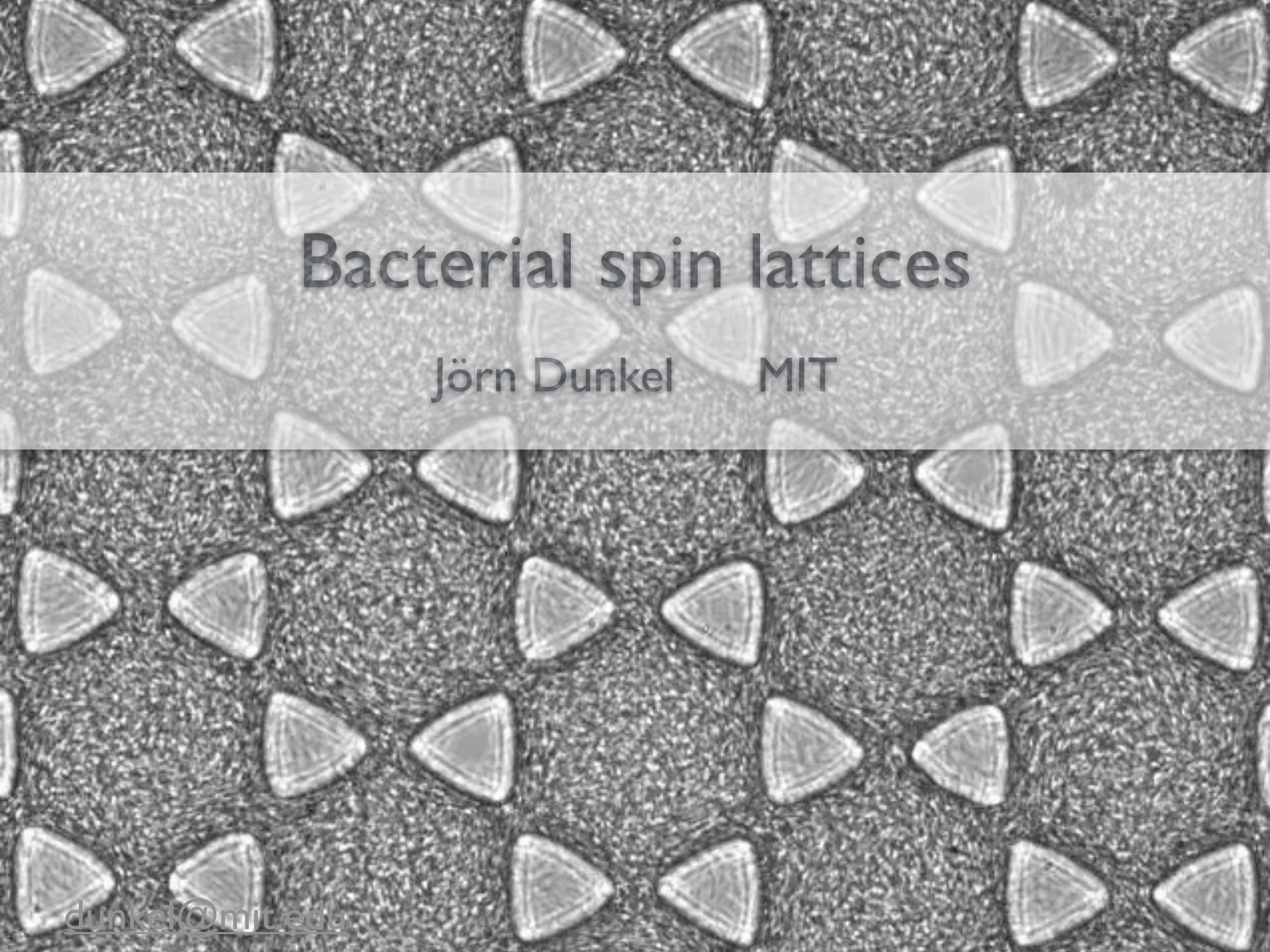


Bacterial spin lattices

Jörn Dunkel MIT

dunkel@mit.edu

Thanks to

Francis Woodhouse (DAMTP)

Norbert Stoop (MIT)

Ray Goldstein (DAMTP)

Marco Polin (Warwick)

Julia Yeomans (Oxford)

Sebastian Heidenreich (PTB)

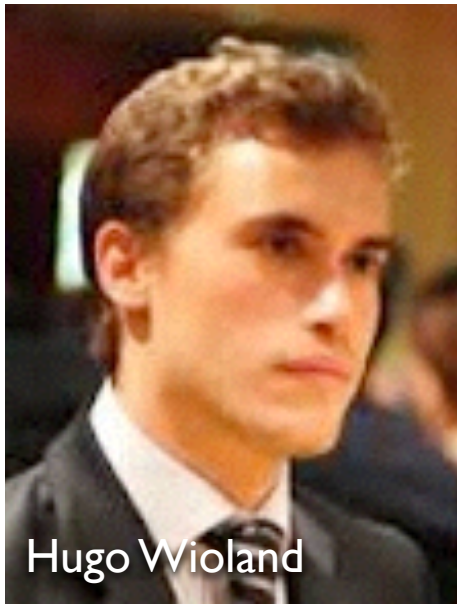
Markus Bär (PTB)

Rik Wensink (CNRS Paris)

Hartmut Löwen (Düsseldorf)

UNIVERSITY OF CAMBRIDGE

THE UNIVERSITY OF WARWICK







erc

ALFRED P. SLOAN FOUNDATION 1934

MIT

How do cells

- swim individually and collectively ?
- navigate in fluid flow ?
- interact with physical structures ?

Can we control microbial swimming ?

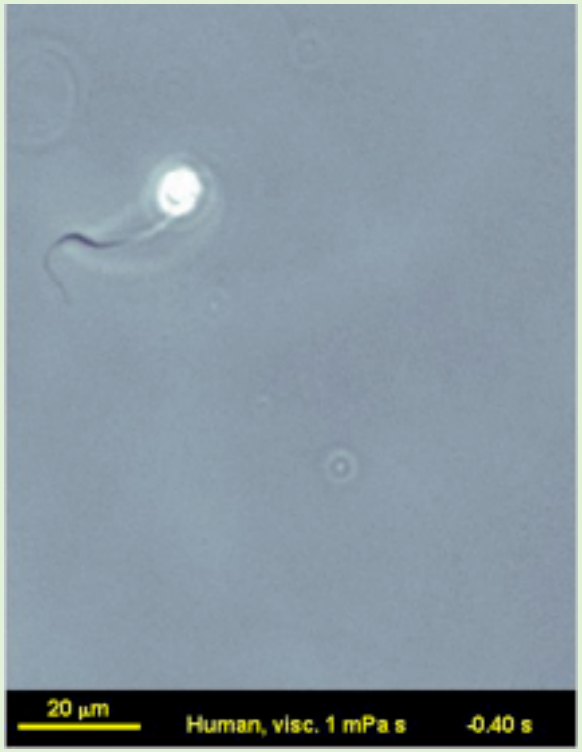
Model organisms

Chlamydomonas alga



Drescher et al (2010) PRL
Guasto et al (2010) PRL

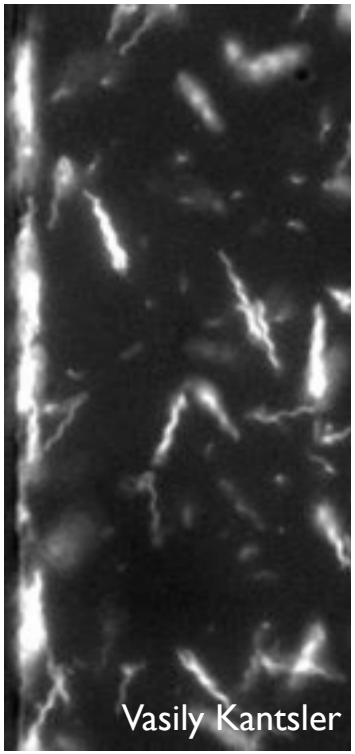
Human sperm cell



Kantsler et al (2014) eLife
Friedrich et al (2010) JEB

Eukaryotes

Bacteria



Drescher et al (2011) PNAS

Prokaryote

How do they swim ?

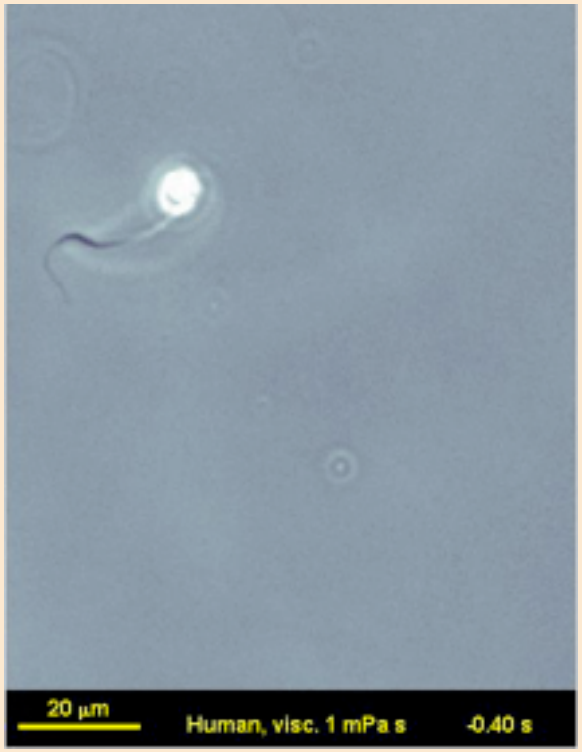
Chlamydomonas alga



Drescher et al (2010) PRL
Guasto et al (2010) PRL

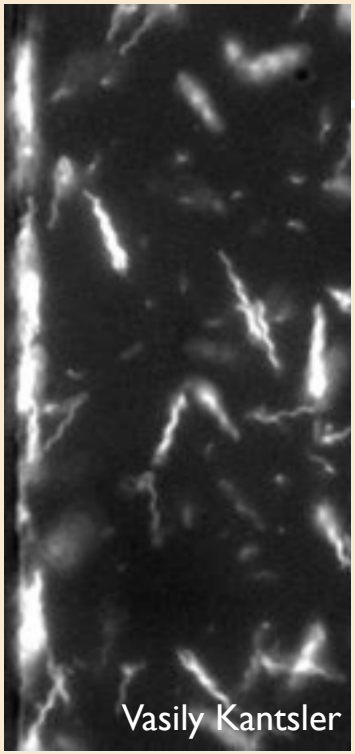
Puller

Human sperm cell



Kantsler et al (2014) eLife
Friedrich et al (2010) JEB

Bacteria



Drescher et al (2011) PNAS

Pusher

Swimming at low Reynolds number

Navier - Stokes:

$$-\nabla p + \eta \nabla^2 \vec{v} = \rho \frac{\partial \vec{v}}{\partial t} + \rho (\vec{v} \cdot \nabla) \vec{v}$$

If $\mathcal{R} \sim UL\rho/\eta \ll 1$

Time doesn't matter. The pattern of motion is the same, whether slow or fast, whether forward or backward in time.

