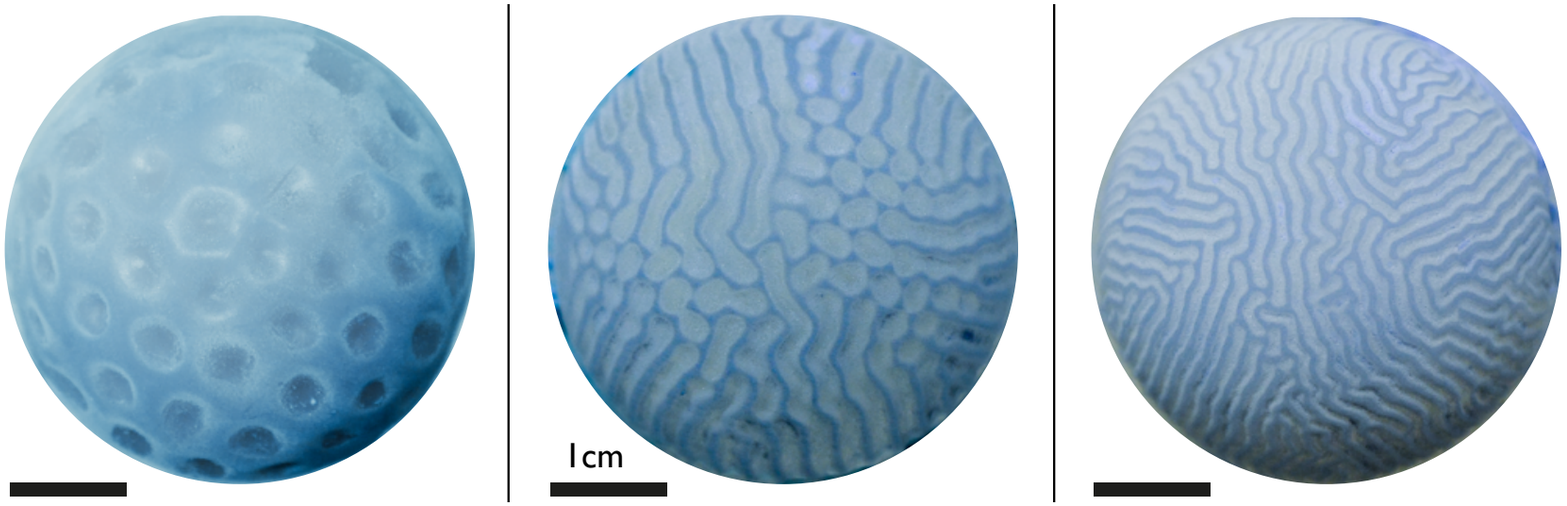
D Terwagne, M Brojan and P M Reis "Smart Morphable Surfaces for Aerodynamic Drag Control" Adv. Mater. 26:38, 6608 (2014)



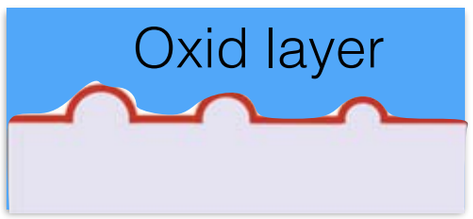


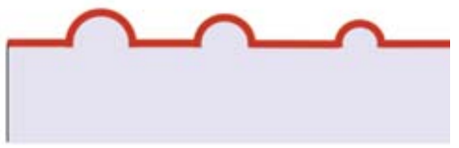


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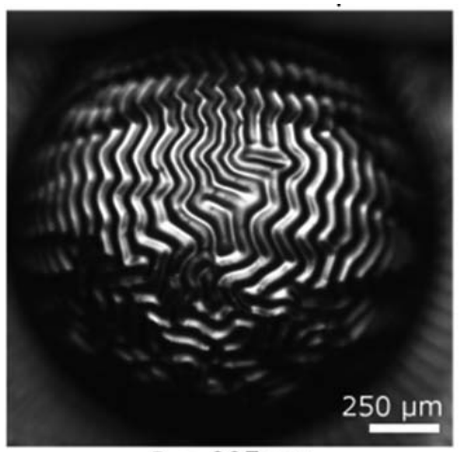




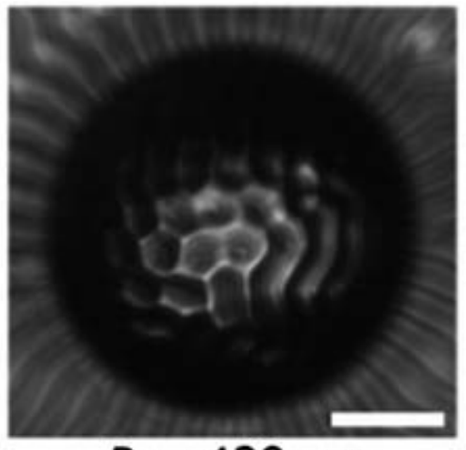




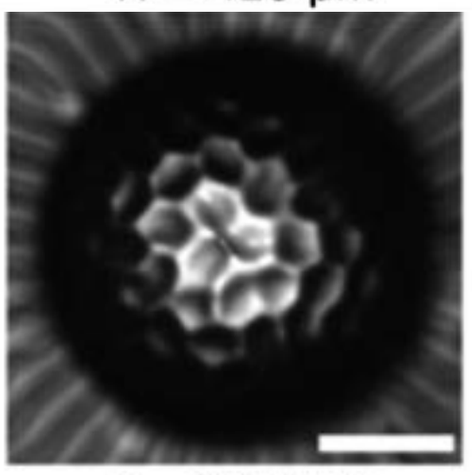
Breid & Crosby (2013) Soft Matter



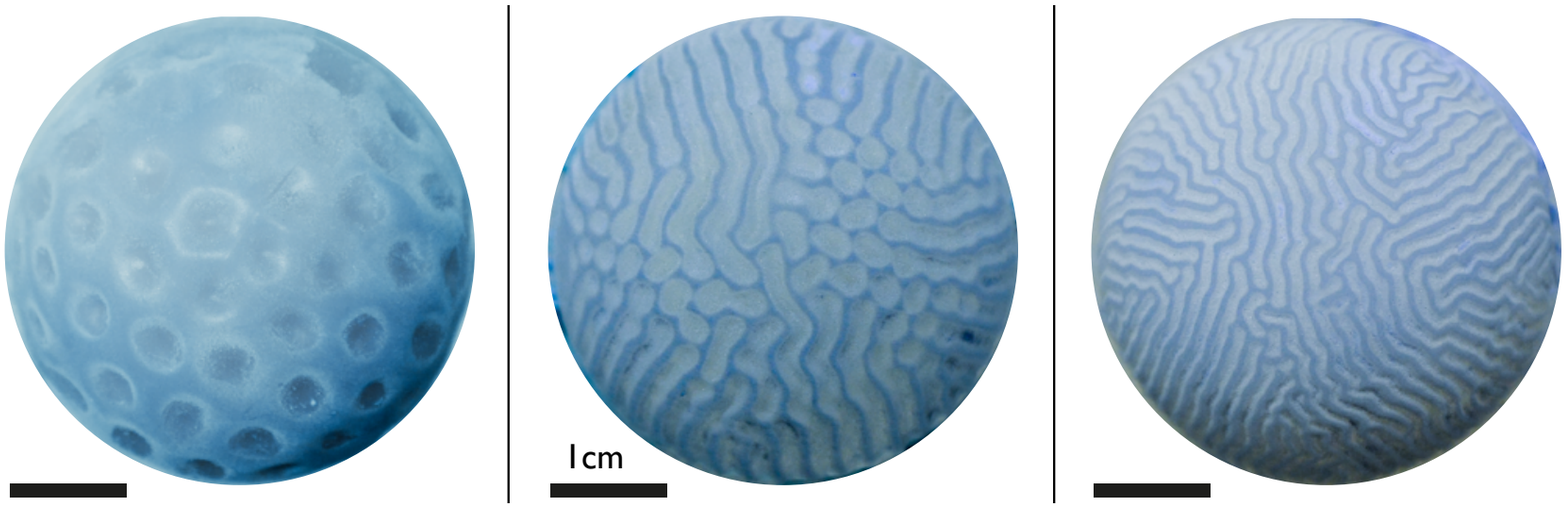
R = 805 μm

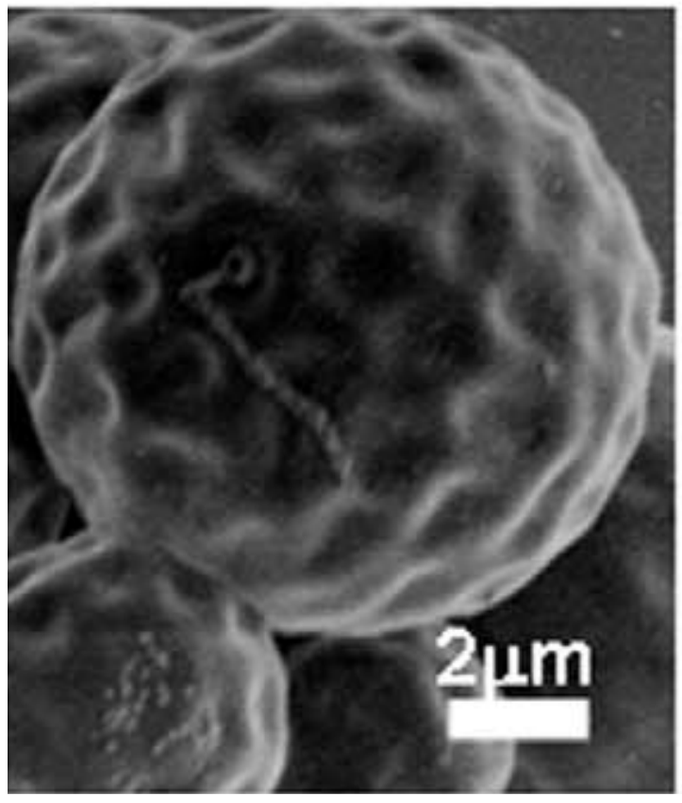


R = 428 μm

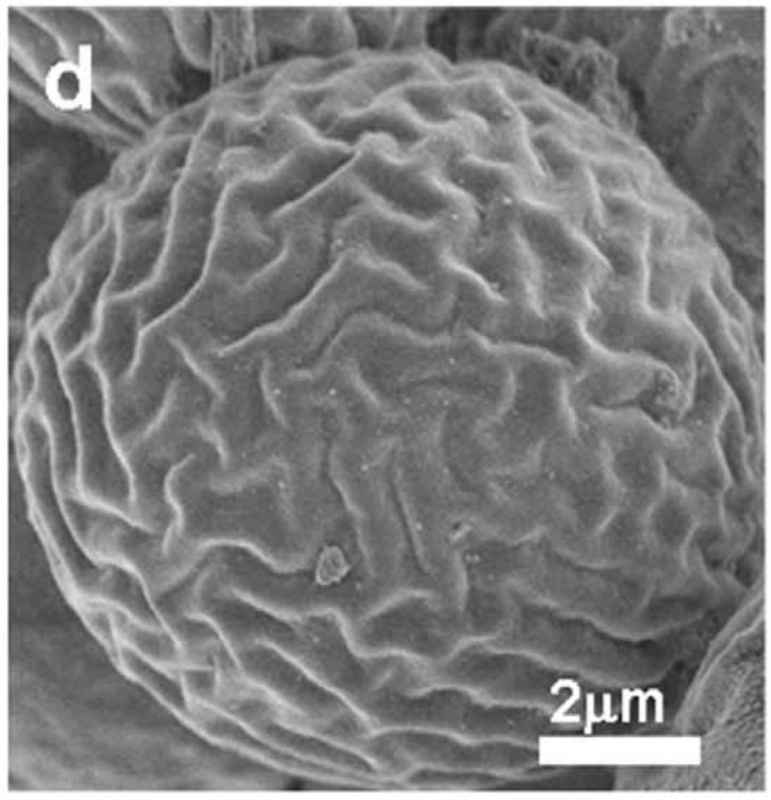


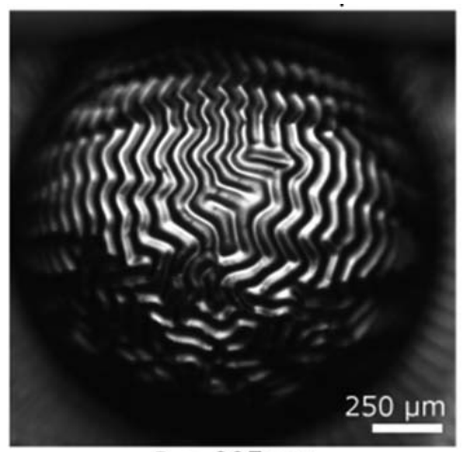
R=381 μm



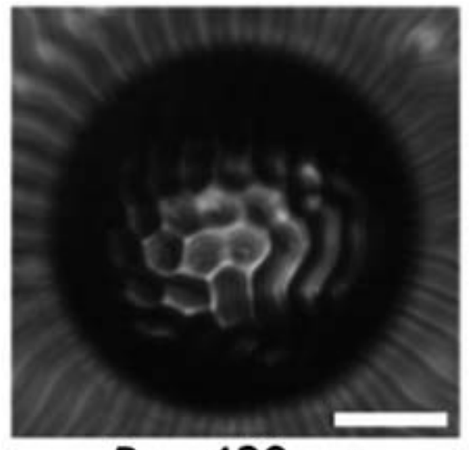


Yin et al (2014) Sci Rep

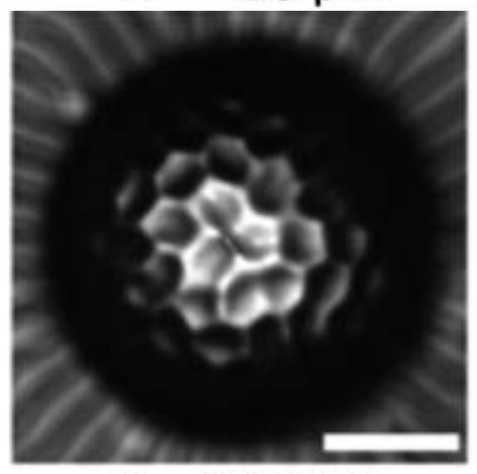




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R = 428 μm



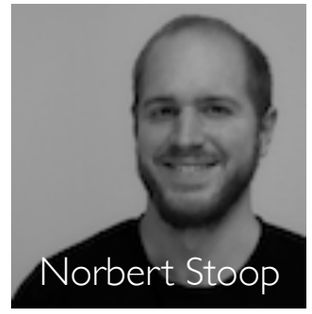
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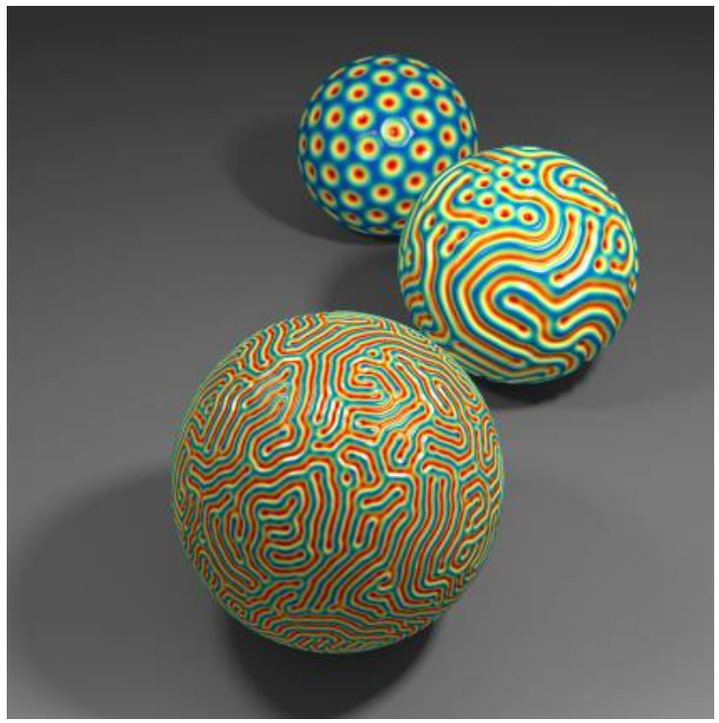
Breid & Crosby (2013) Soft Matter

5 orders of magnitude in length … similar patterns & transitions

Mechanical pattern formation on curved surfaces

$$
\begin{aligned}
\mu\partial_t u = \; & \gamma_0 \triangle u - \gamma_2 \triangle^2 u - au - bu^2 - cu^3 + \\
& \frac{h}{2}\left\{(\nu-1)\left[b^{\alpha\beta}\nabla_\alpha u \nabla_\beta u + 2u\nabla_\beta\left(b^{\alpha\beta}\nabla_\alpha u\right)\right] + \right. \\
& \left. 2\nu\left[\mathcal{H}(\nabla u)^2 - 2\nabla\cdot(\mathcal{H}u\nabla u)\right]\right\} + \\
& \frac{h}{2}\left[(1-\nu)u\nabla_\beta\left(uc^{\alpha\beta}\nabla_\alpha u\right) - \nu\mathcal{R}u(\nabla u)^2 + \right. \\
& \left. \nu\nabla\cdot(\mathcal{R}u^2\nabla u)\right]
\end{aligned}
$$







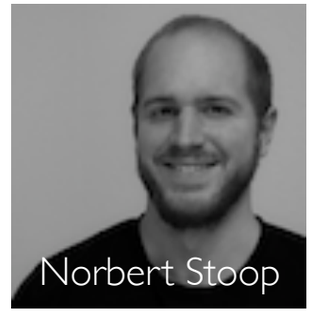


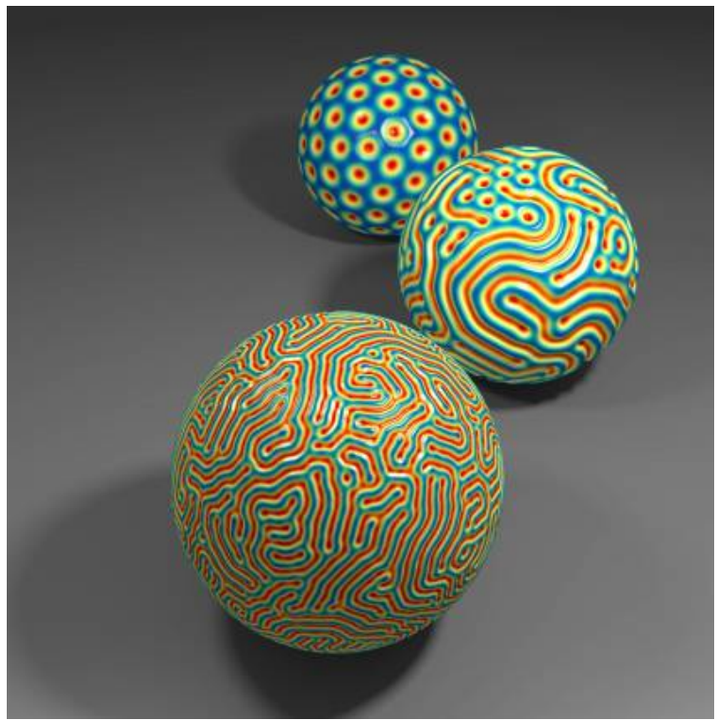


Nature Materials 2015
PRL 2016

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& \left. \nu\nabla\cdot(\mathcal{R}u^2\nabla u)\right]
\end{aligned}
$$











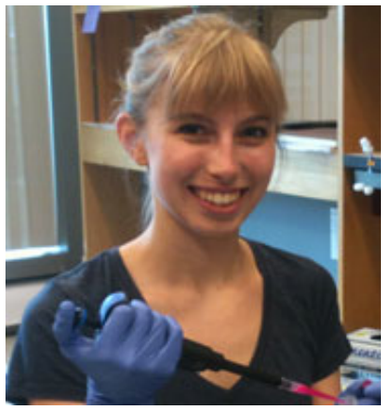
Nature Materials 2015
PRL 2016

Tissue folding in fly embryos

In collaboration with Adam Martin's lab (MIT Biology)







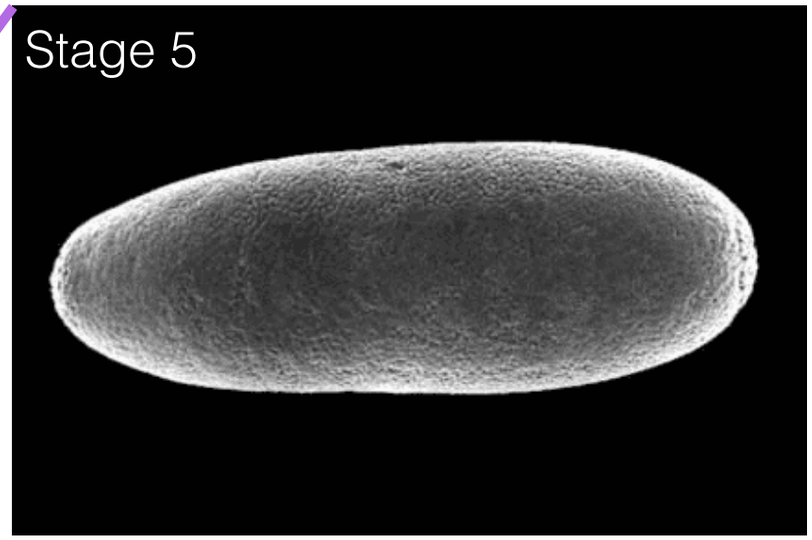
Natalie Heer



Tissue folding in fly embryos

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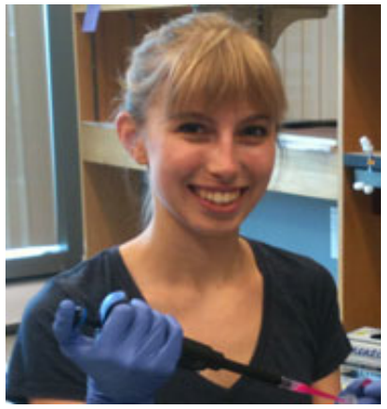