The Scallop Theorem



American Journal of Physics, Vol. 45, No. 1, January 1977



G I Taylor



James Lighthill



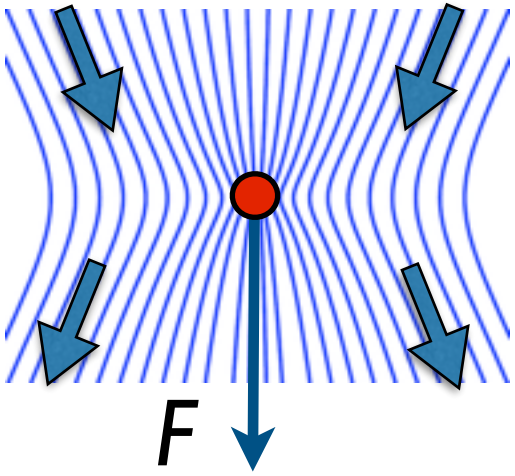
Edward Purcell

$$0 = \mu \nabla^2 \boldsymbol{u} - \nabla p + \boldsymbol{f},$$
$$0 = \nabla \cdot \boldsymbol{u}.$$

+ time-dependent BCs

Superposition of singularities

stokeslet



$$p(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{F}}{4\pi r^2} + p_0$$

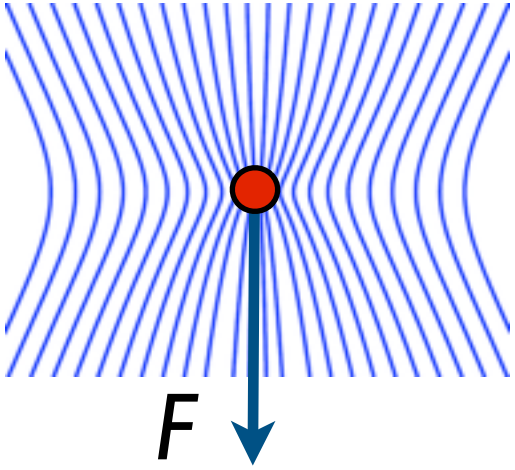
$$v_i(\boldsymbol{r}) = \frac{(8\pi\mu)^{-1}}{r}[\delta_{ij} + \hat{r}_i \hat{r}_j]F_j$$

flow ~ r^{-1}

in 3D

Superposition of singularities

stokeslet

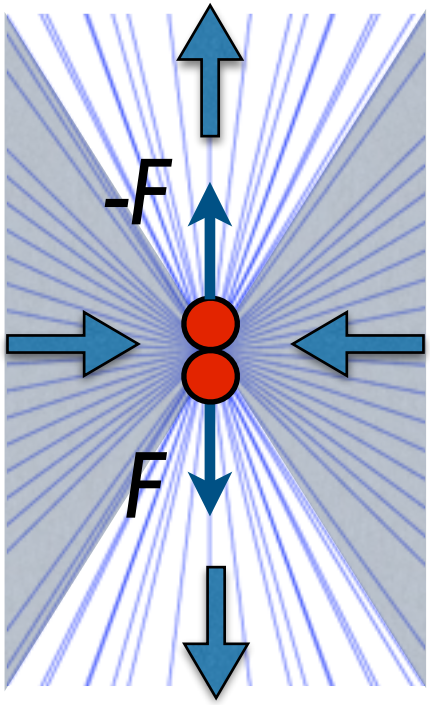


$$p(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{F}}{4\pi r^2} + p_0$$

$$v_i(\boldsymbol{r}) = \frac{(8\pi\mu)^{-1}}{r}[\delta_{ij} + \hat{r}_i \hat{r}_j]F_j$$

flow ~ r^{-1}

2 x stokeslet =
symmetric dipole

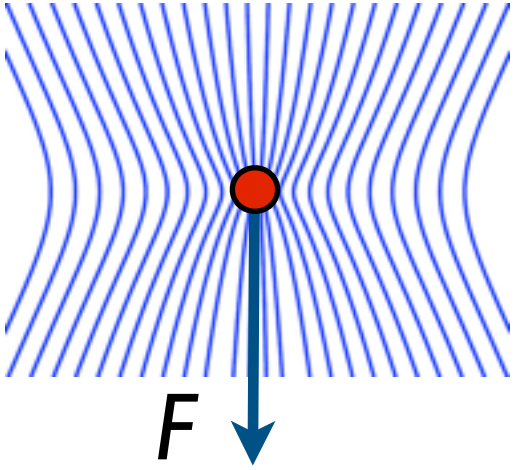


r^{-2}

'pusher' dipole

Superposition of singularities

stokeslet

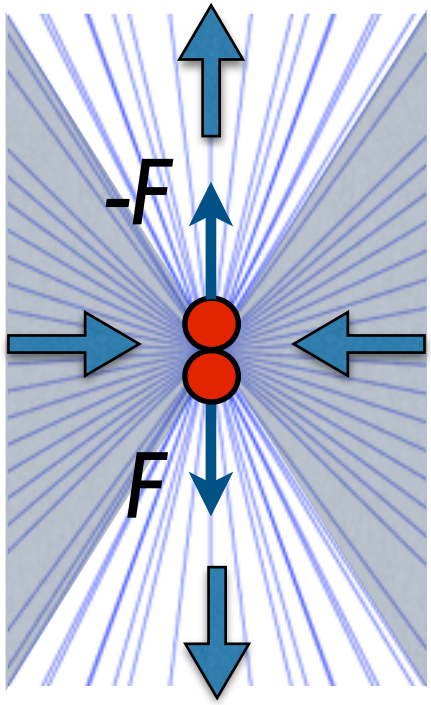


$$p(\boldsymbol{r}) = \frac{\hat{\boldsymbol{r}} \cdot \boldsymbol{F}}{4\pi r^2} + p_0$$

$$v_i(\boldsymbol{r}) = \frac{(8\pi\mu)^{-1}}{r}[\delta_{ij} + \hat{r}_i \hat{r}_j]F_j$$