flybase.org

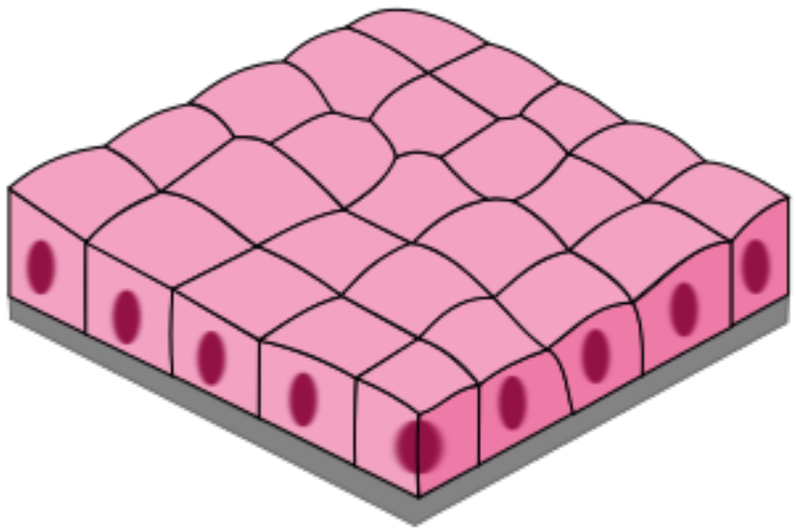




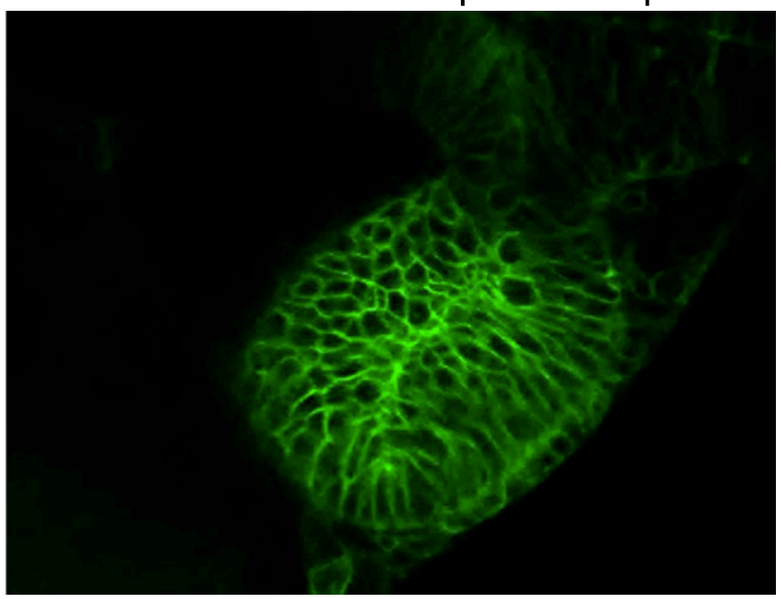
Natalie Heer



Folding of epithelial sheets widespread

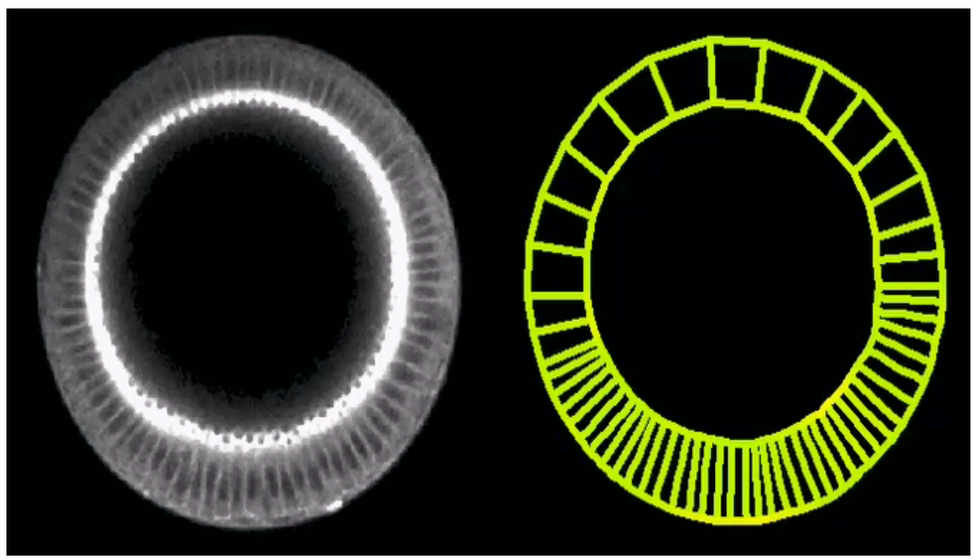


Zebra fish: Optic cup



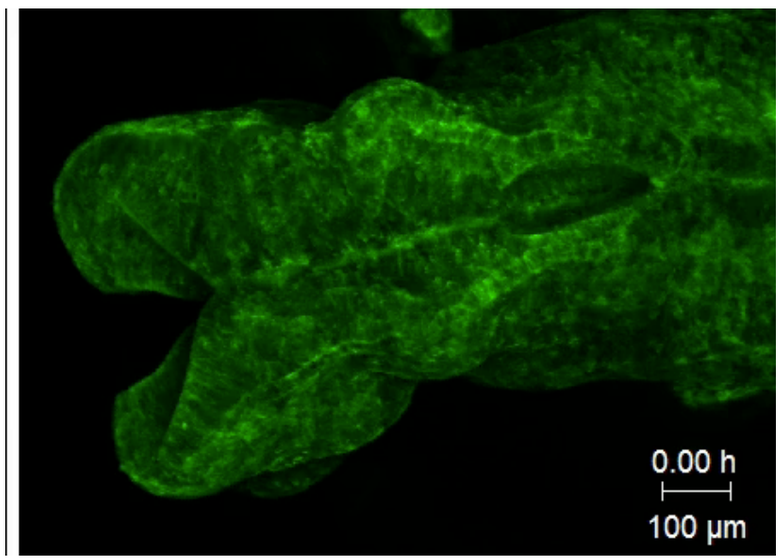
Heermann et al 2015

Drosophila: Ventral furrow



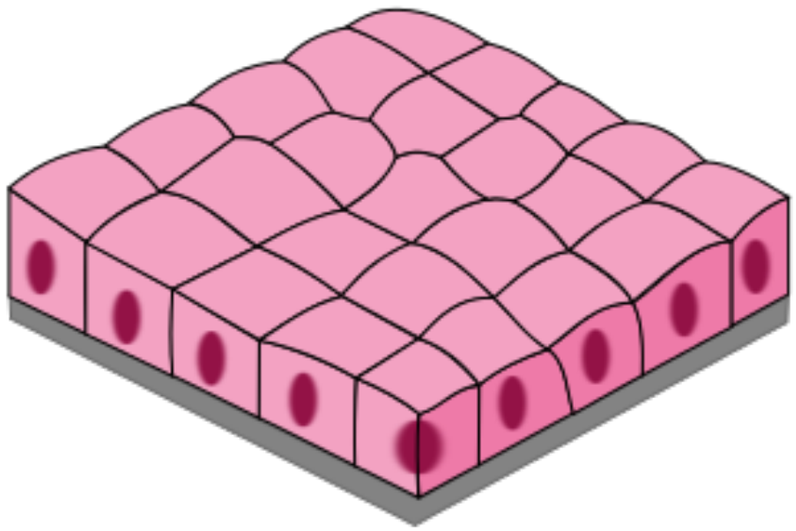
Brodland et al 2010

Mouse: Neural tube

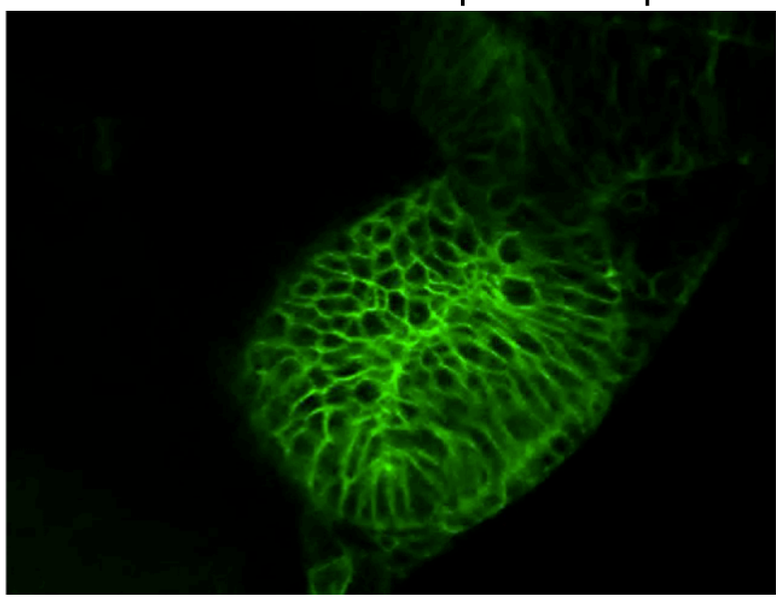


Massarwa and Niswander 2013

Folding of epithelial sheets widespread

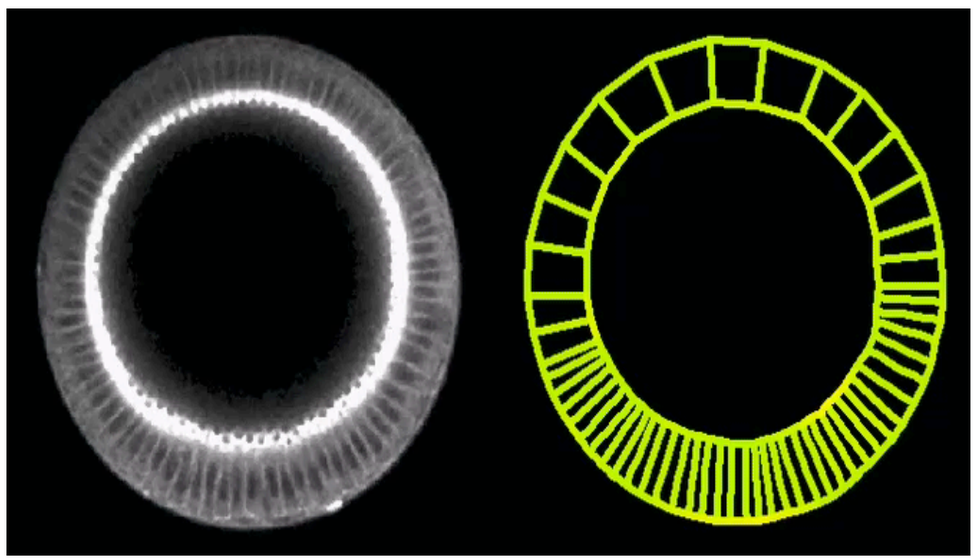


Zebra fish: Optic cup



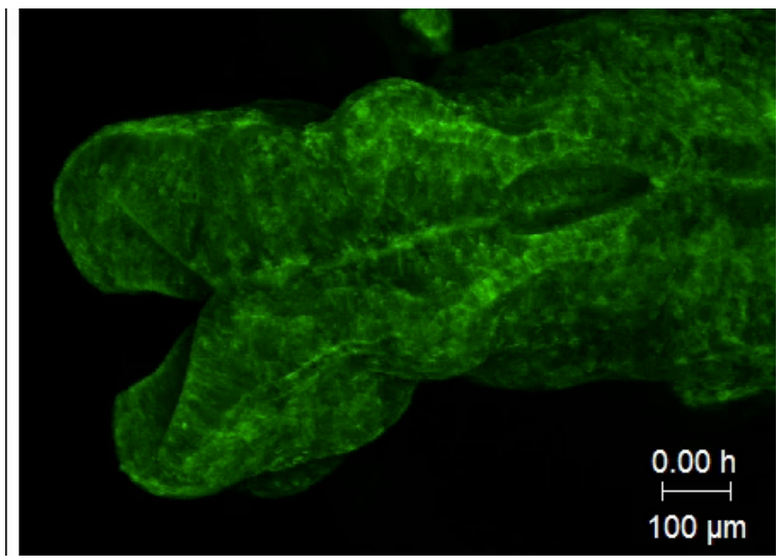
Heermann et al 2015

Drosophila: Ventral furrow



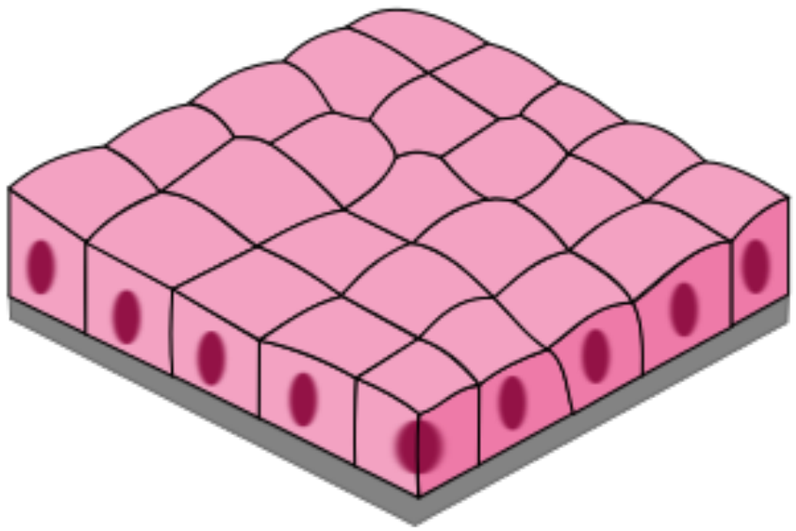
Brodland et al 2010

Mouse: Neural tube

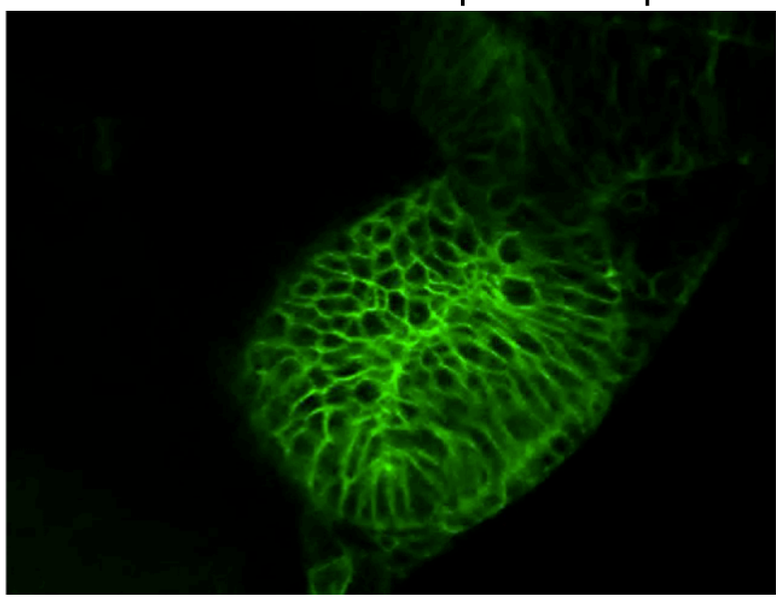


Massarwa and Niswander 2013

Folding of epithelial sheets widespread

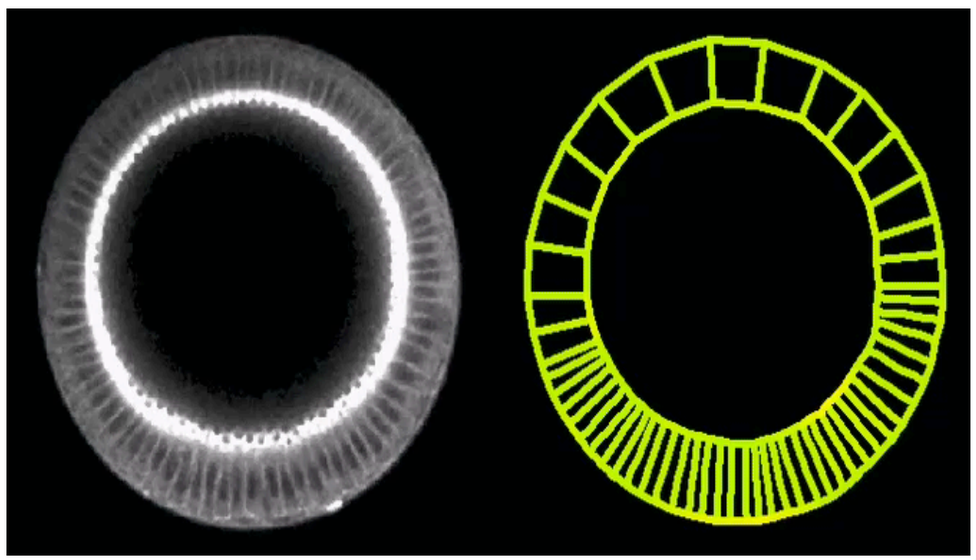


Zebra fish: Optic cup



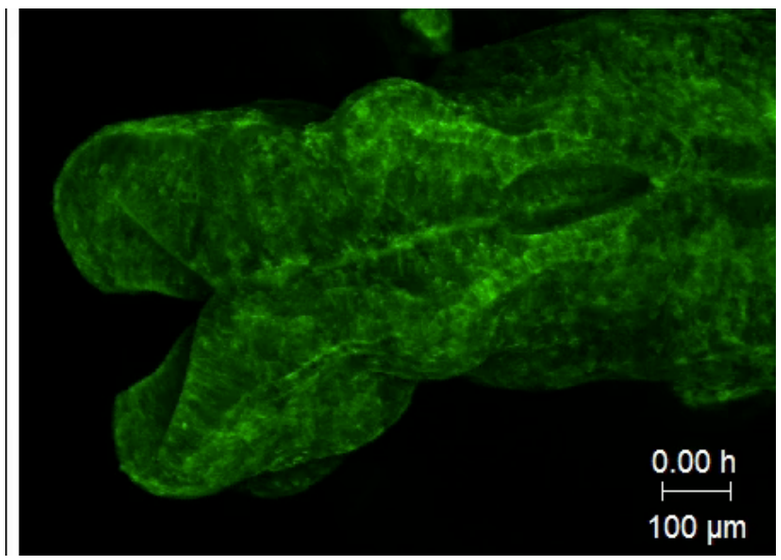
Heermann et al 2015

Drosophila: Ventral furrow



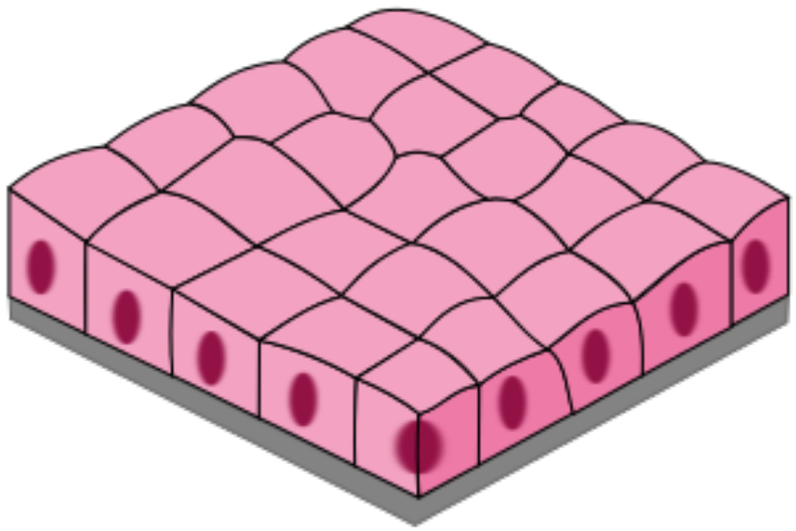
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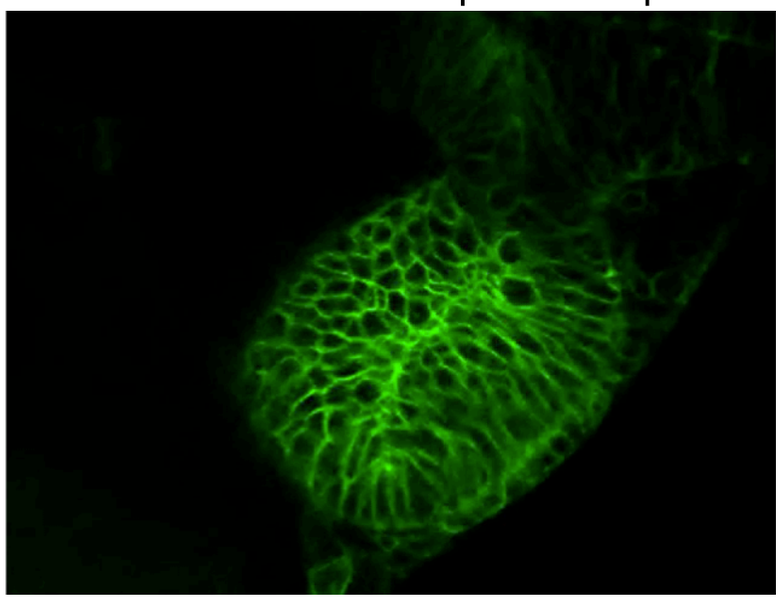


Massarwa and Niswander 2013

Folding of epithelial sheets widespread

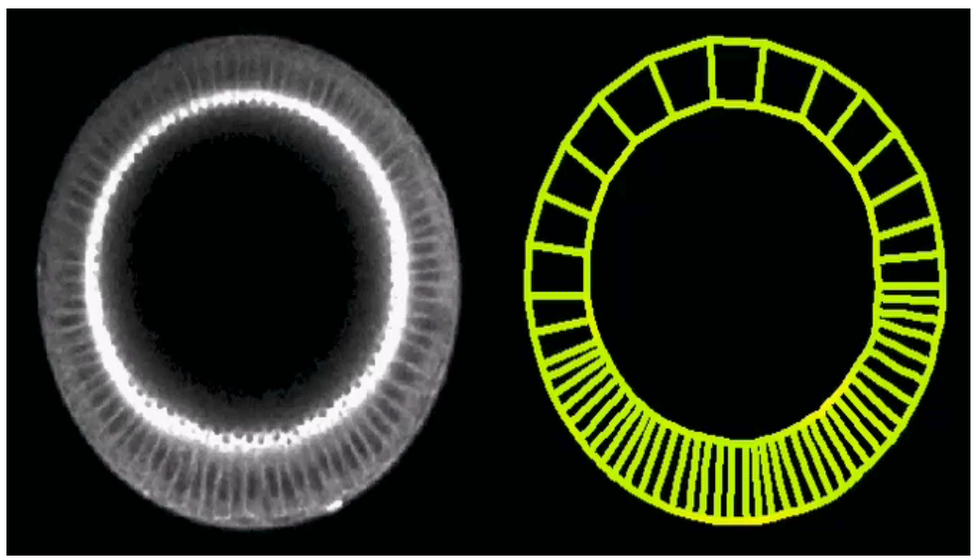


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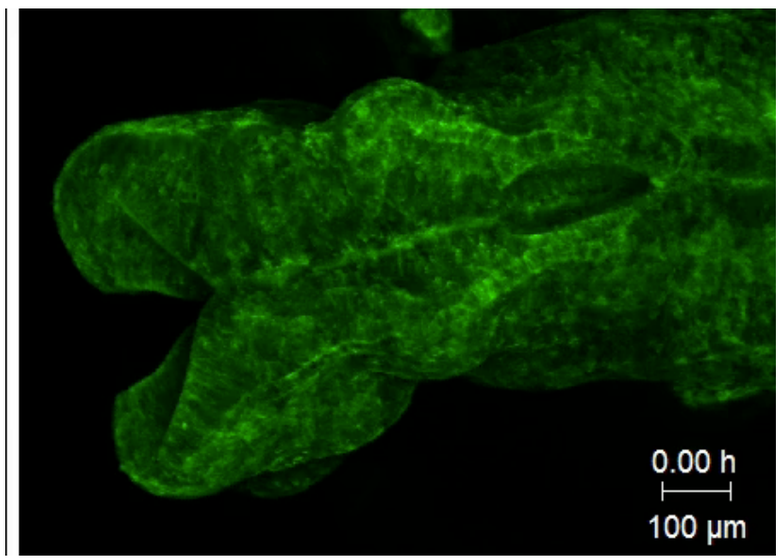
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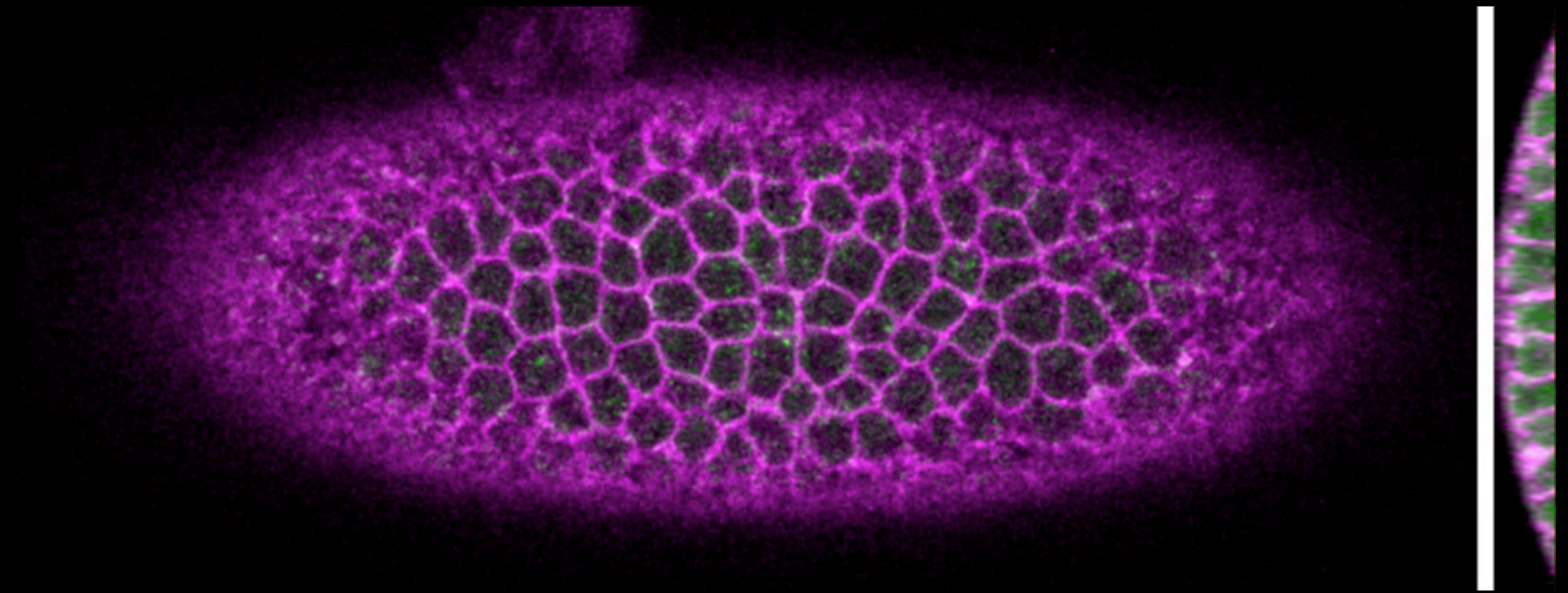


Brodland et al 2010

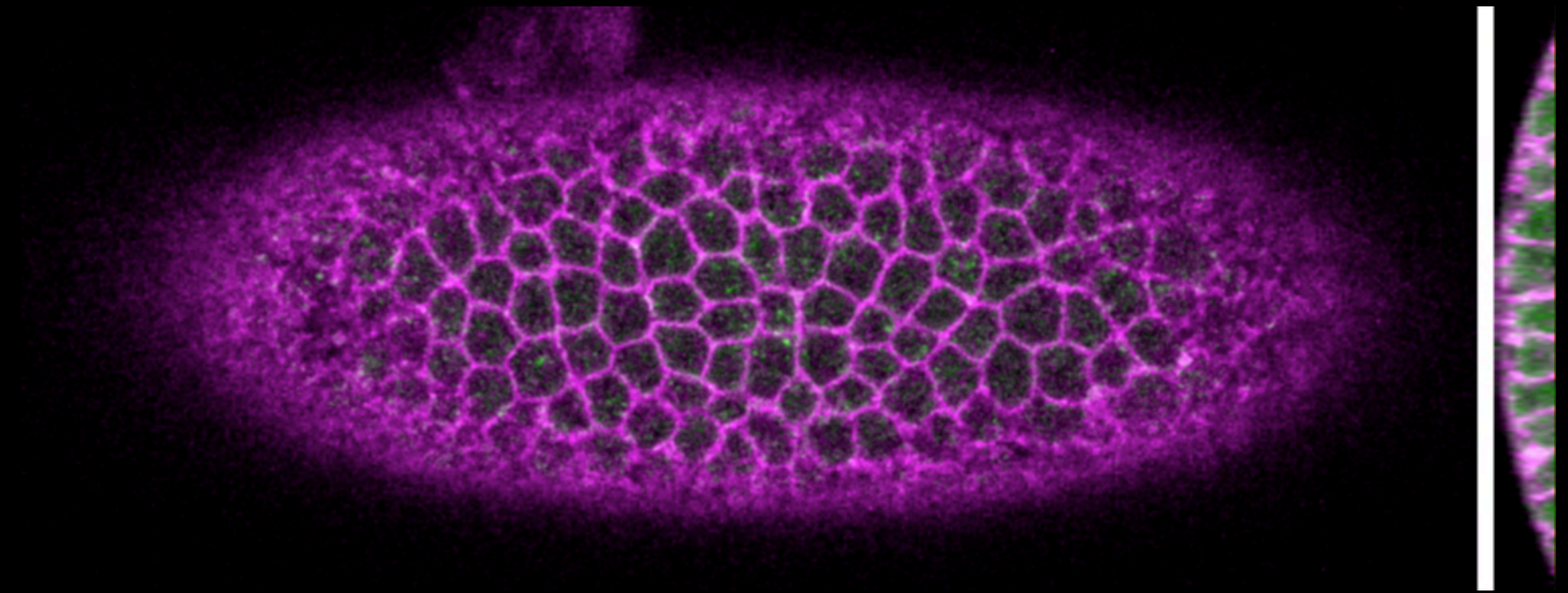
Mouse: Neural tube



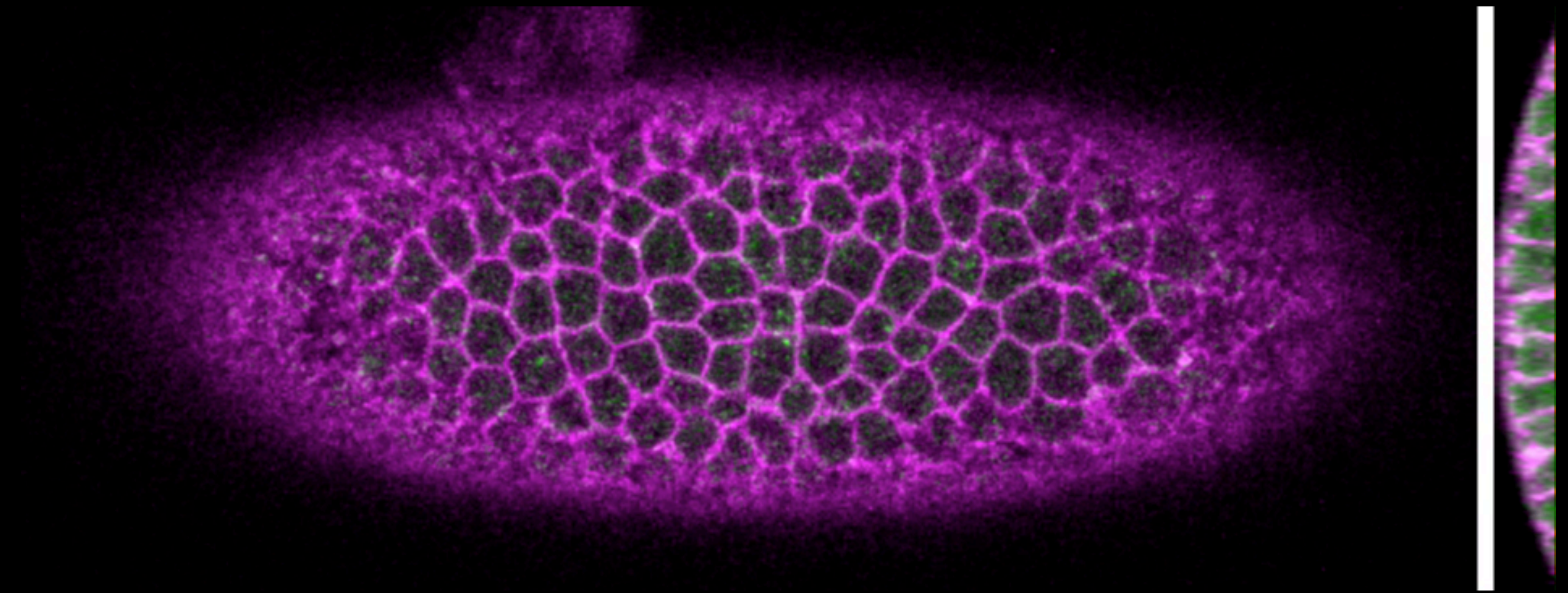
Massarwa and Niswander 2013



Movie: Adam Martin’s Lab (MIT Biology)

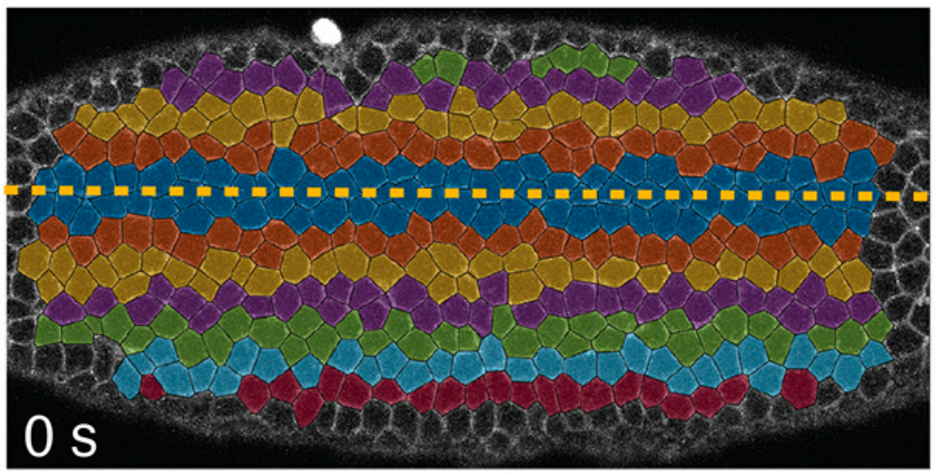


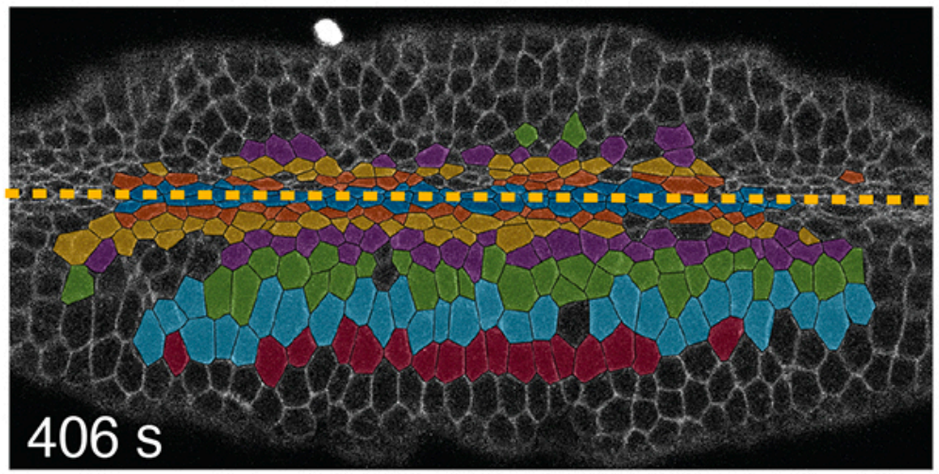
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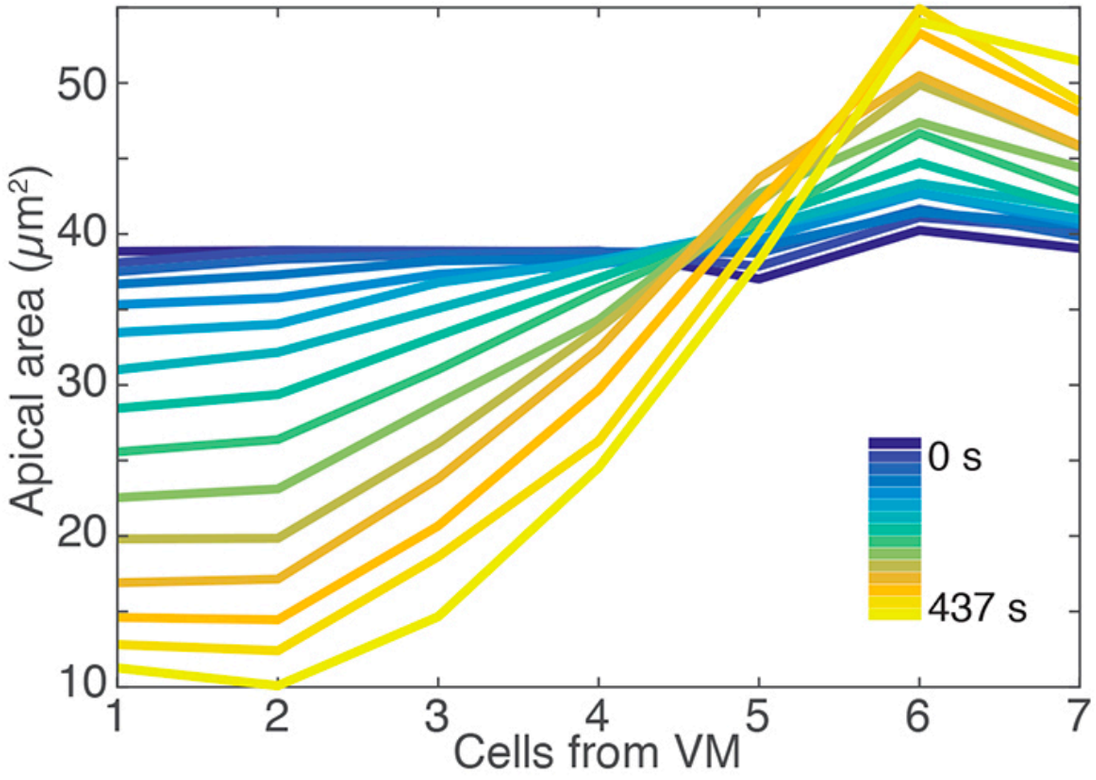
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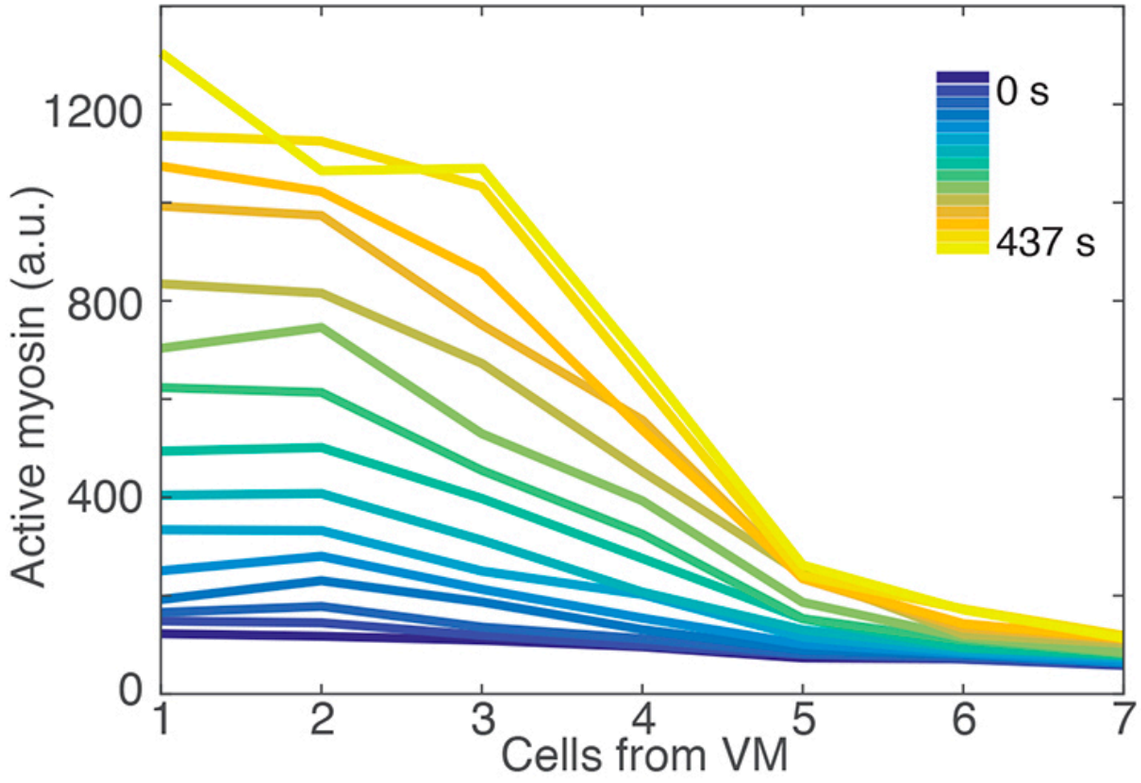
Apical cell area and myosin











Natalie Heer *et al*, Development 2017



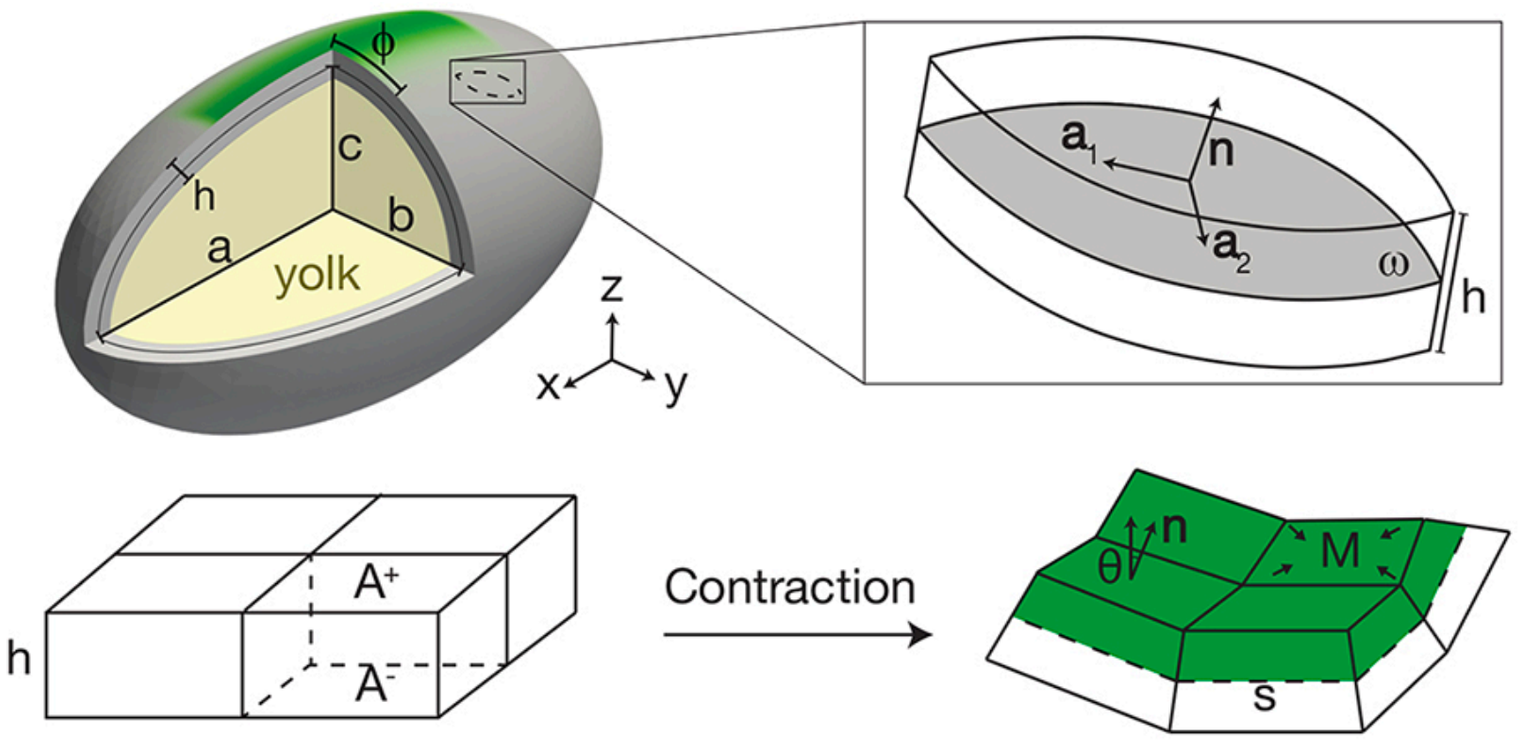
Pearson Miller

3D chemo-mechanical model



Norbert Stoop

$$E(A^+, A^-) = C_M \rho_M A^{+2} + K(A^+ - A^0)^2 + K(A^- - A^0)^2$$



3D chemo-mechanical model

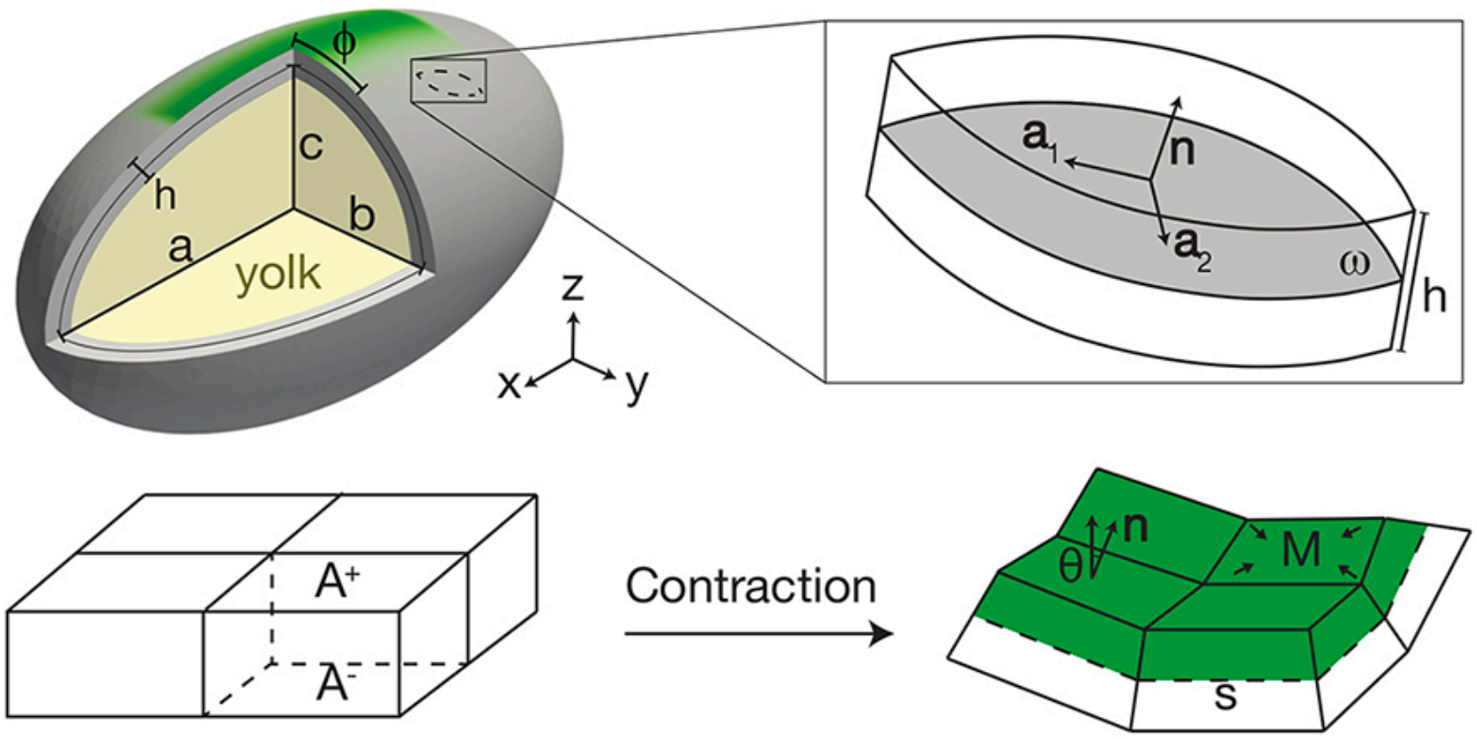


Pearson Miller



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$$\mathcal{E} = \frac{Yh}{8(1-\nu^2)} \int_{\bar{\omega}} S\Big\{ (1-\nu)\mathrm{Tr}[(\mathbf{a} - \bar{\mathbf{a}})^2] + \nu \mathrm{Tr}(\mathbf{a} - \bar{\mathbf{a}})^2 + \frac{h^2}{3}[(1-\nu)\mathrm{Tr}[(\mathbf{b} - \bar{\mathbf{b}})^2] + \nu \mathrm{Tr}(\mathbf{b} - \bar{\mathbf{b}})^2] \Big\} d\bar{\omega}$$

+ volume constraints, …



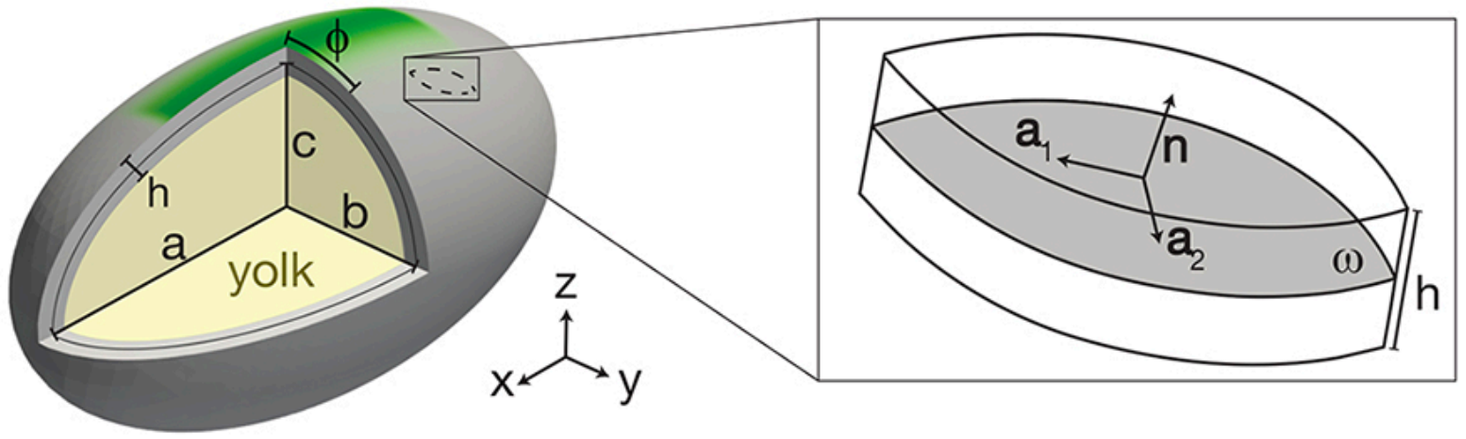
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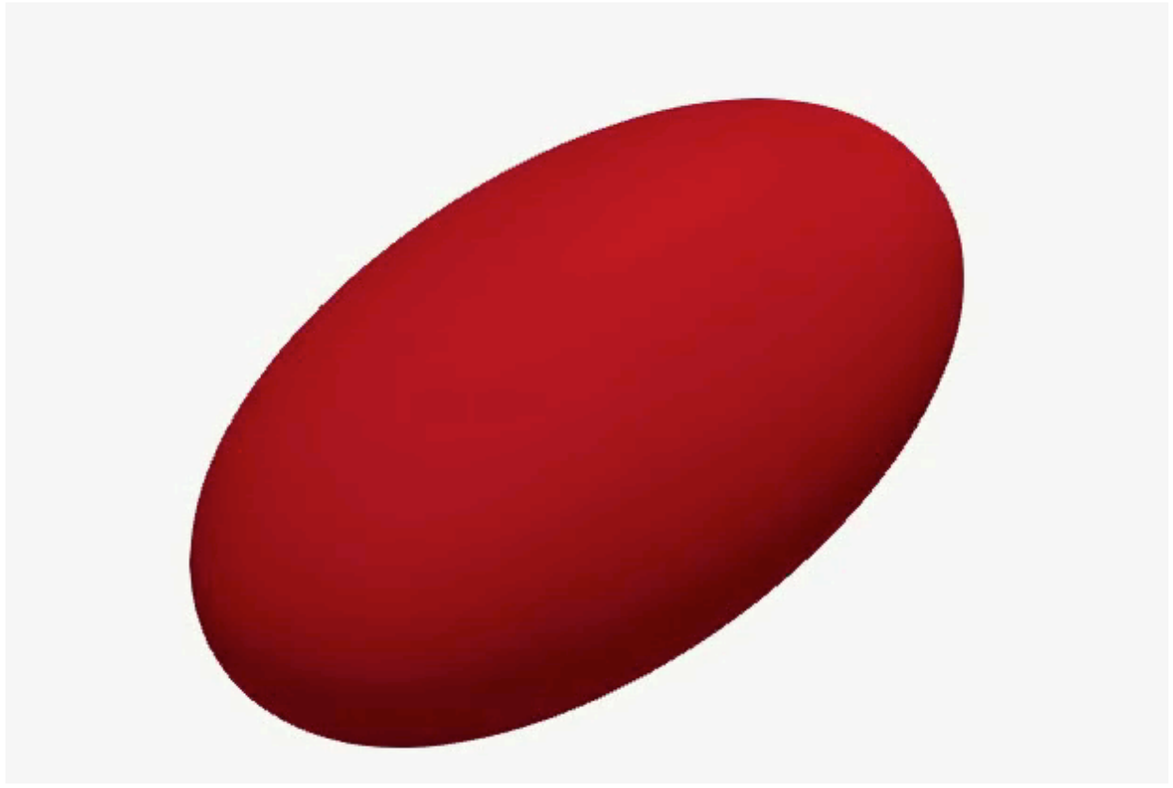