flow ~ r^{-1}

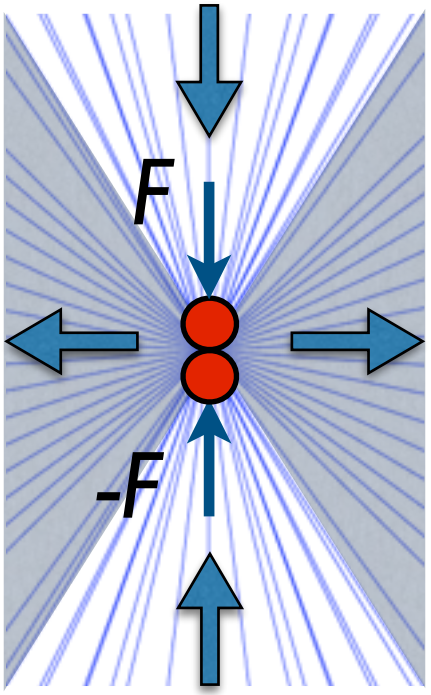
2 x stokeslet = symmetric dipole



r^{-2}

'pusher' dipole

2 x stokeslet = symmetric dipole

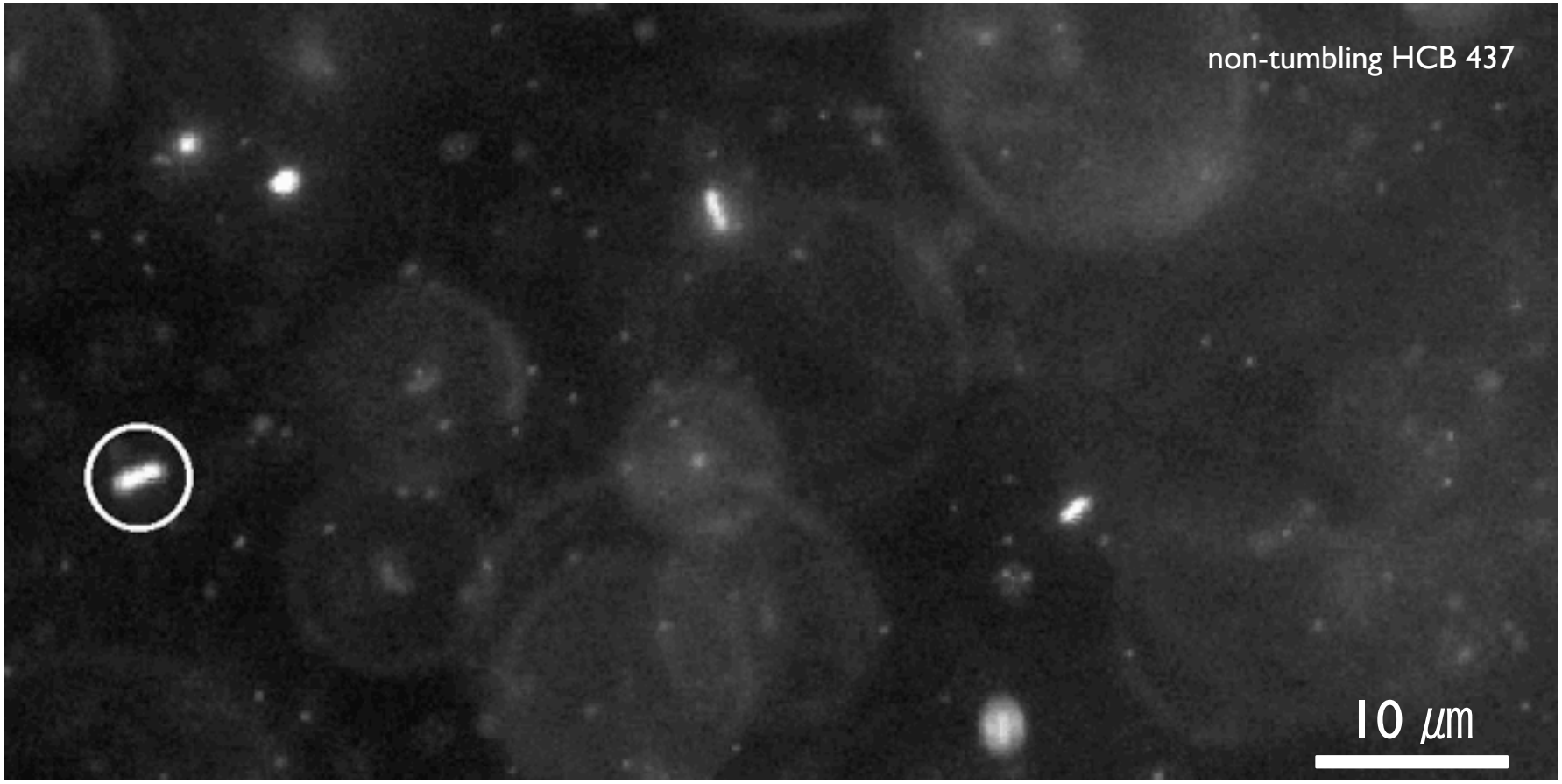


r^{-2}

'puller' dipole

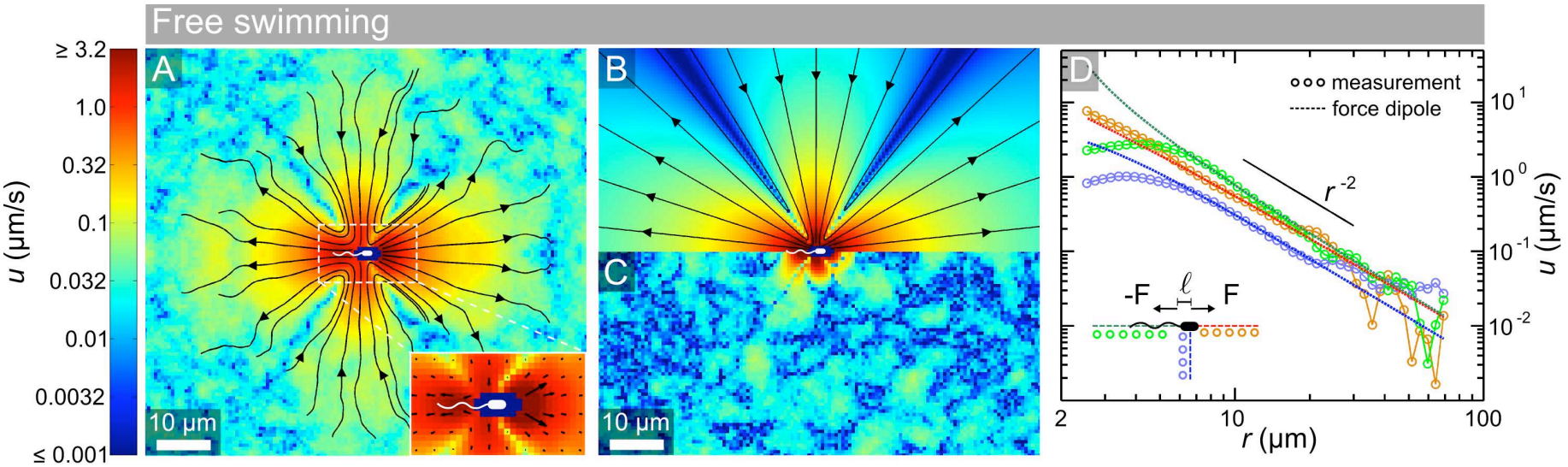
E. coli (non-tumbling)





Drescher et al (2011) PNAS

E.coli (non-tumbling HCB 437)



$$\boldsymbol{u}(\boldsymbol{r}) = \frac{A}{|\boldsymbol{r}|^2}\left[3(\hat{\boldsymbol{r}}.\hat{\boldsymbol{d}})^2 - 1\right]\hat{\boldsymbol{r}}, \quad A = \frac{\ell F}{8\pi\eta}, \quad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$V_0 = 22 \pm 5 \ \mu\text{m/s}$$
$$\ell = 1.9 \ \mu\text{m}$$
$$F = 0.42 \ \text{pN}$$

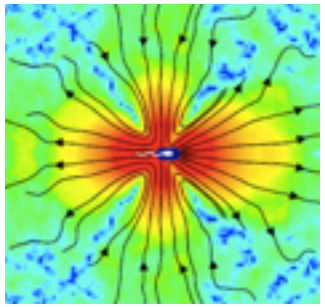
'pusher' dipole

Drescher et al (2011) PNAS

Implications for collective swimming

- hydrodynamic advection

$$\boldsymbol{v} \sim \frac{A}{r^2}$$



- steric alignment
- hydrodynamic alignment less important

$$\omega = \nabla \times \boldsymbol{v} \sim \frac{A}{r^3}$$

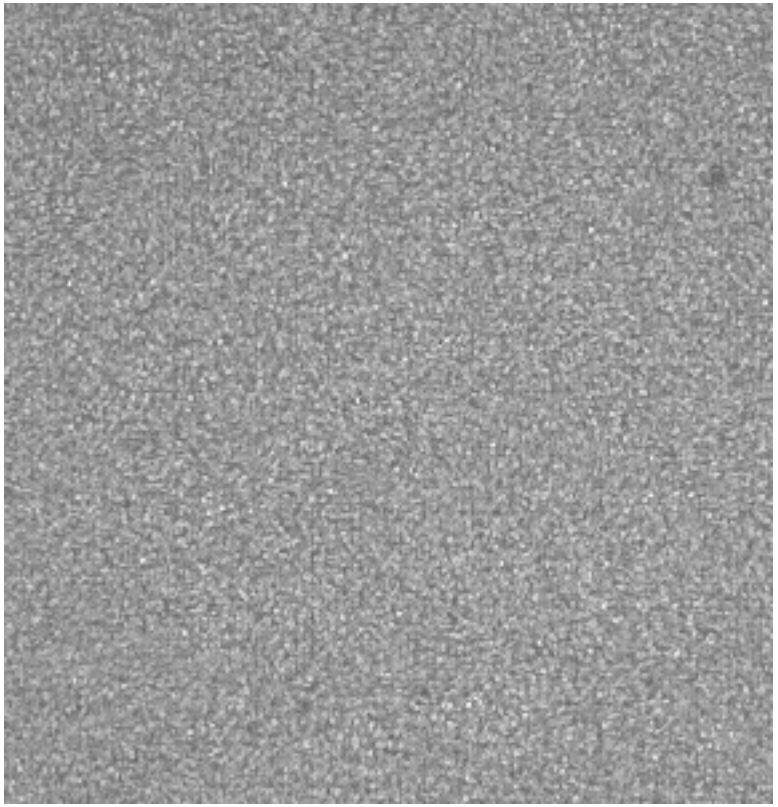
- intrinsic rotational noise
  much larger than thermal noise

$$D_r = 0.057 \text{ rad}^2/\text{s}$$

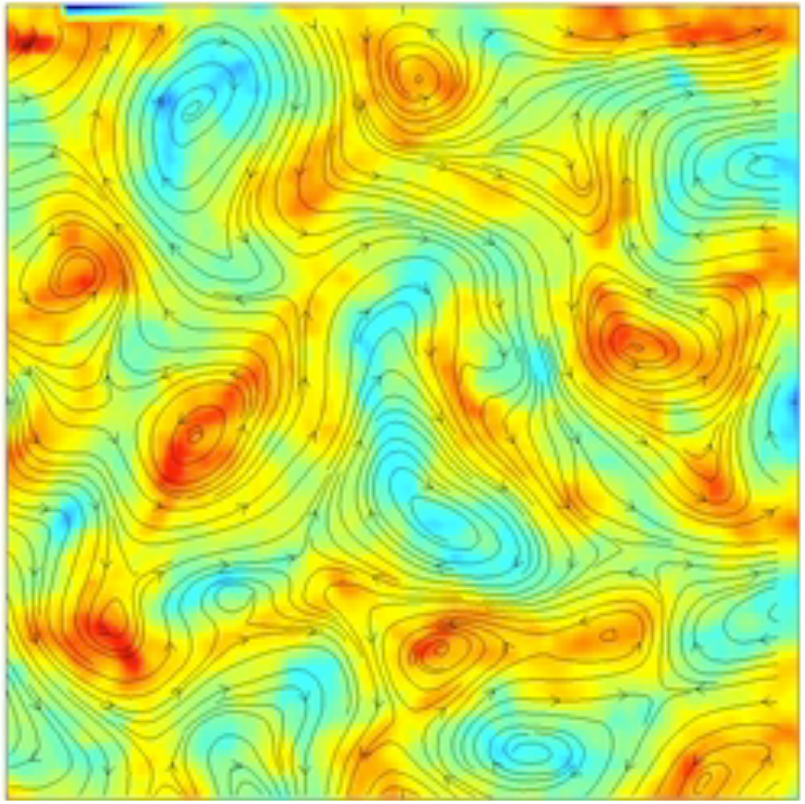
Drescher et al (2011) PNAS

Bacterial ‘turbulence’

Bacillus subtilis



bright field



PIV

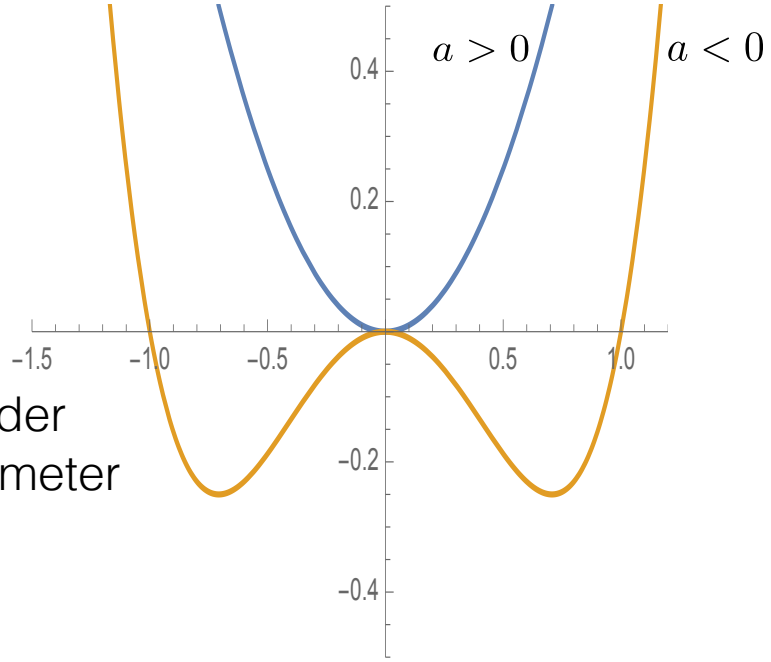
Dunkel et al (2013) PRL

see also:
Sokolov & Aronson (2012) PRL

Theory: basic idea

Order-parameter space

potential

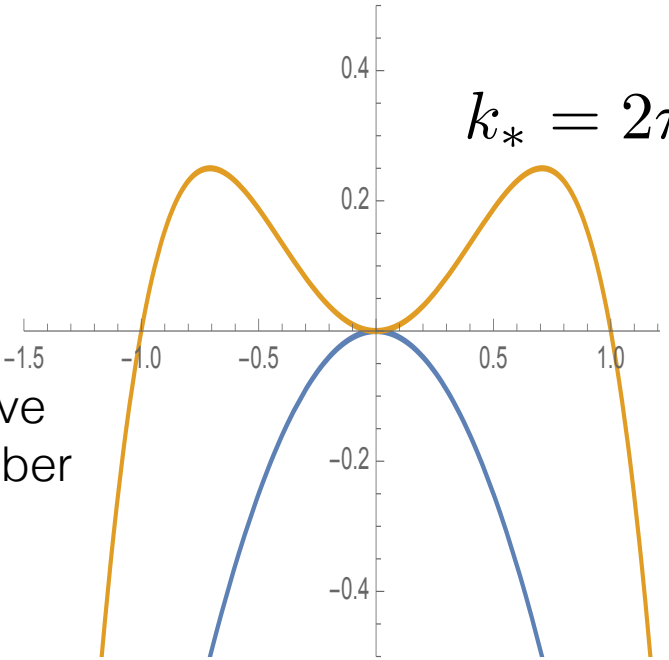


order parameter

$$a\phi^2 + b\phi^4$$

Fourier space

dispersion



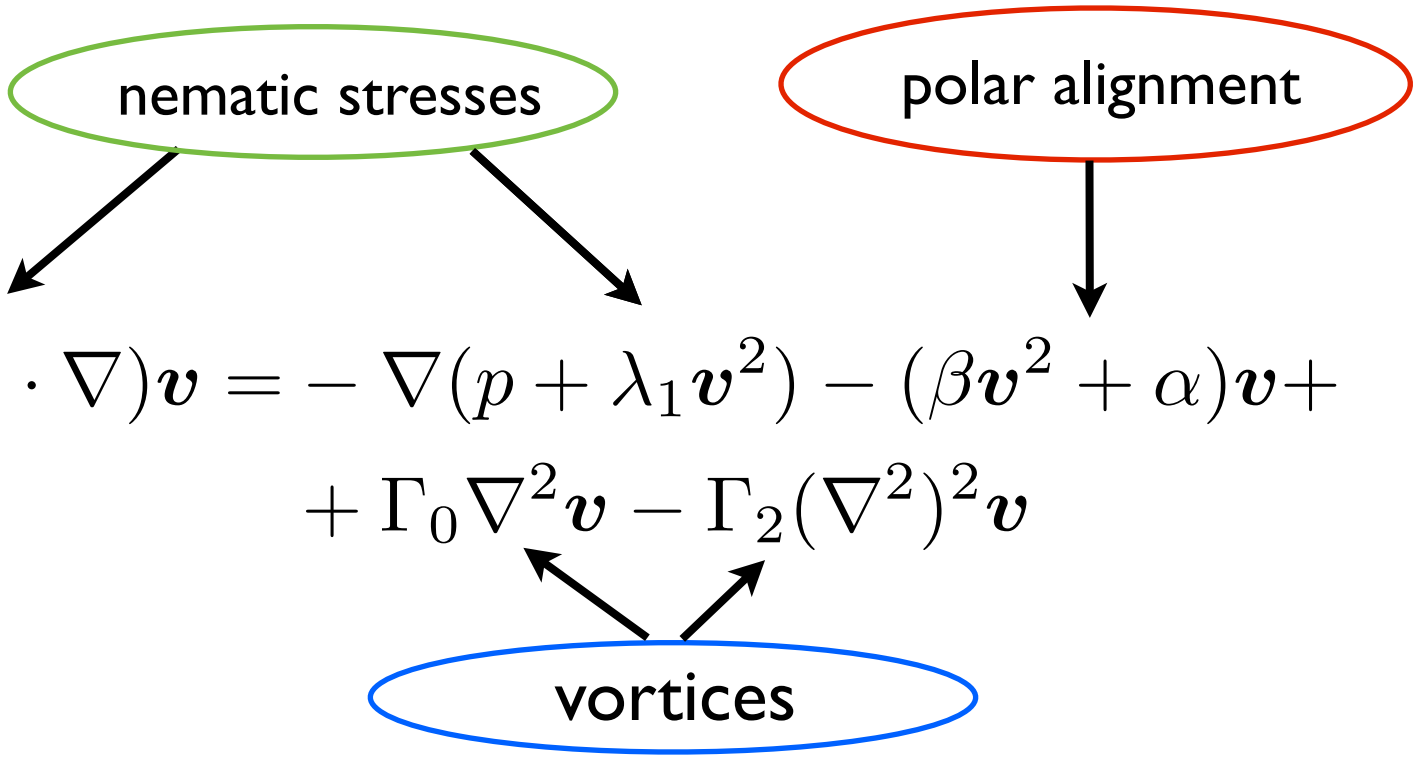
wave number

$$\alpha k^2 + \beta k^4$$

$$-\alpha\nabla^2 + \beta(\nabla^2)^2$$

Minimal continuum theory for bacterial velocity field

incompressibility $\nabla \cdot \boldsymbol{v} = 0$