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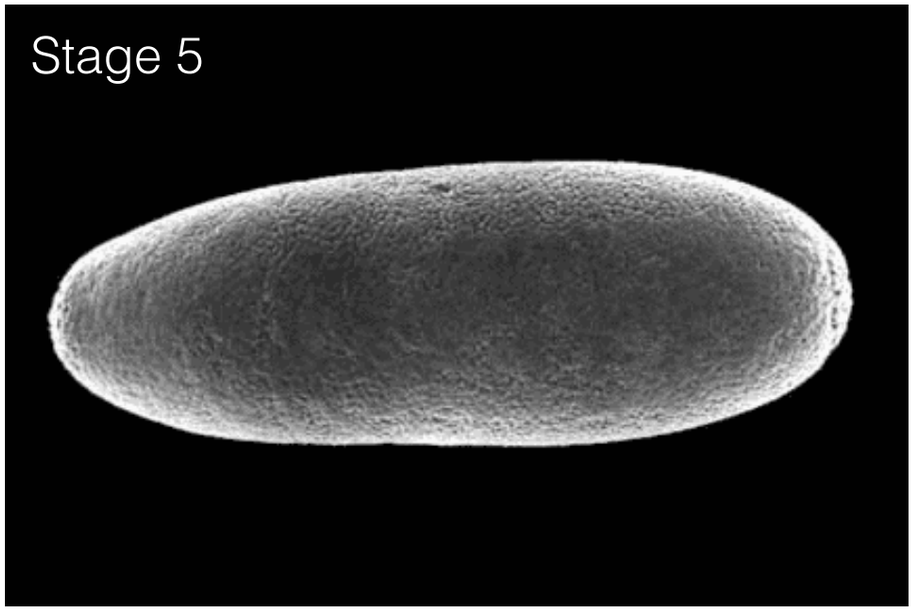
+ volume constraints, …

$$M(\varphi) = M_0(1 + e^{-kw})/(1 + e^{k(|\varphi|-w)})$$

Wildtype simulation



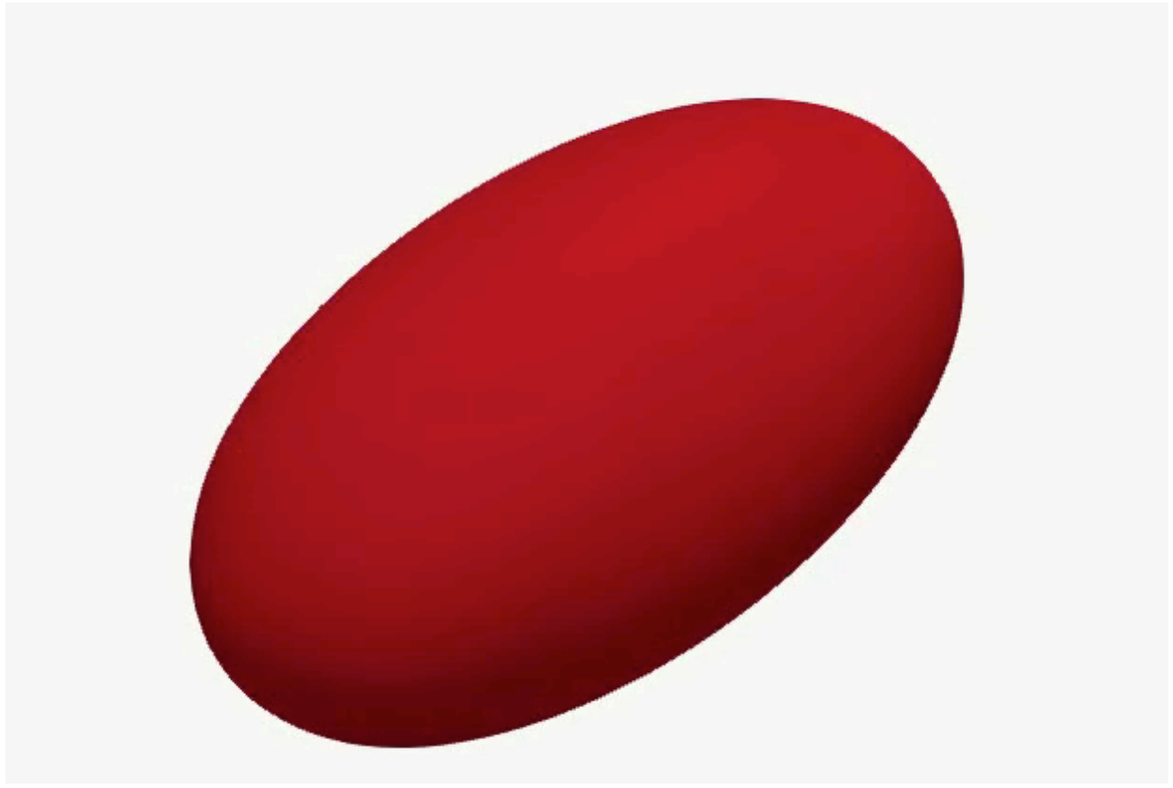
http://flybase.org



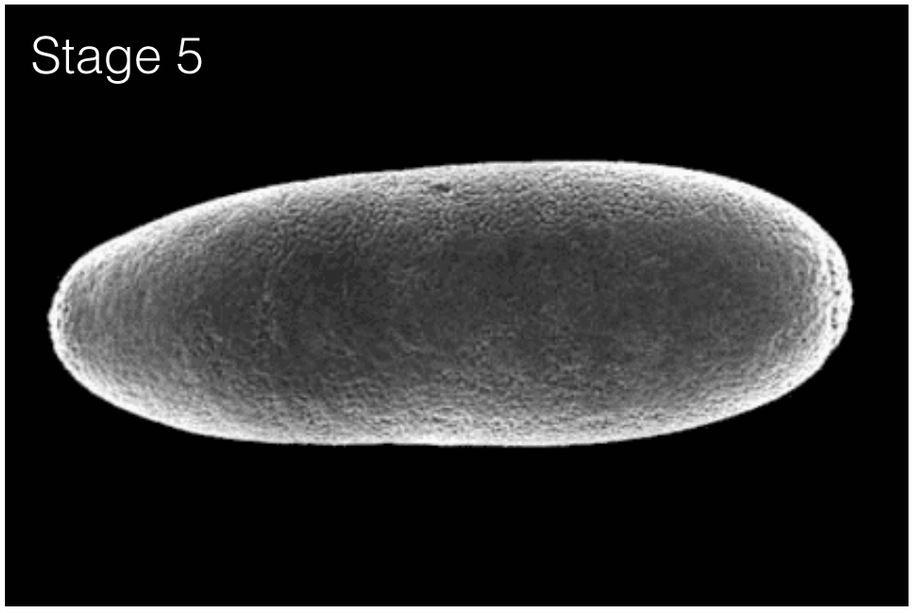


Natalie Heer *et al,* Development 2017

Wildtype simulation



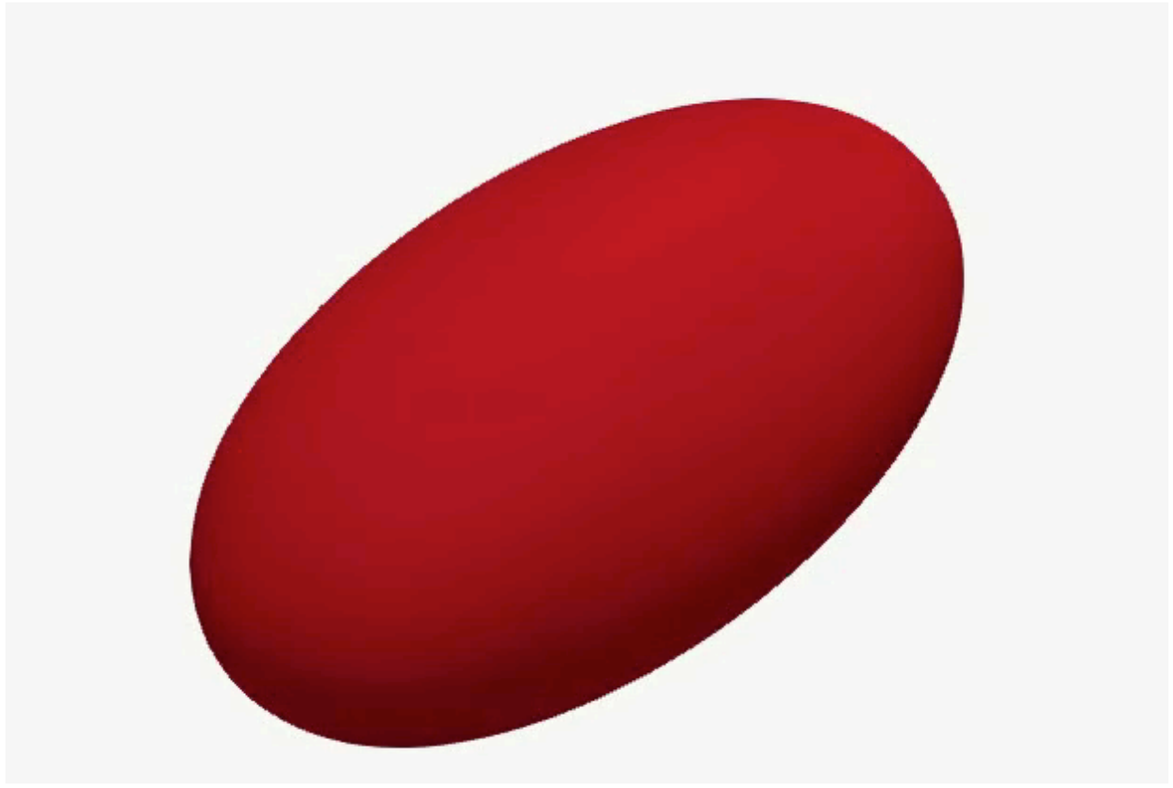
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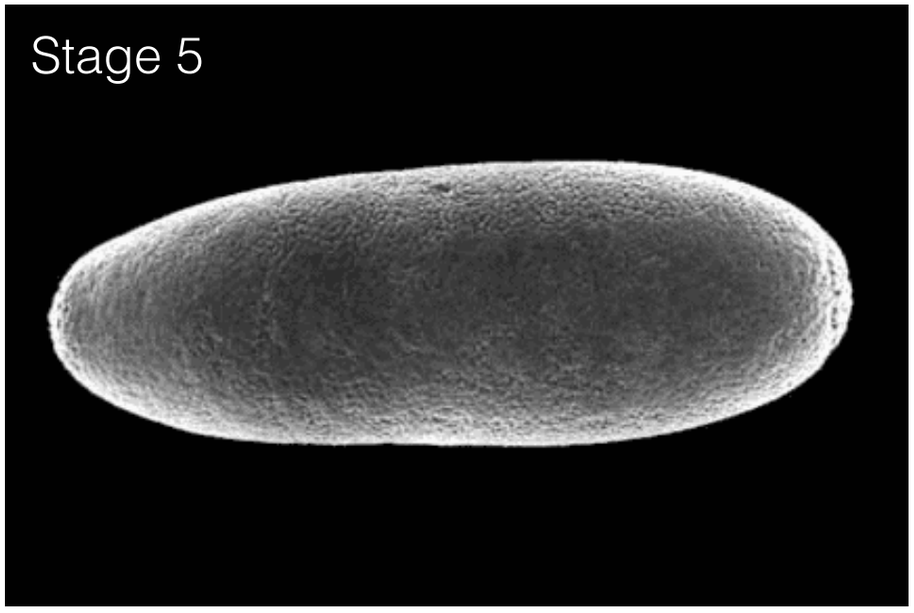
Natalie Heer *et al,* Development 2017

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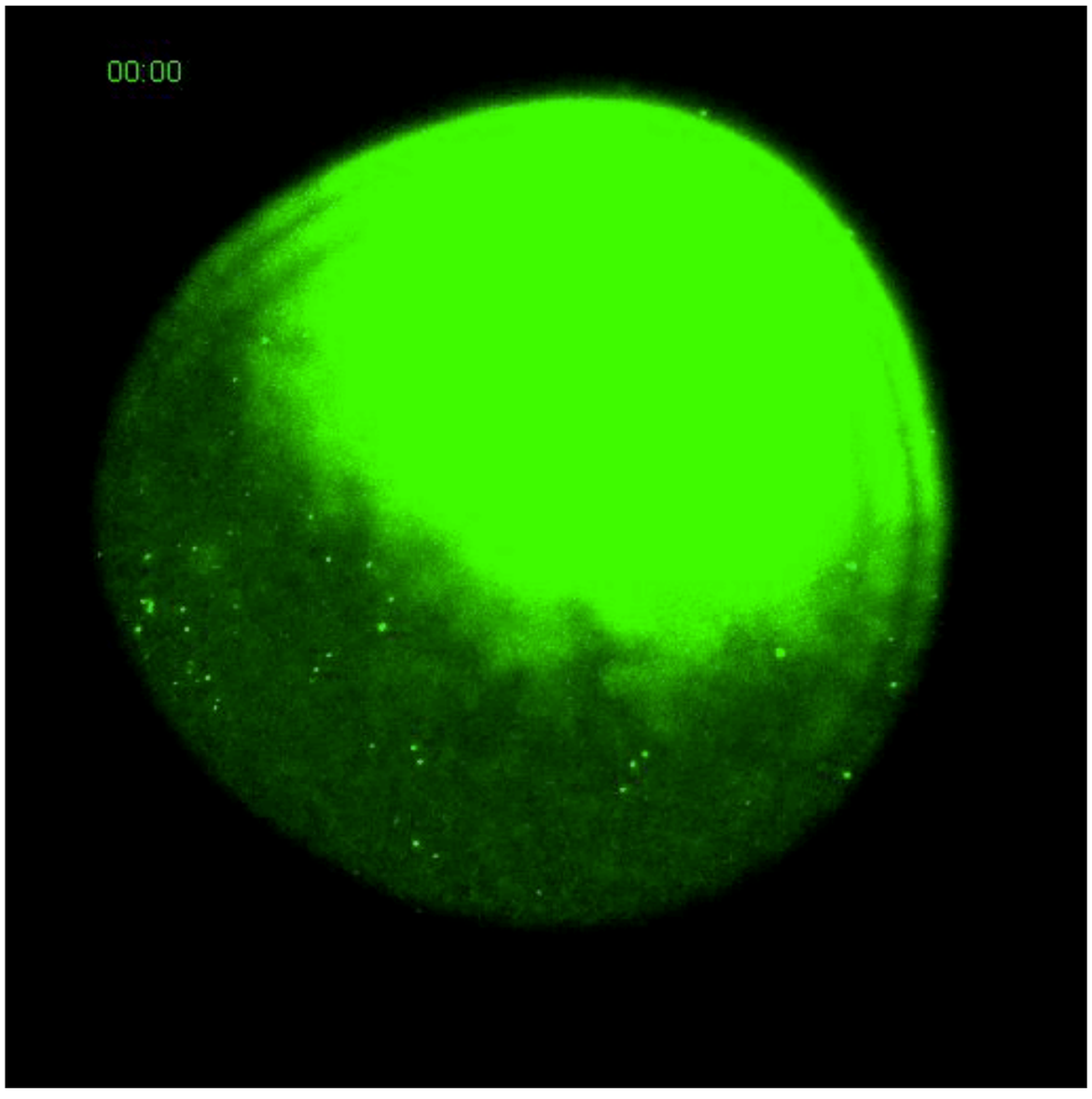
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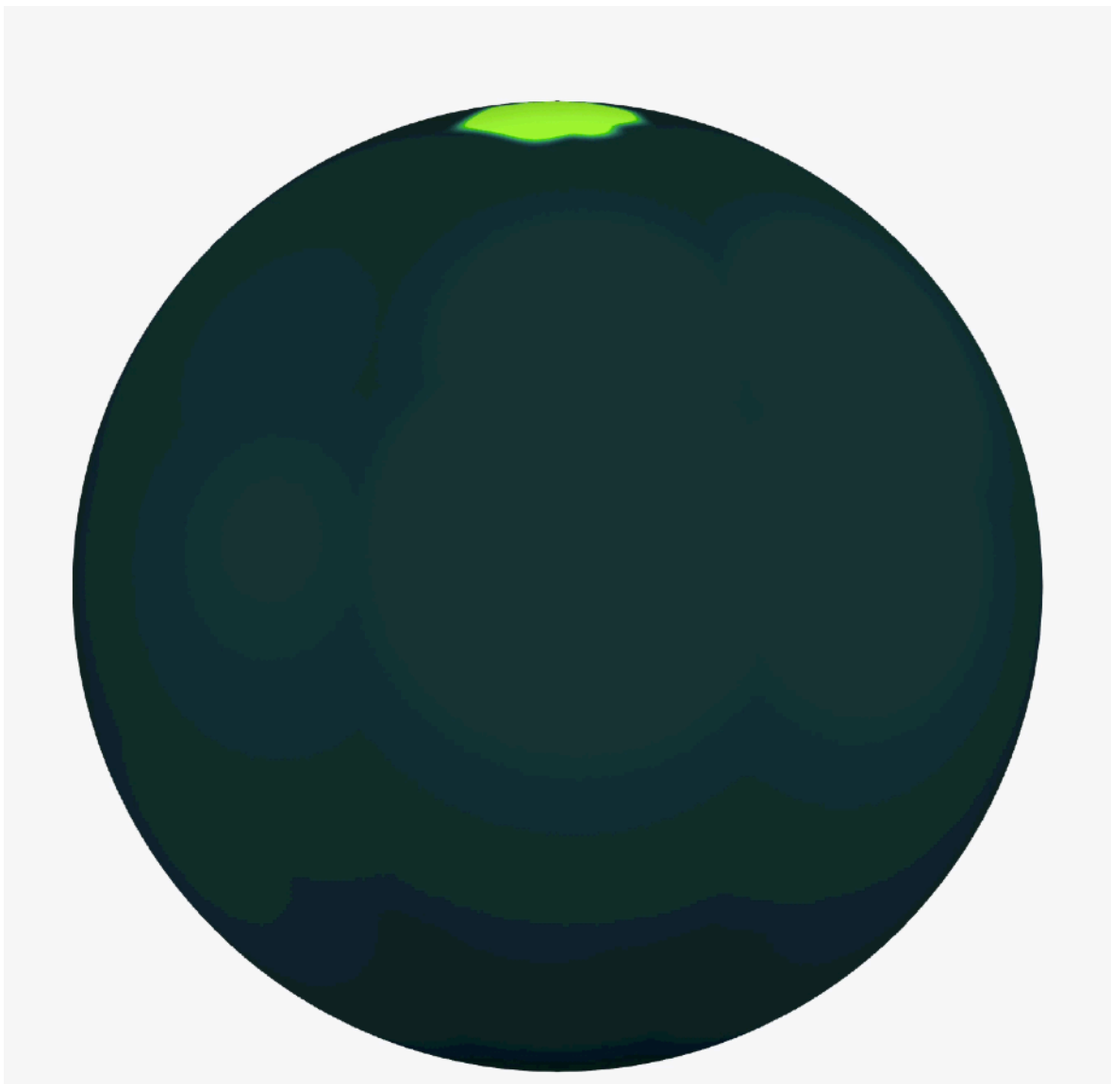
Turing pattern in starfish oocytes



In collaboration with Nikta Fakhri's lab (MIT Physics)