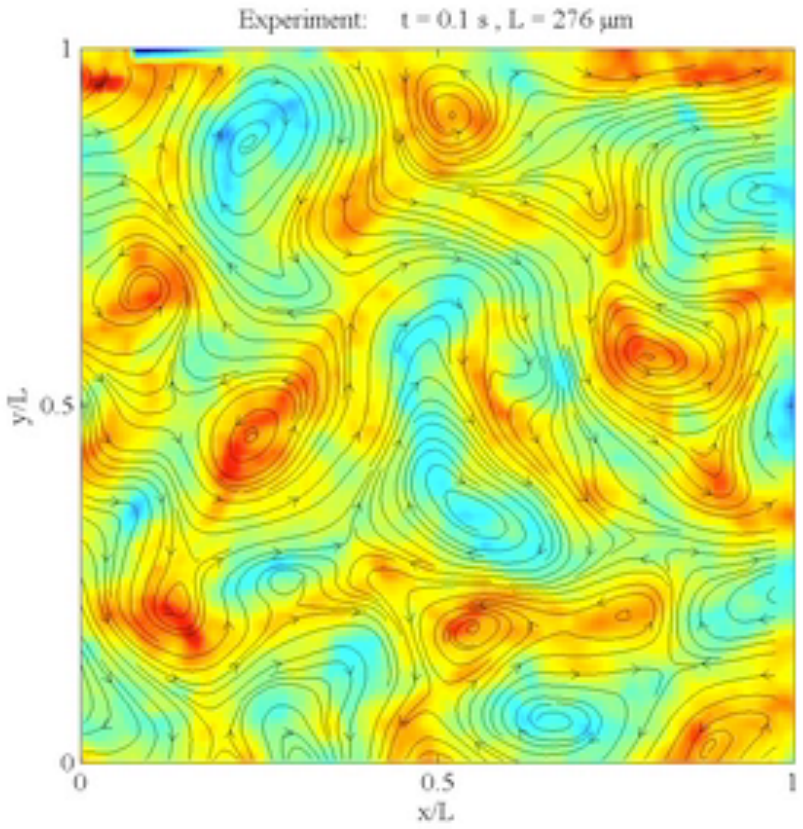


Dunkel et al (2014) PRL

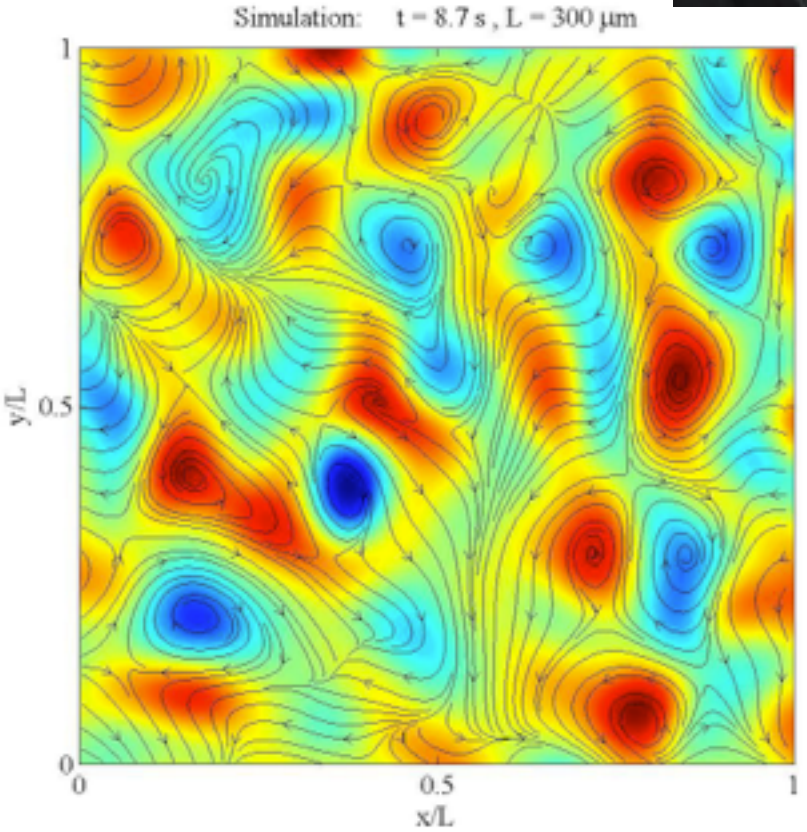
Wensink et al (2012) PNAS

experiment *vs.* theory





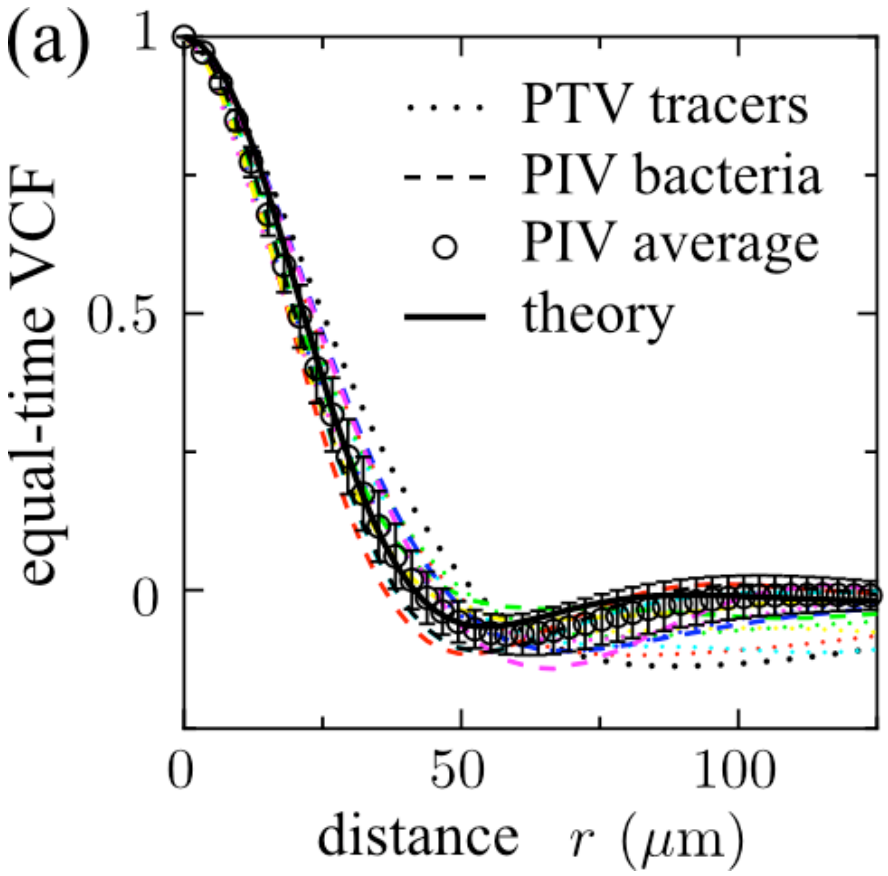
quasi-2D slice



2D slice
from 3D simulation

PRL 2013

Velocity correlations



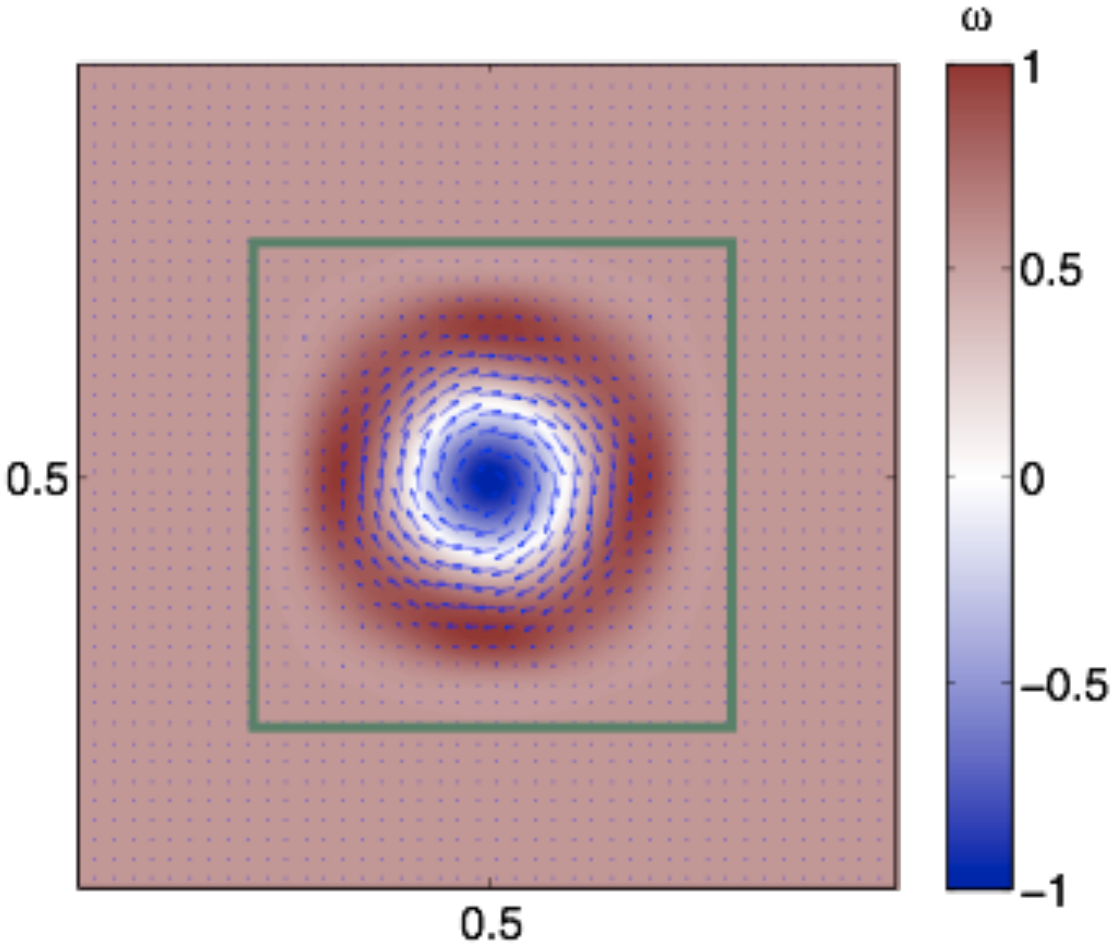
Vortex diameter ~ 70μm

PRL 2013

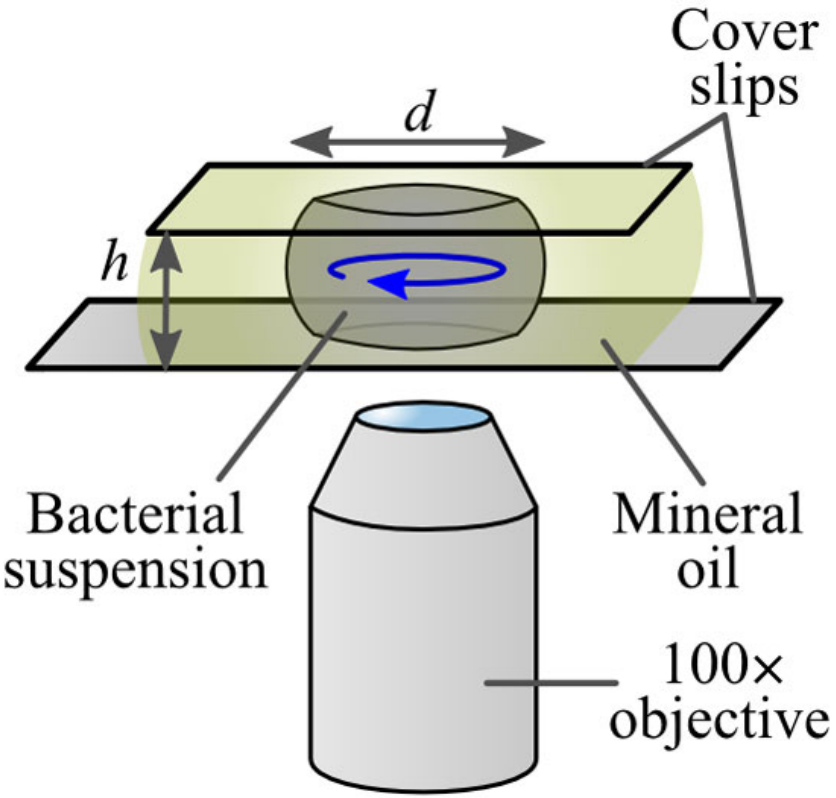
consistent with Sokolov & Aronson (2012) PRL

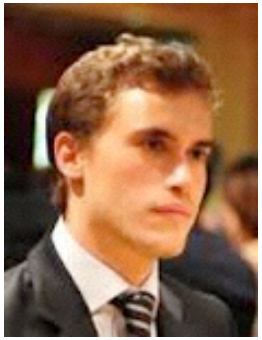
Can we stabilize vortices ?

‘prediction’ of many models



Can we stabilize vortices ?

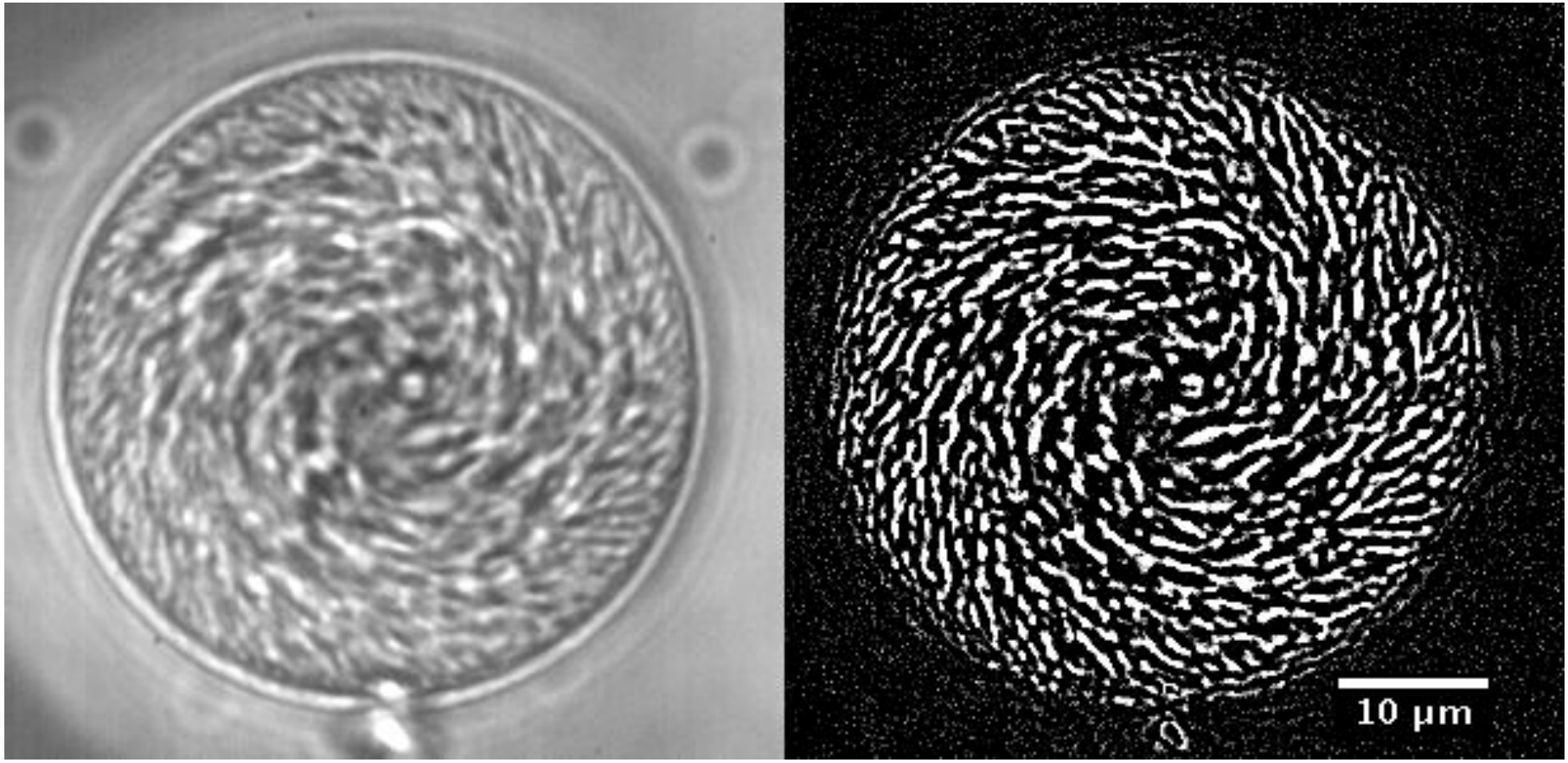




Wioland et al (2013) PRL

Stable bacterial spiral vortex

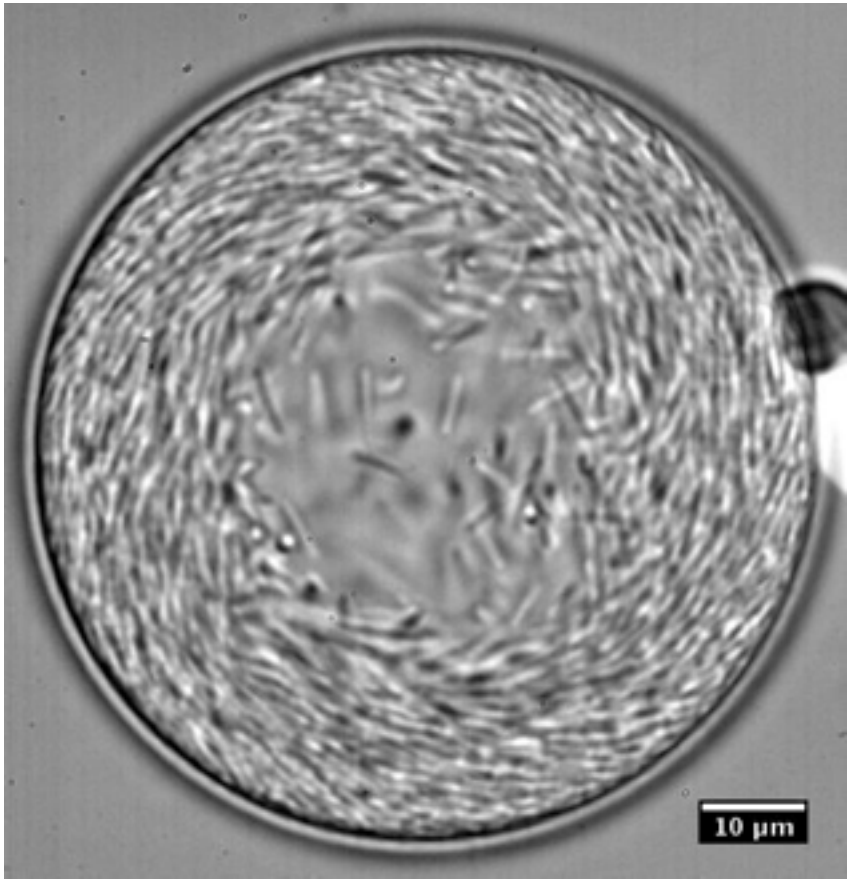
Vortex life time ~ minutes



Wioland et al (2013) PRL
Lushi et al (2014) PNAS

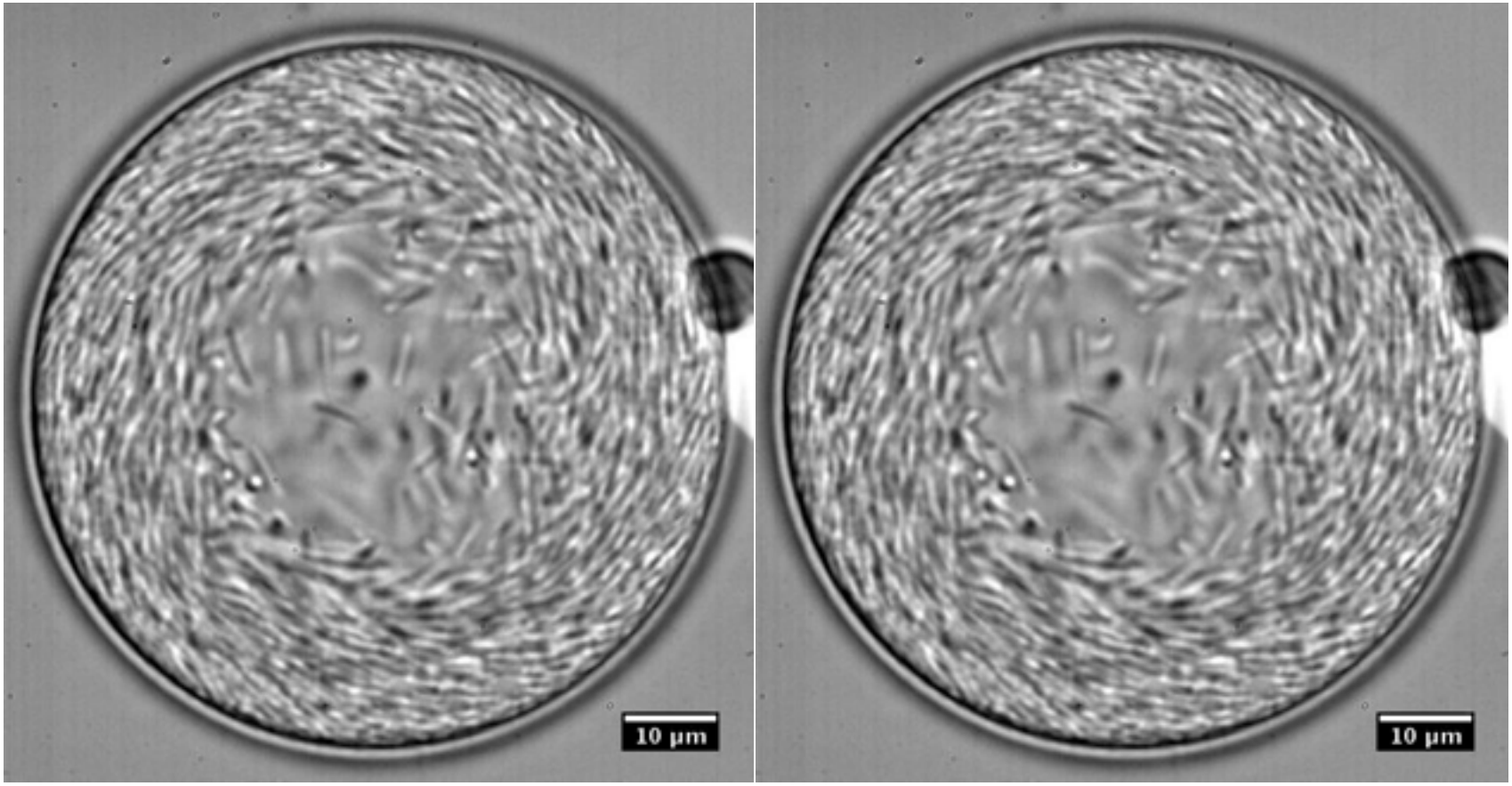
Edge currents !

Edge currents



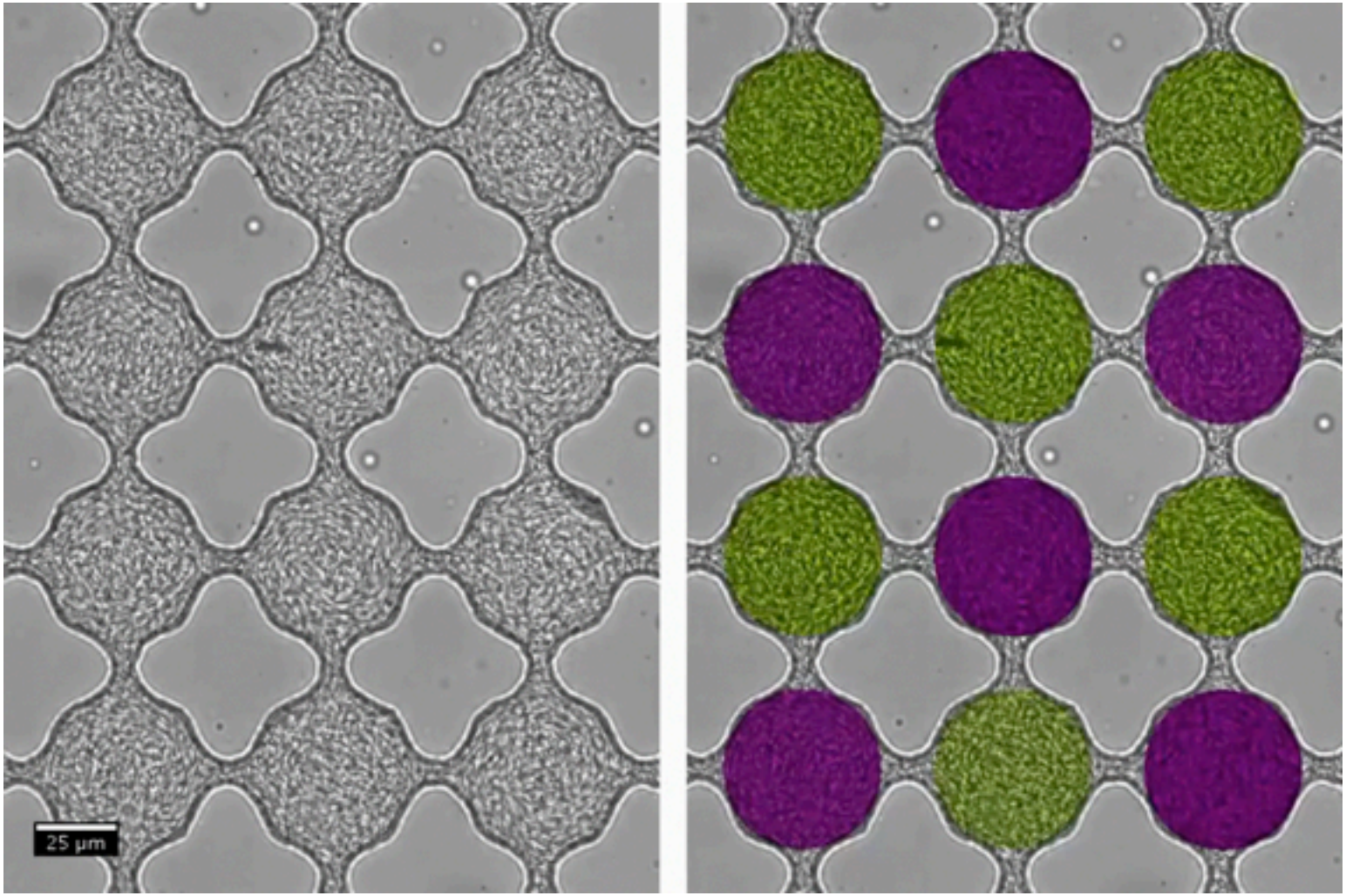
Wioland et al (2013) PRL
Lushi et al (2014) PNAS