Thanks to Nikta's students
Melis Tekant, Tzer Han Tan & Hridesh Kedia



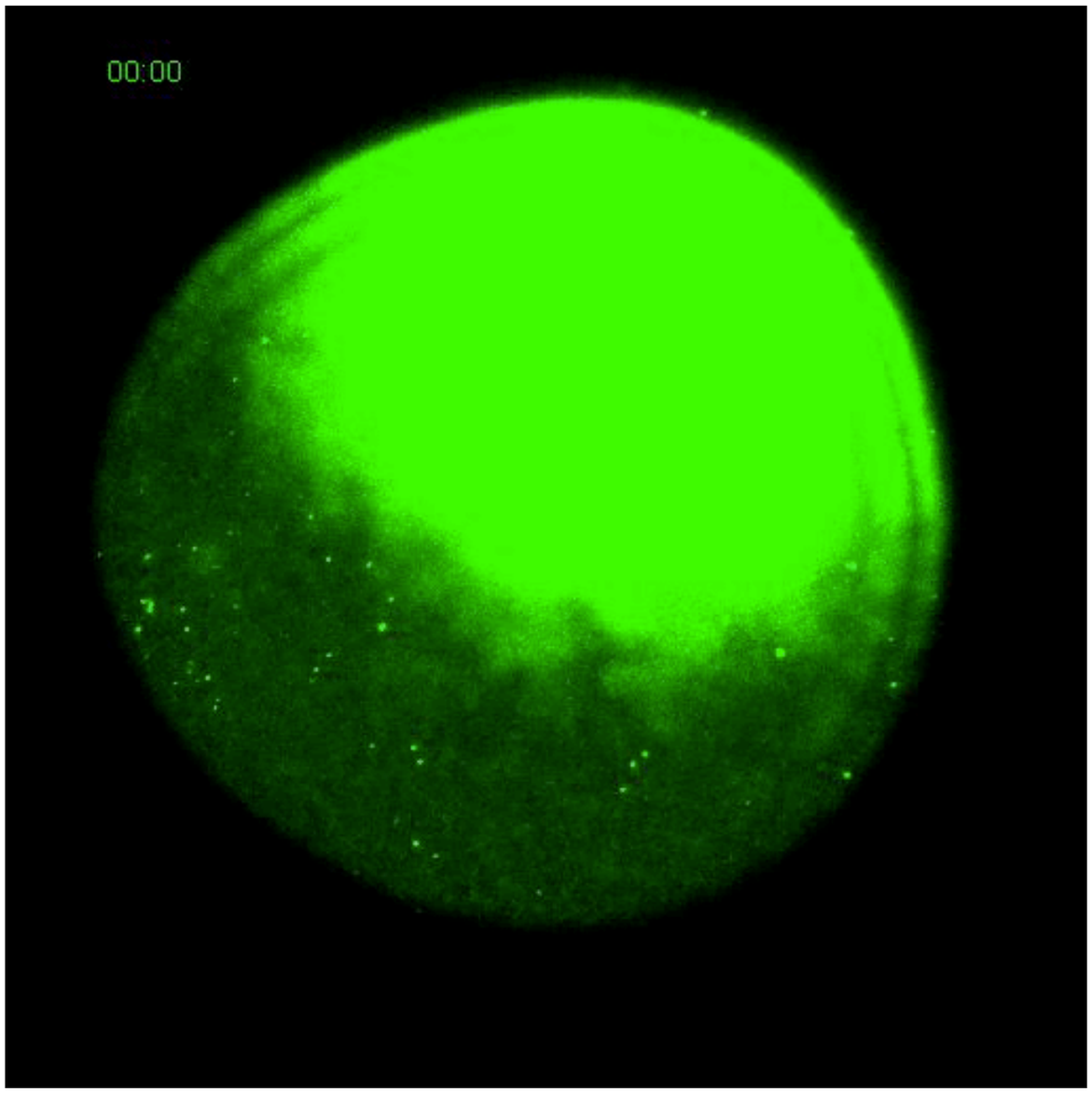
Gray-Scott reaction-diffusion model



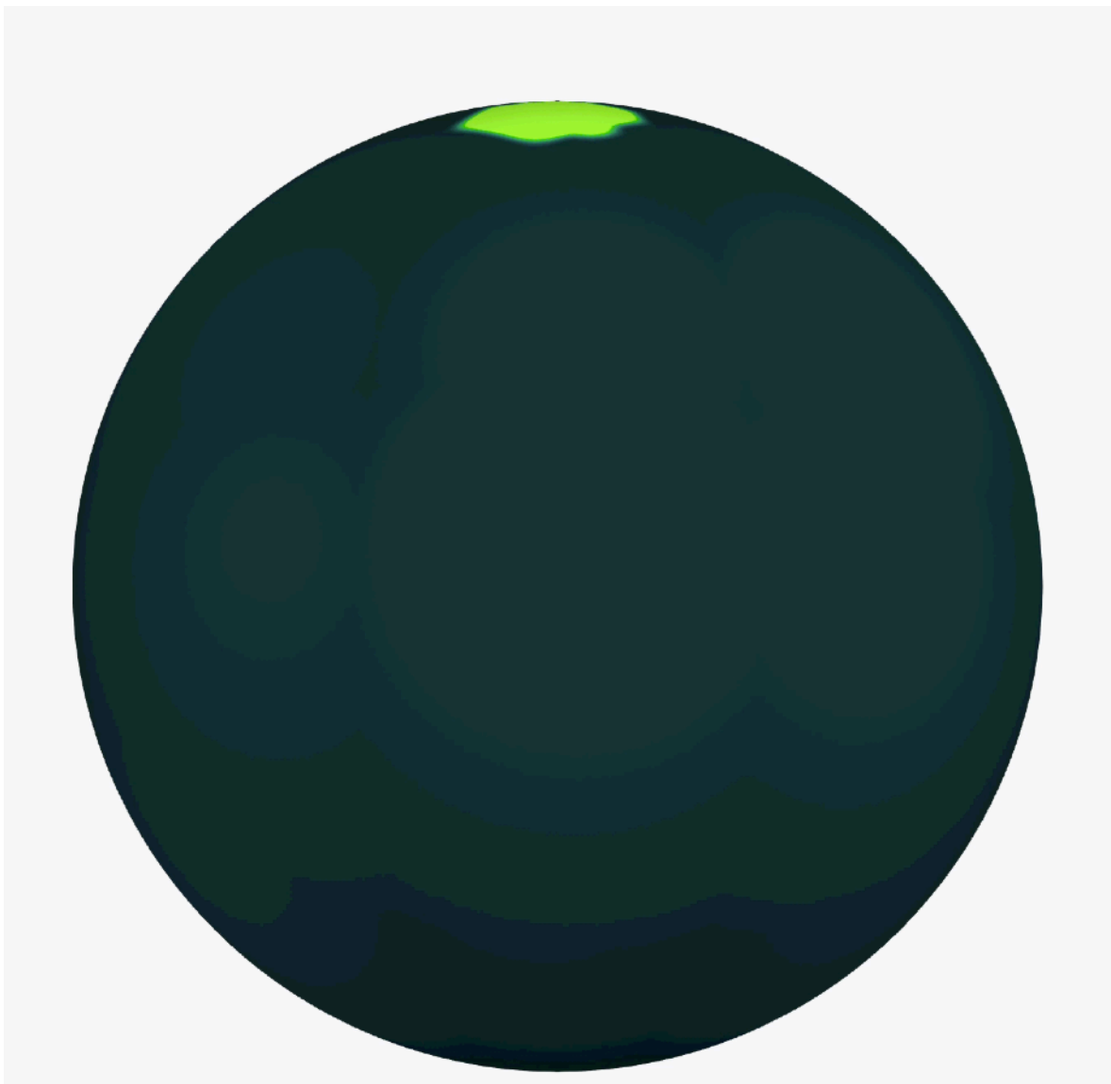
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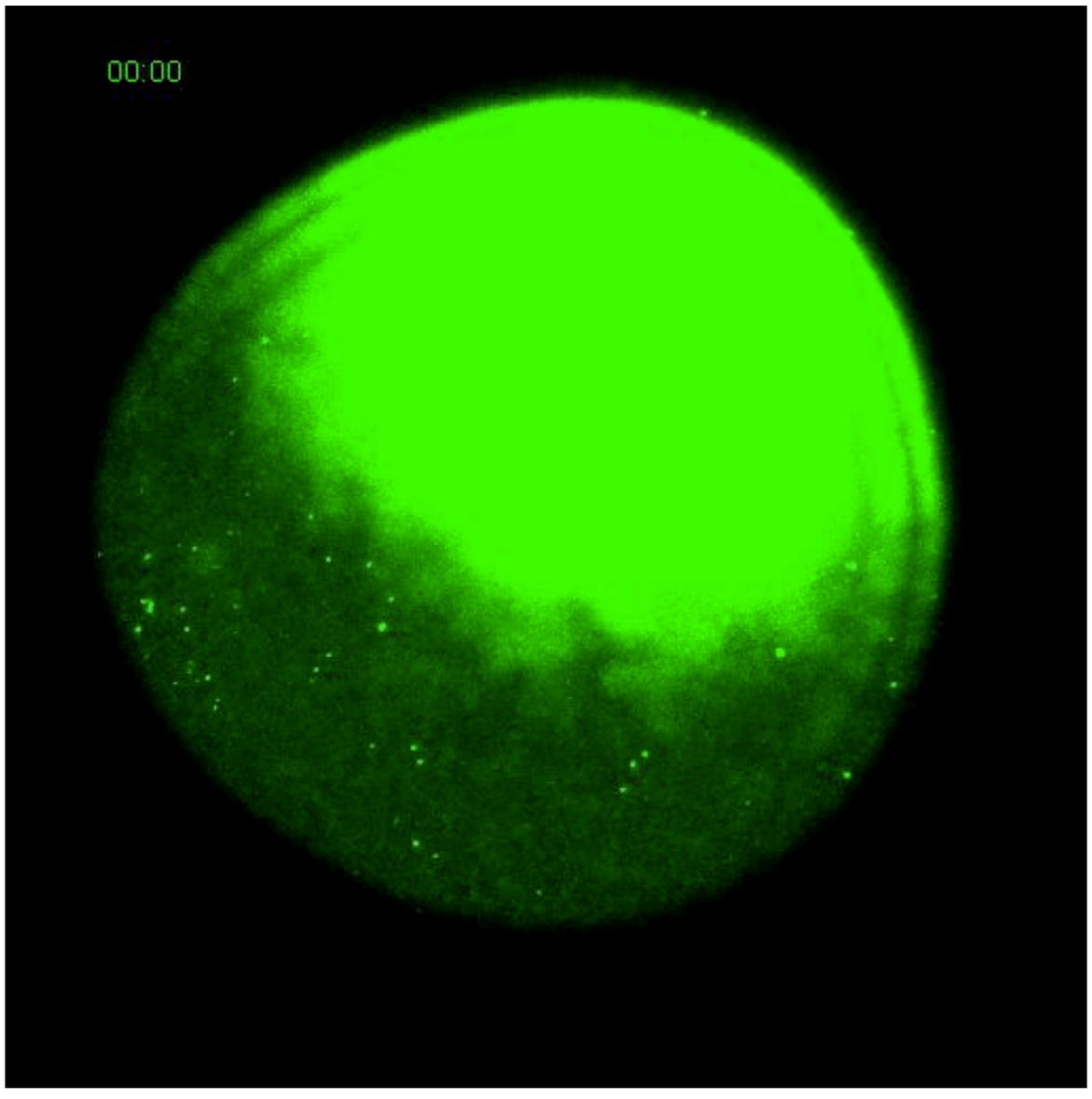
Gray-Scott reaction-diffusion model



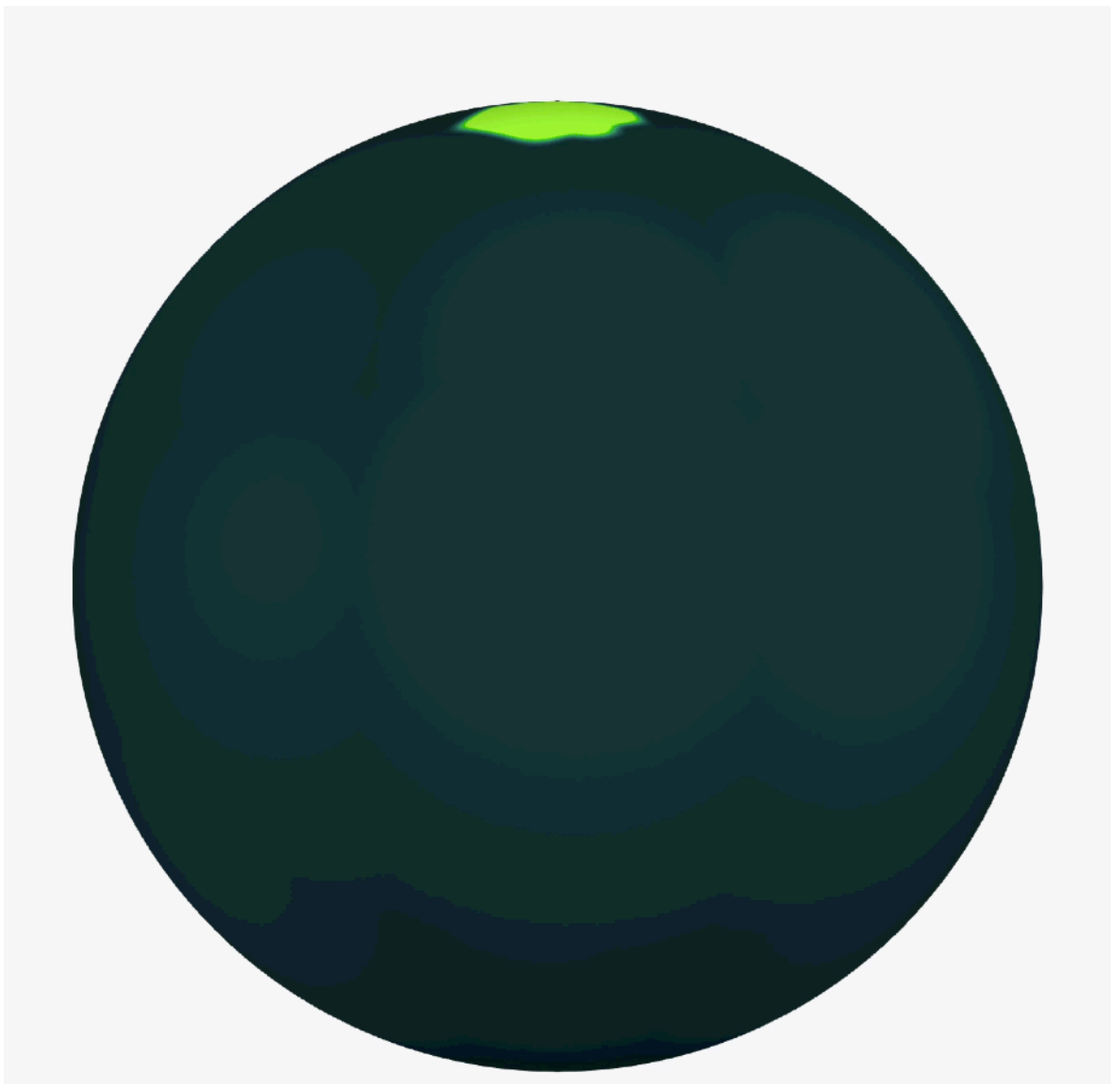
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Gray-Scott reaction-diffusion model

Part III:
How spaghetti break

MechE undergraduate Ronald Heisser (experiments, 18.354 course project)

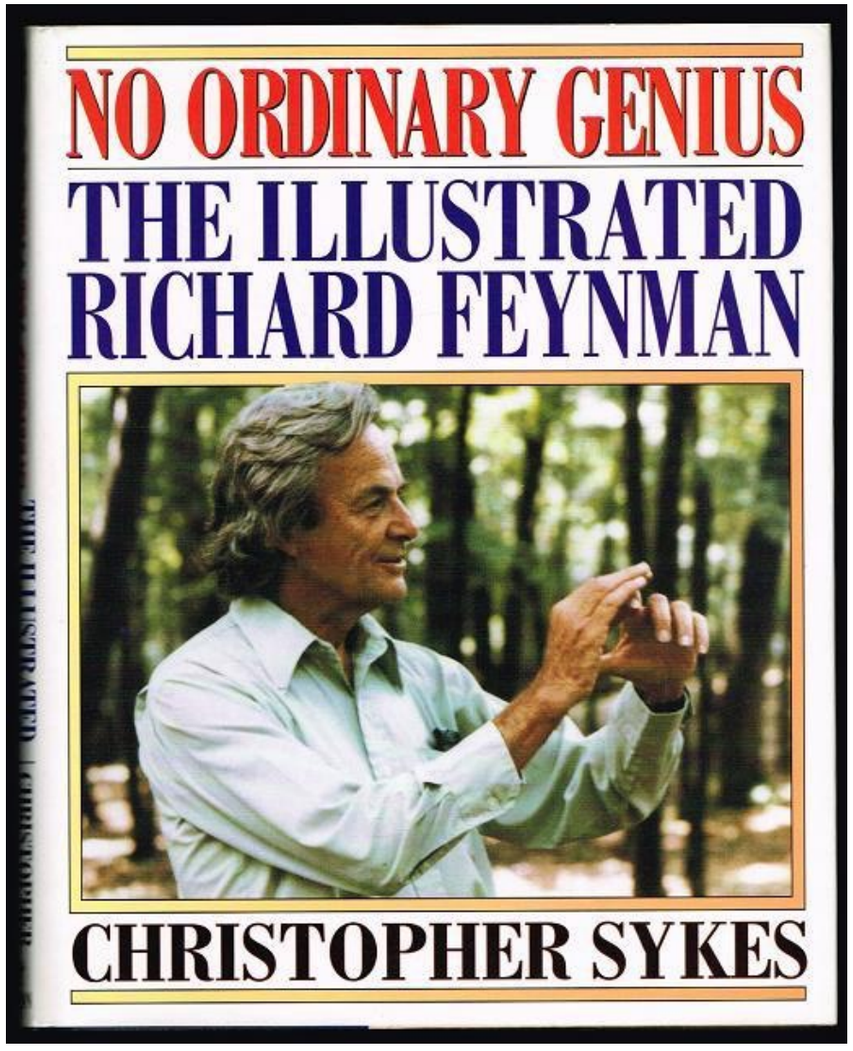
2nd year Math PhD student Vishal Patil (theory)





Feynman's puzzle

"Why does it break into three pieces? We spent the next two hours coming up with crazy theories…well, we ended up at the end of a couple of hours with broken spaghetti all over the kitchen and no real good theory about why spaghetti breaks in three."

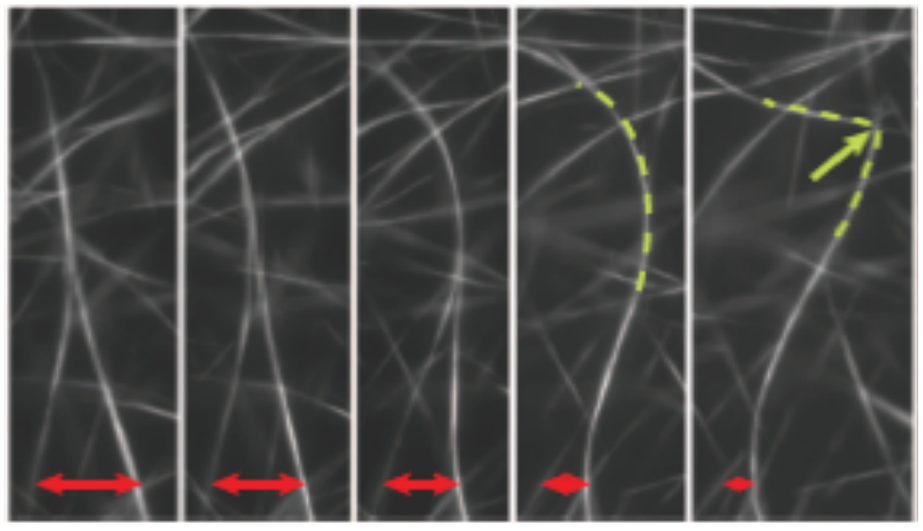


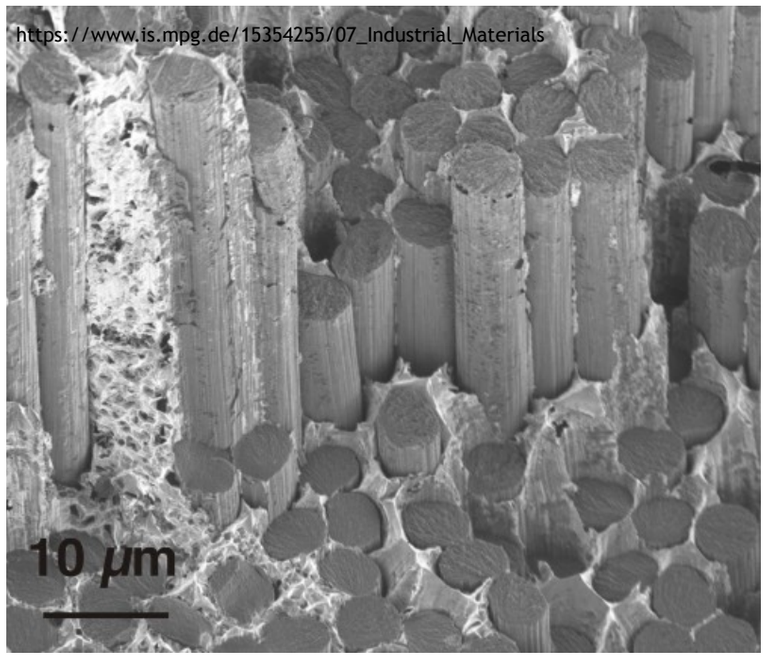
Why study 1D fracture ?





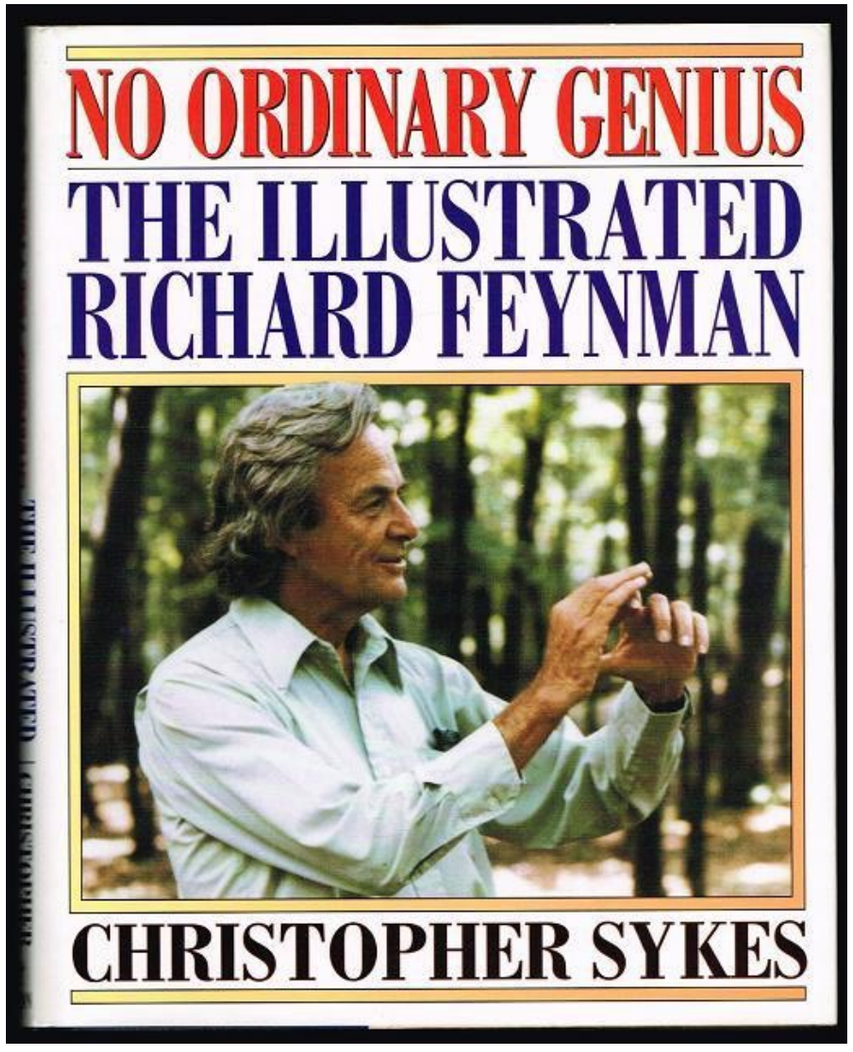






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PRL **95,** 095505 (2005) PHYSICAL REVIEW LETTERS week ending 26 AUGUST 2005

Fragmentation of Rods by Cascading Cracks: Why Spaghetti Does Not Break in Half

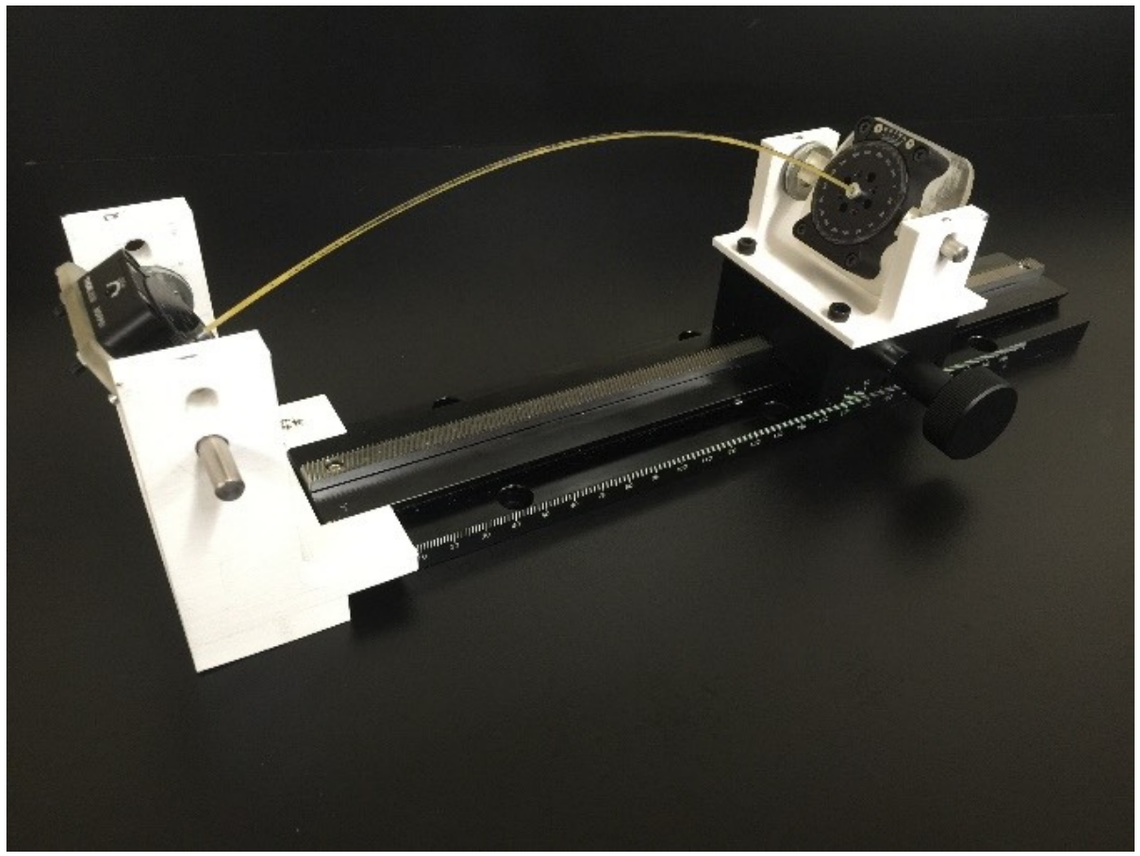
Basile Audoly and Sébastien Neukirch
Laboratoire de Modélisation en Mécanique, CNRS/Université Paris VI, 4 place Jussieu, Paris, France
(Received 22 December 2004; published 25 August 2005)

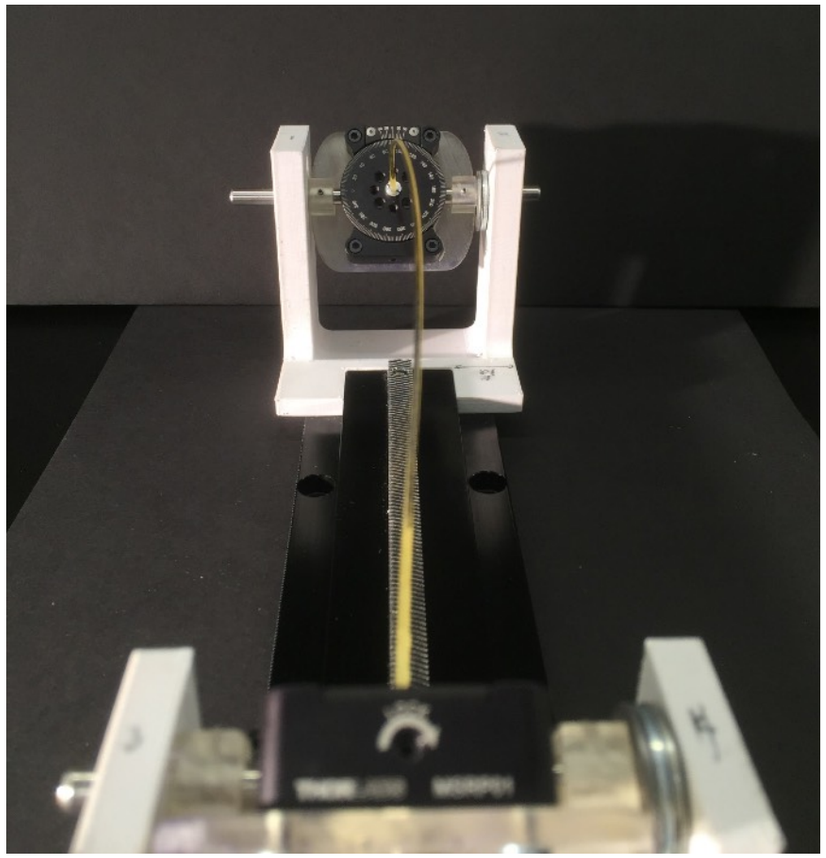


Ronald’s puzzle



“How can we break it into 2 pieces?”