Can we couple vortices ?



Wioland et al (2013) PRL
Lushi et al (2014) PNAS

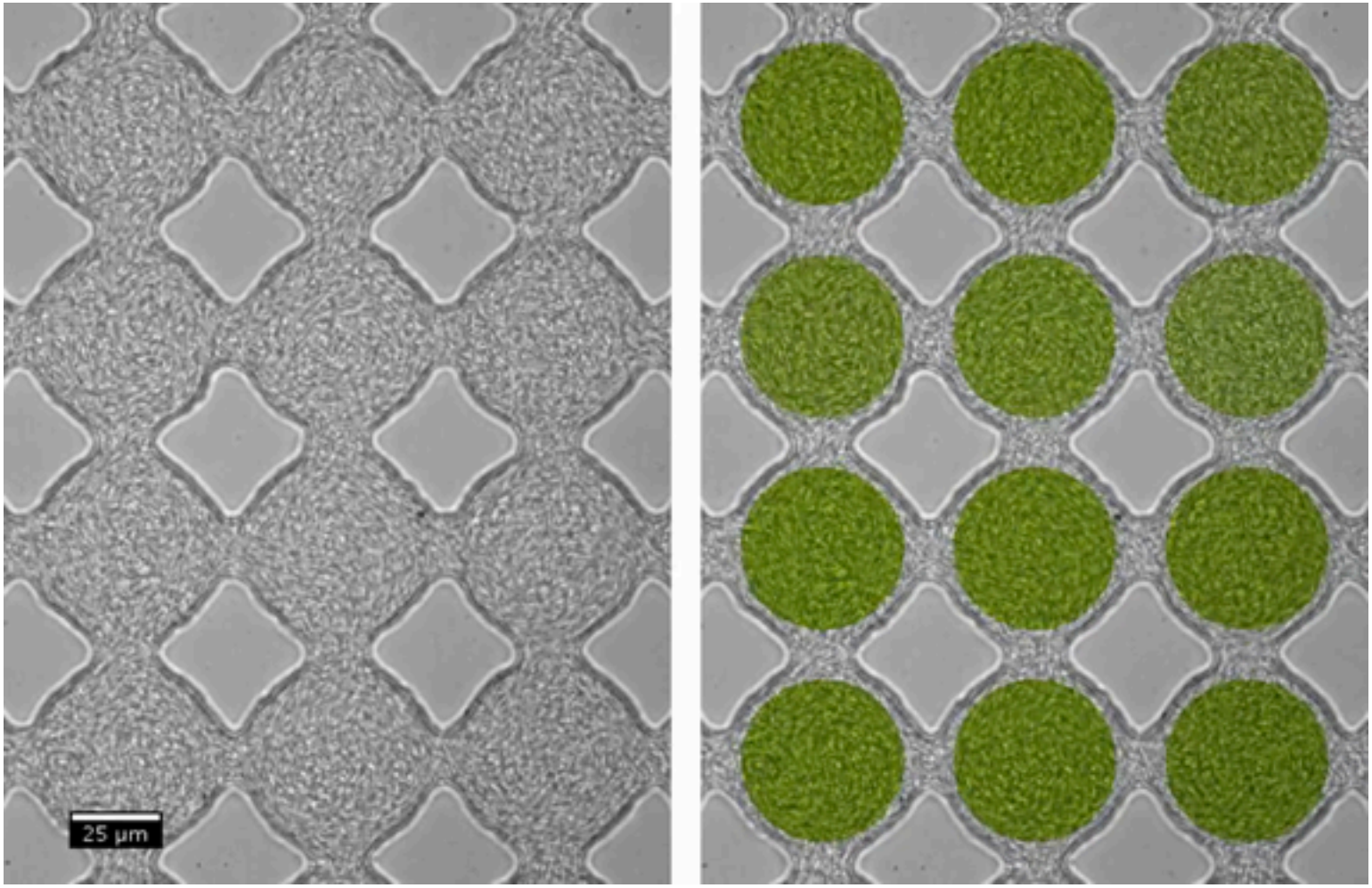
Anti-ferromagnetic order



Wioland et al (2013)

but only for weak coupling (small gaps)

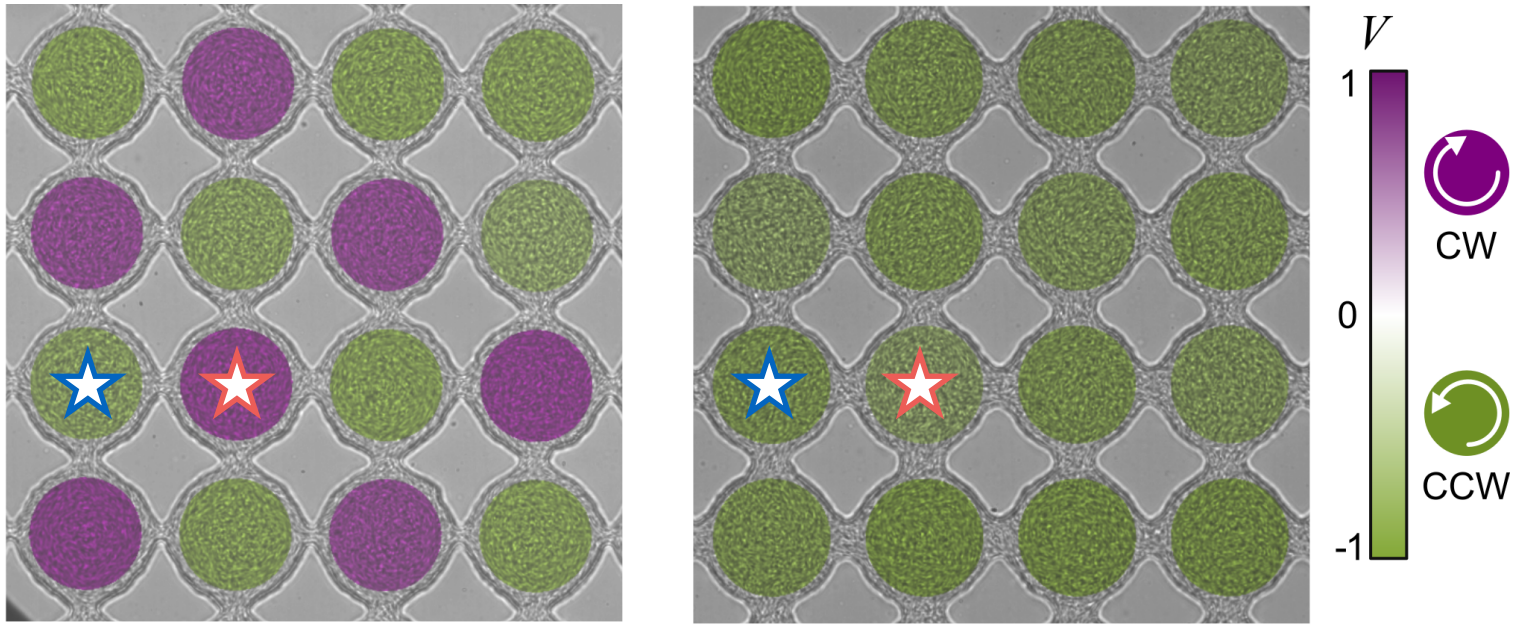
Ferromagnetic order

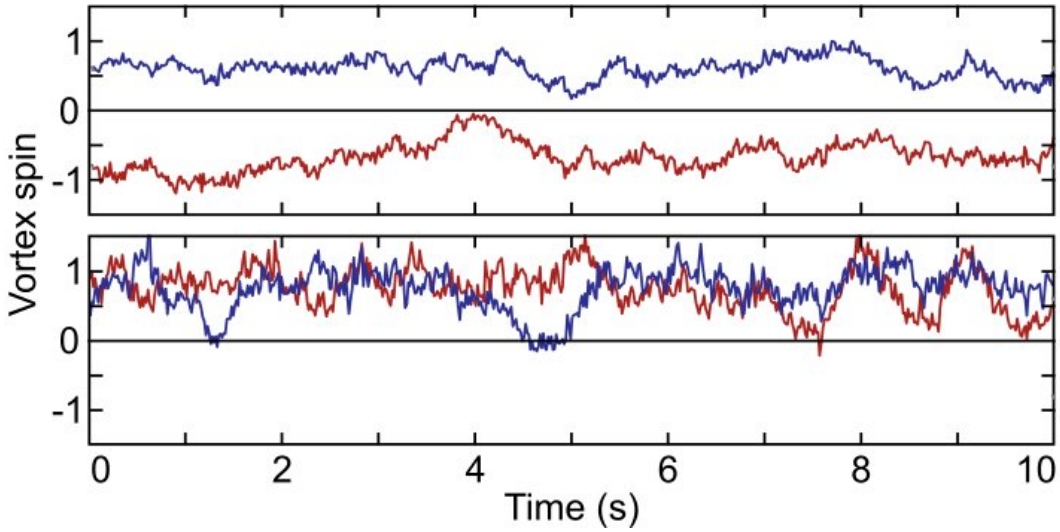


Wioland et al (2013)