low twist



low twist



low twist



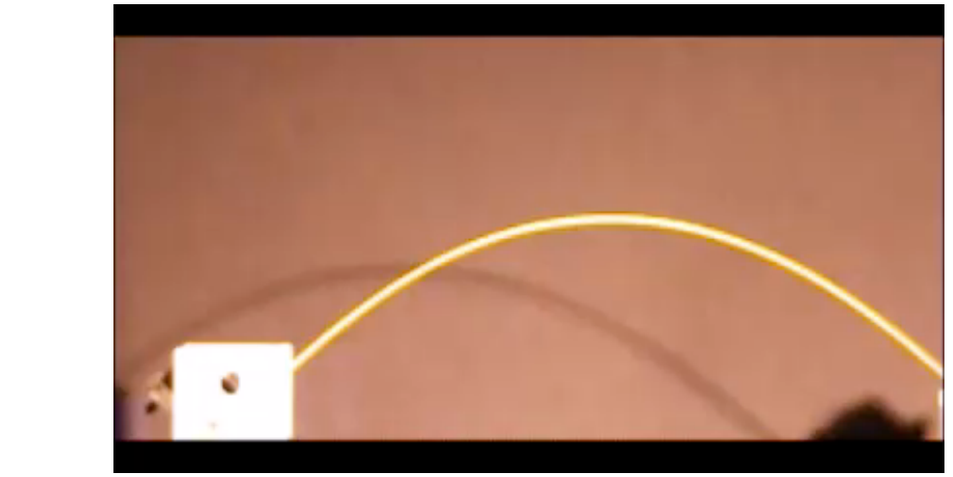
high twist

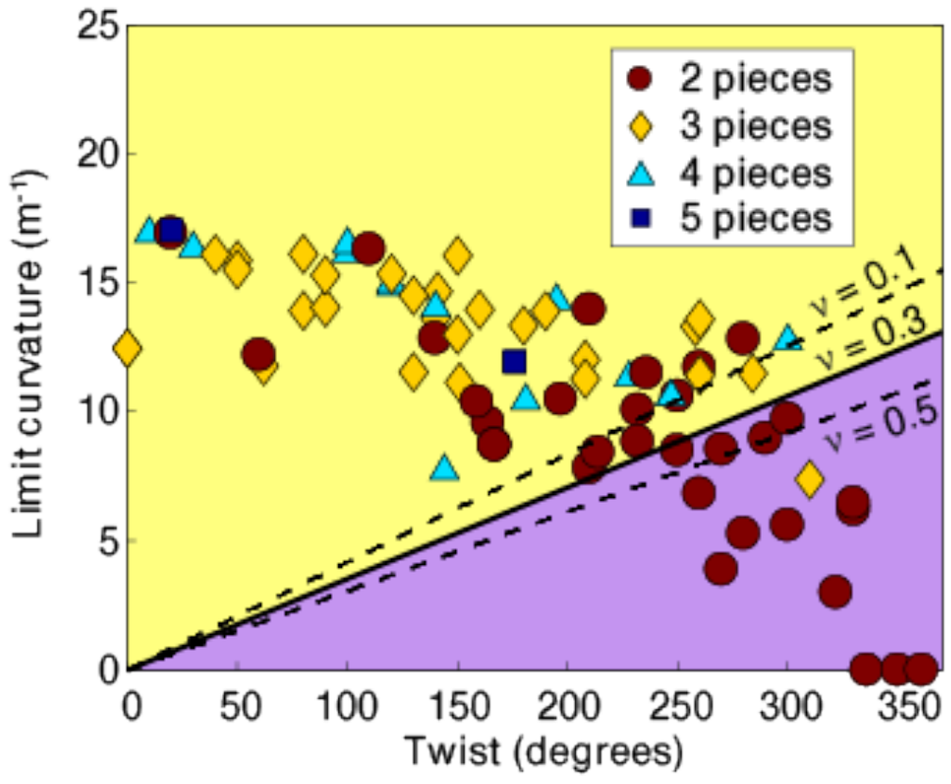


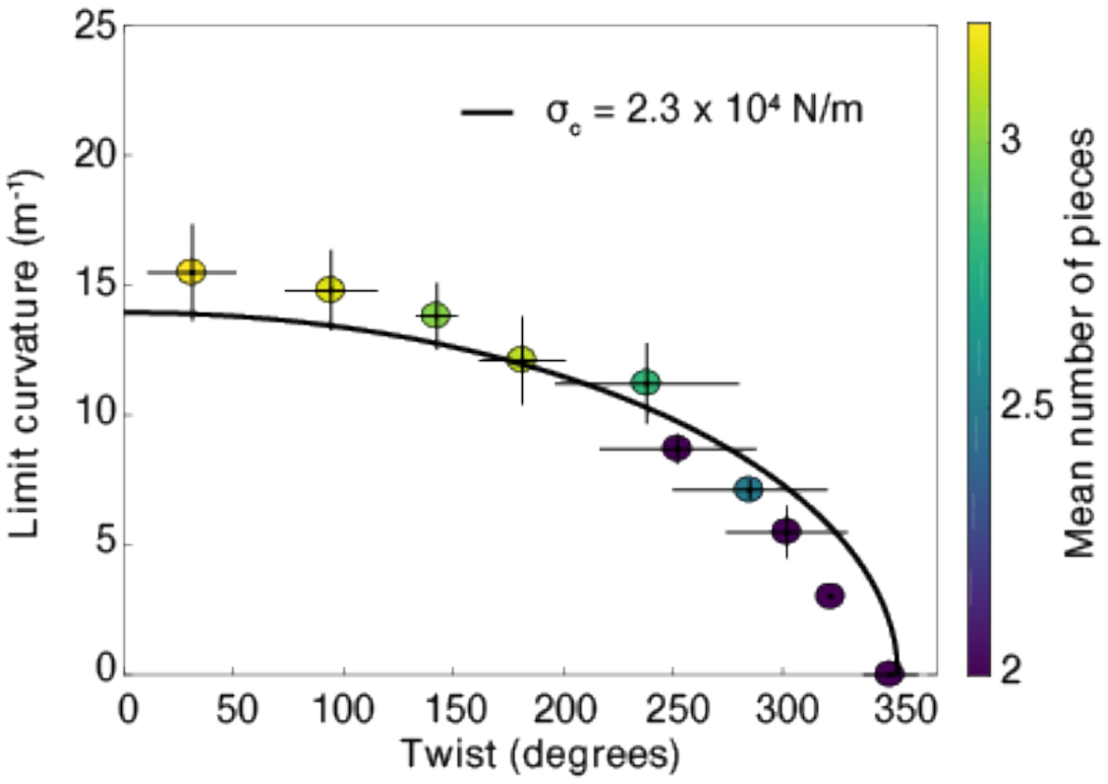
low twist

high twist





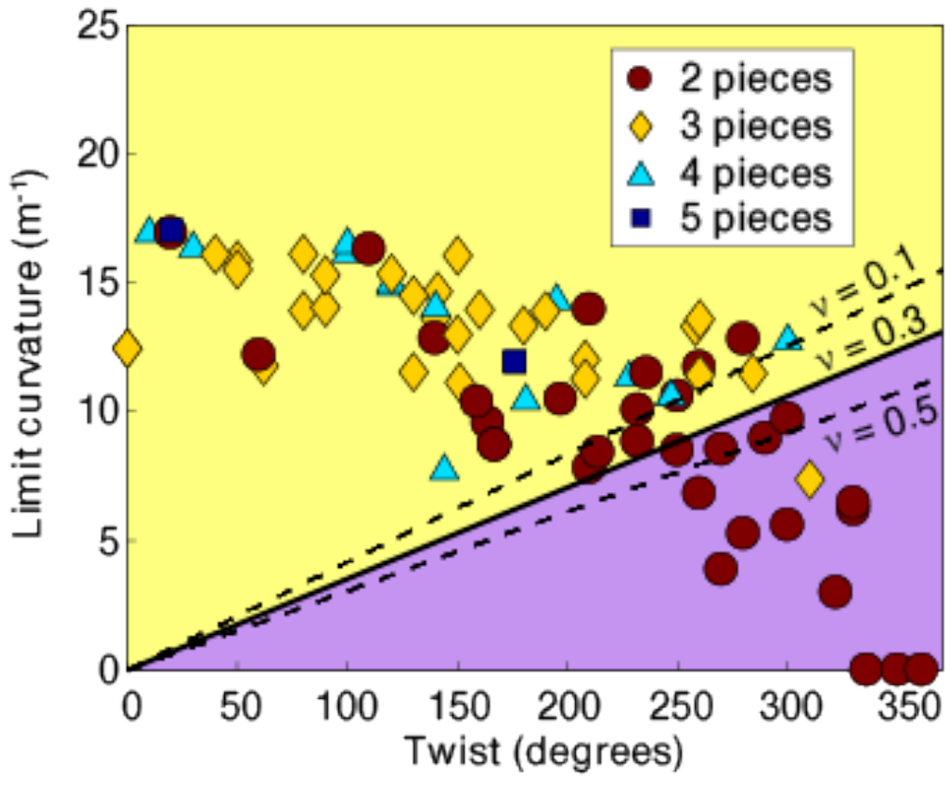


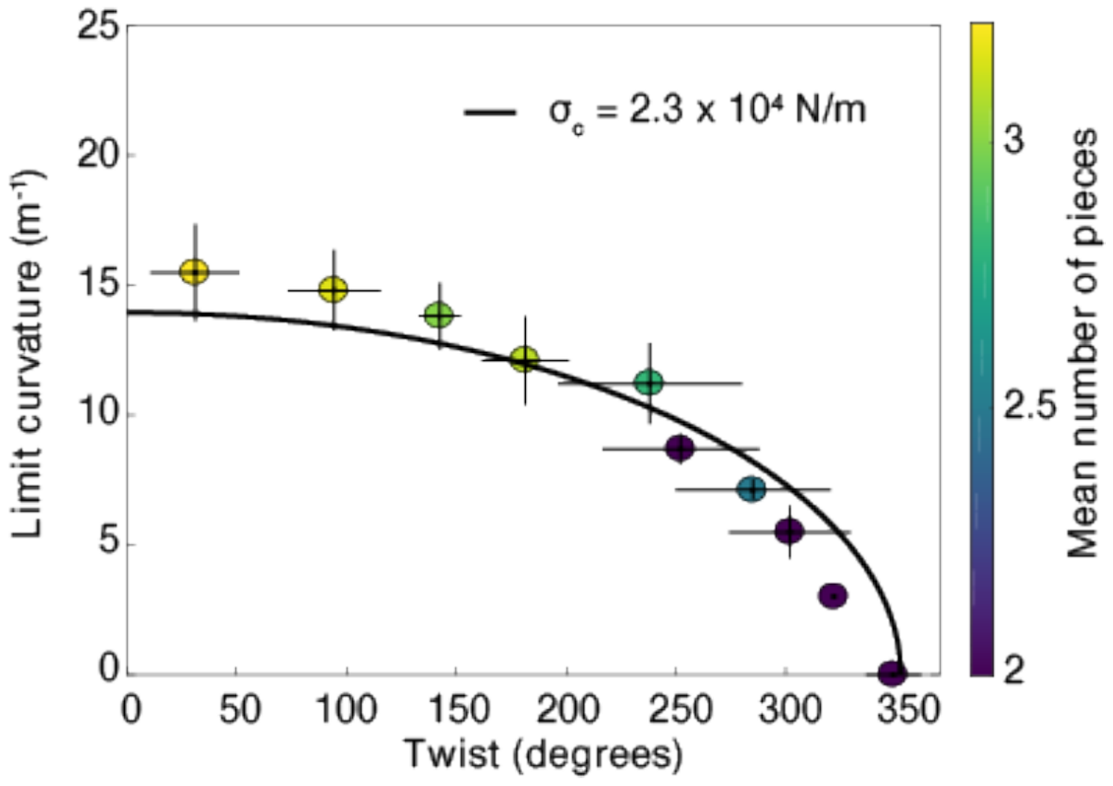


low twist

high twist







Twist criterion for binary fracture

$$\kappa_c < \frac{Tw}{\sqrt{2}L(1+\nu)\sqrt{C^2-1}}$$

Modified von-Mises ellipse

$$\sigma_c^2 = E^2 I \kappa_c^2 + \mu^2 J \left(\frac{Tw}{L}\right)^2$$

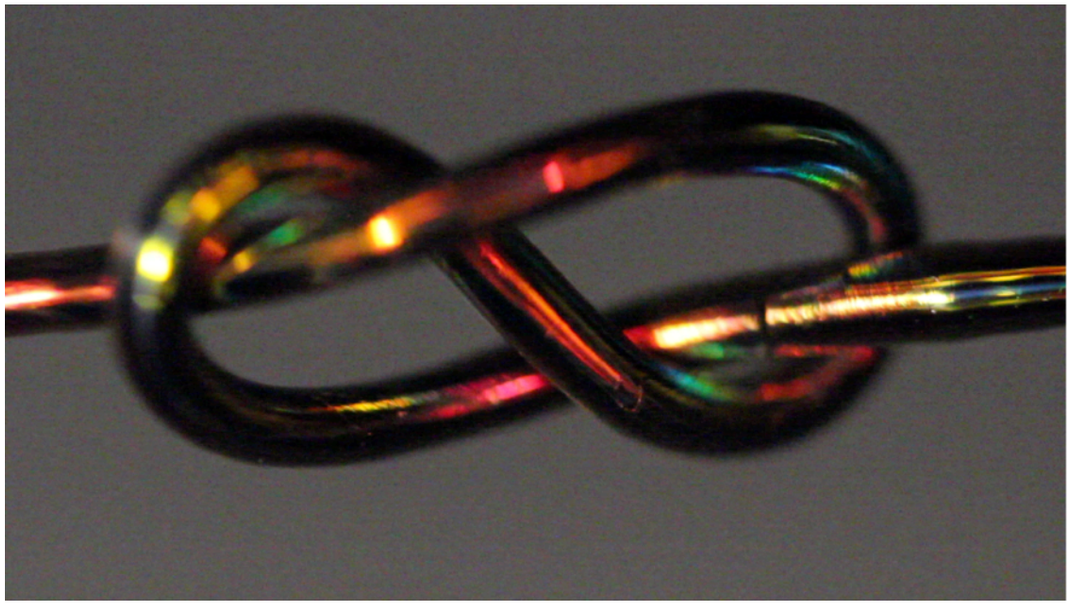


Seeing the stress in knots

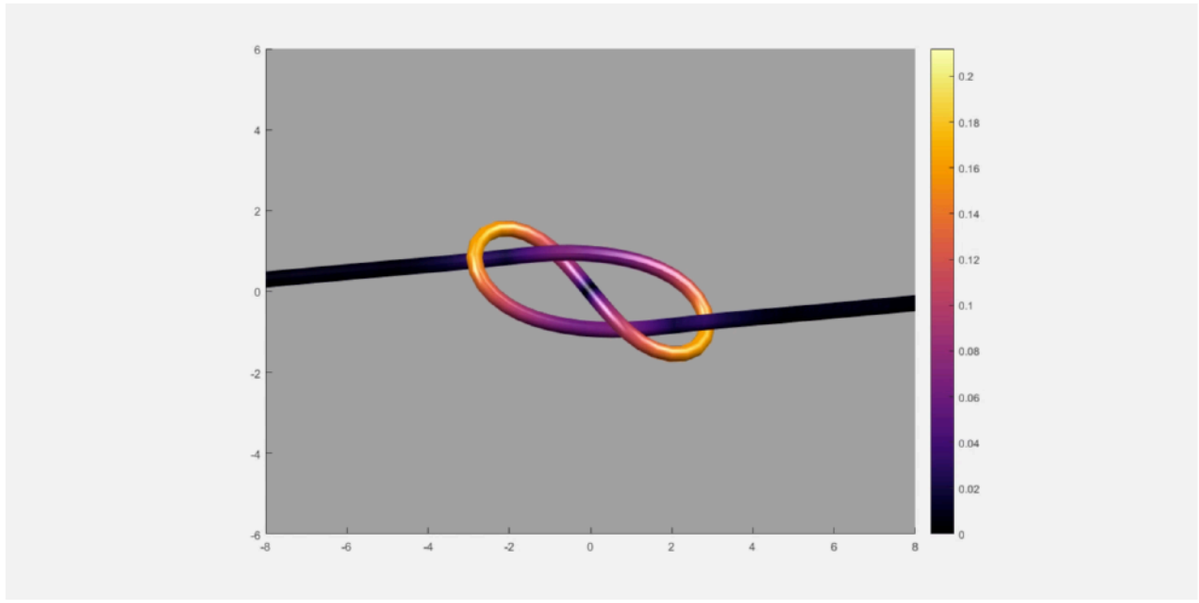
In collaboration with Mathias Kolle's lab (MIT MechE)

$$\mathcal{E} = \frac{1}{2}\int_0^L ds\left[EI\kappa_1(s,t)^2 + EI\kappa_2(s,t)^2 + \mu J\kappa_3(s,t)^2 + EA(|\mathbf{x}'(s,t)| - 1)^2\right]$$

$$\dot{\mathbf{x}} = -\delta\mathcal{E}/\delta\mathbf{x}$$



Experiment: Joseph Sandt



Simulation: Vishal Patil

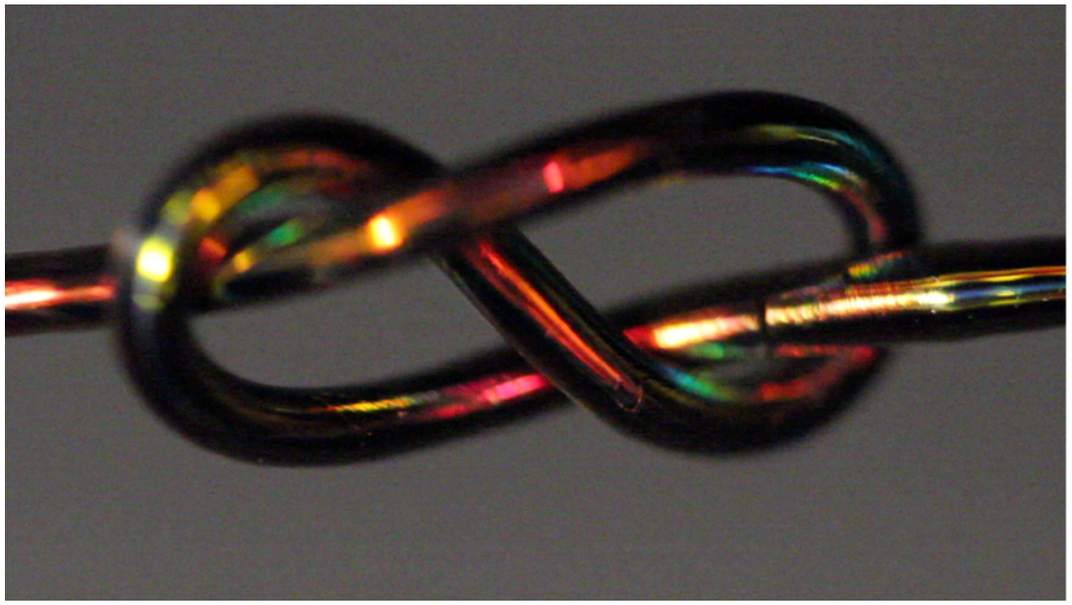


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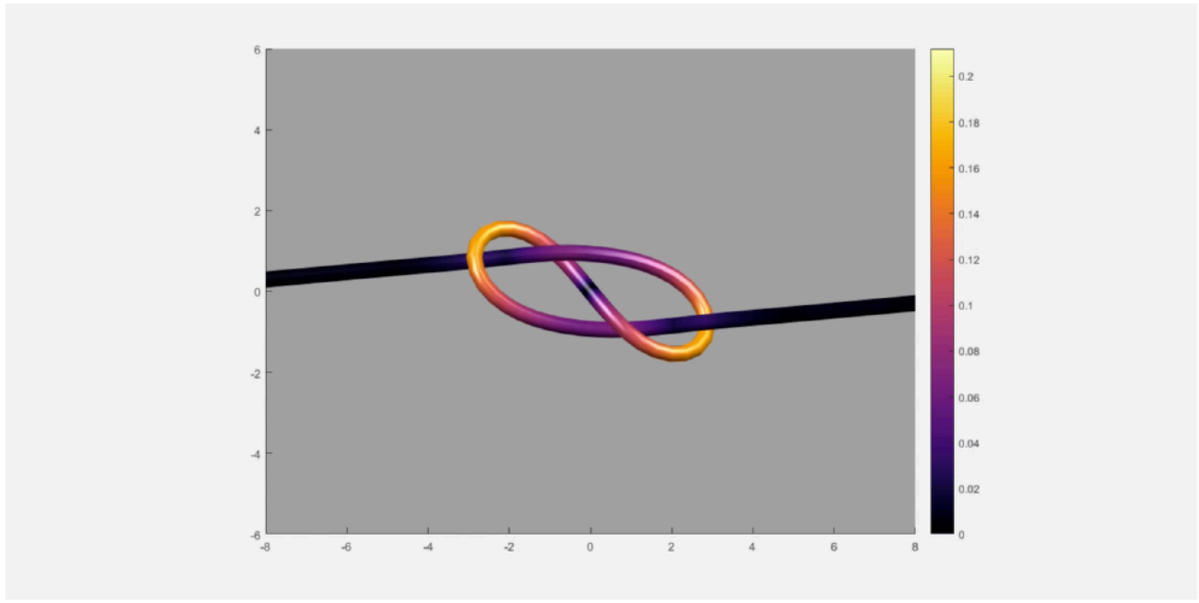
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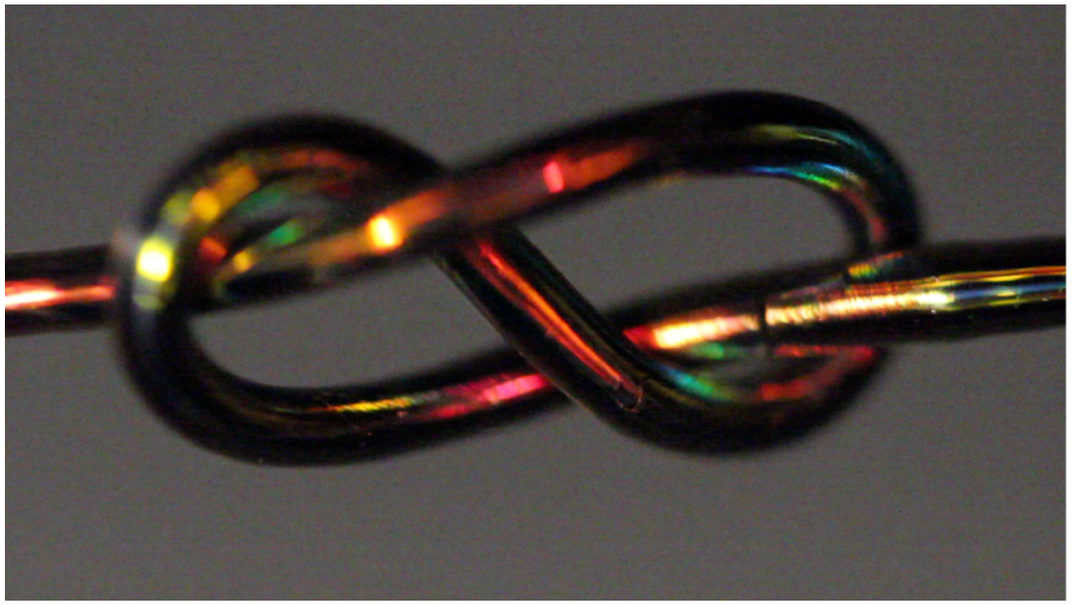


Seeing the stress in knots

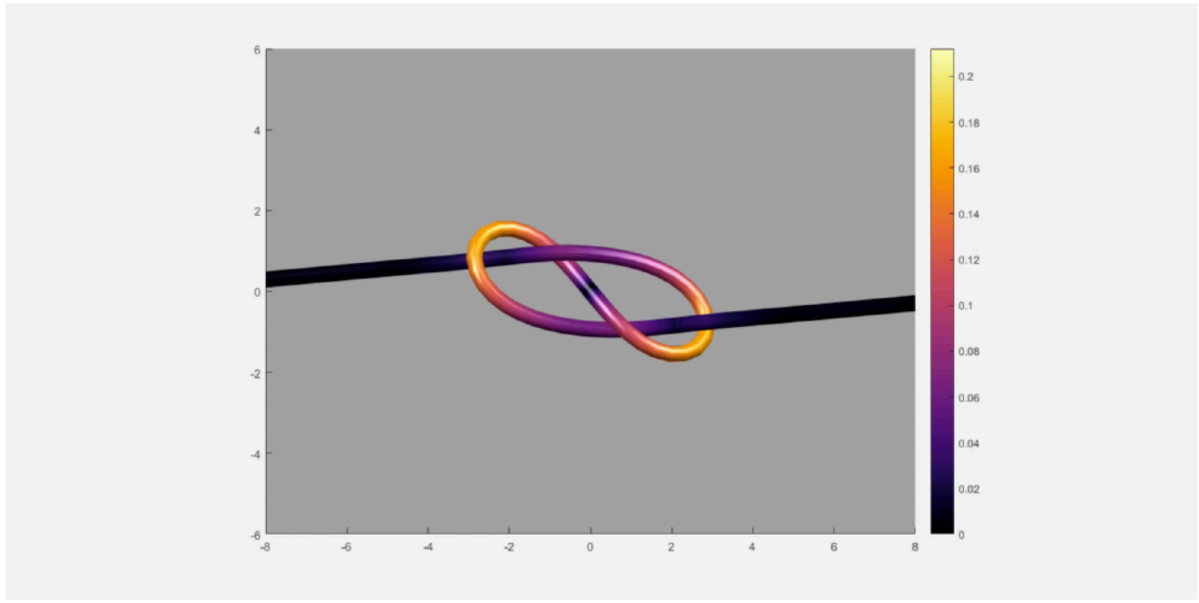
In collaboration with Mathias Kolle's lab (MIT MechE)

$$\mathcal{E} = \frac{1}{2}\int_0^L ds\left[EI\kappa_1(s,t)^2 + EI\kappa_2(s,t)^2 + \mu J\kappa_3(s,t)^2 + EA(|\mathbf{x}'(s,t)| - 1)^2\right]$$

$$\dot{\mathbf{x}} = -\delta\mathcal{E}/\delta\mathbf{x}$$



Experiment: Joseph Sandt



Simulation: Vishal Patil

Thanks to my group



Norbert Stoop
Postdoc



Philip Pearce
Postdoc/Instructor



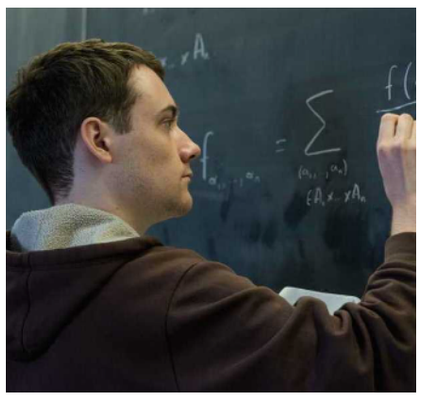
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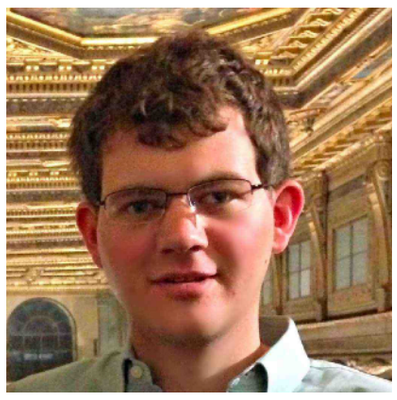
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