driven by edge currents around pillars

Vortex-pair dynamics

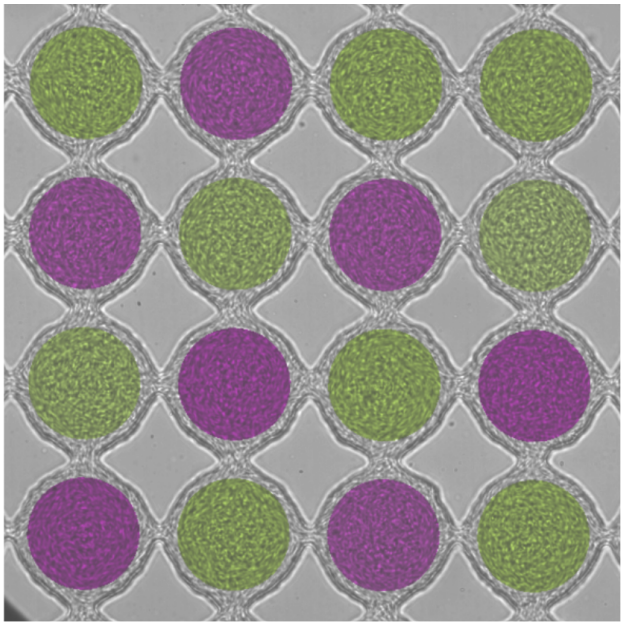


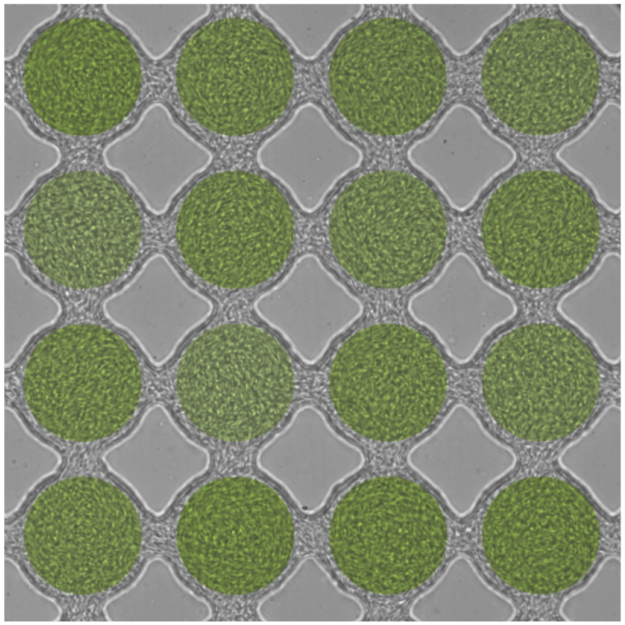


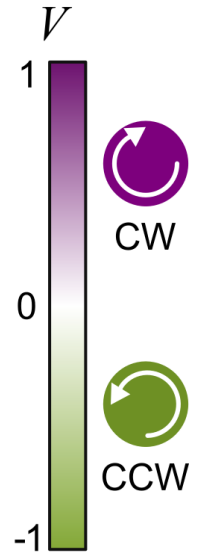
Anti-Ferro

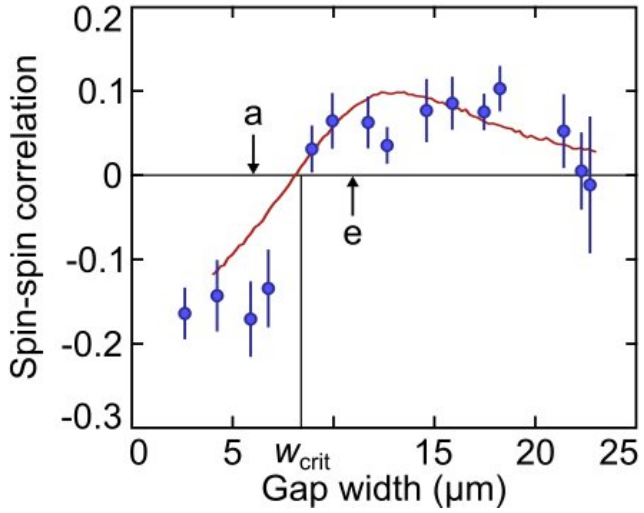
Ferro

Vortex-pair correlation



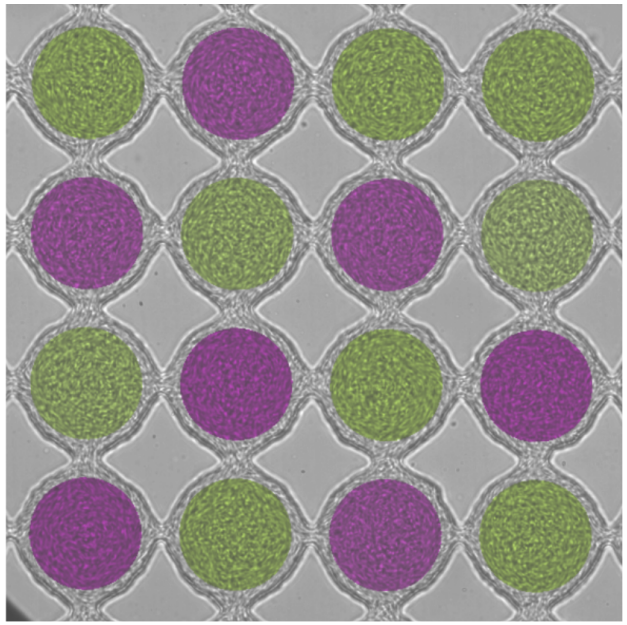


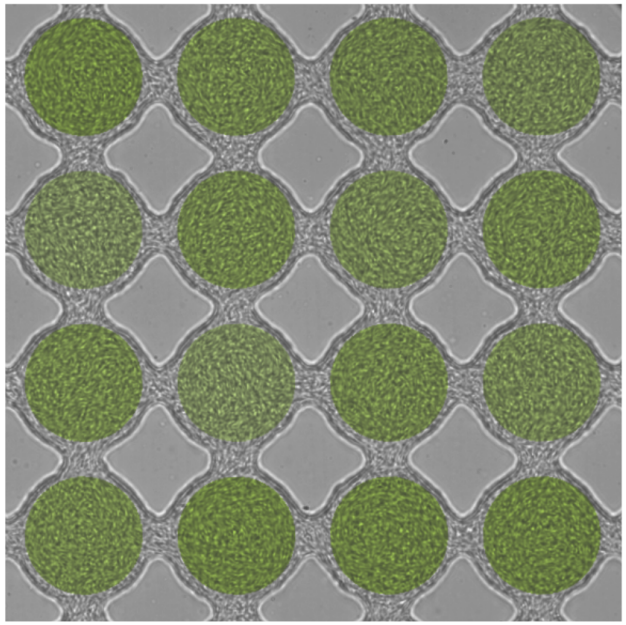


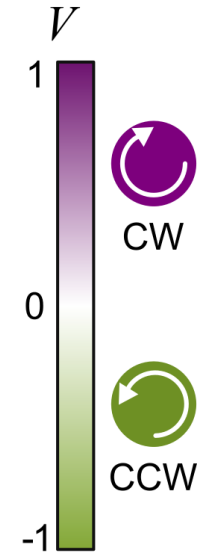


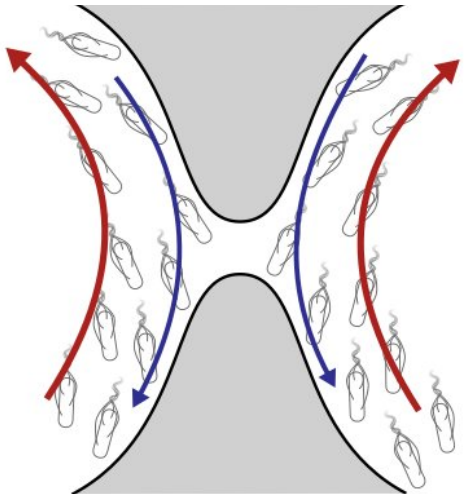
Wioland et al
Nature Physics 2016

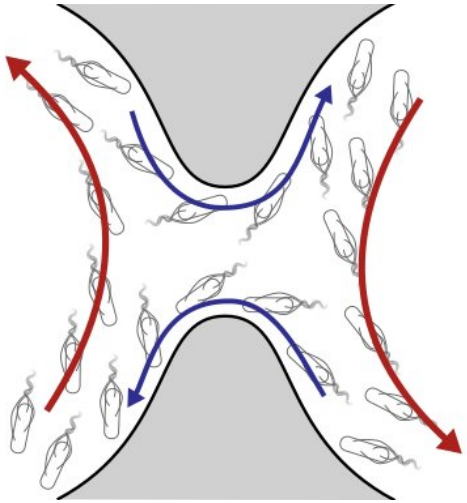
Vortex & edge-current coupling



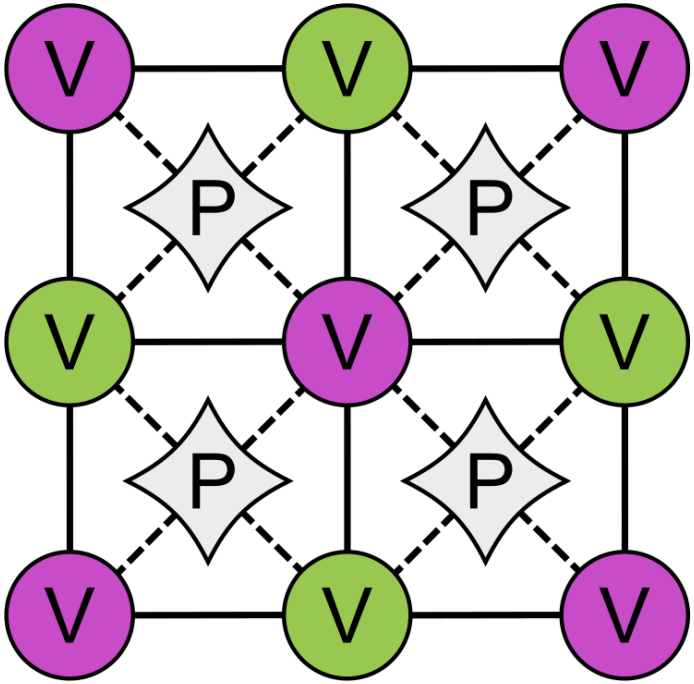


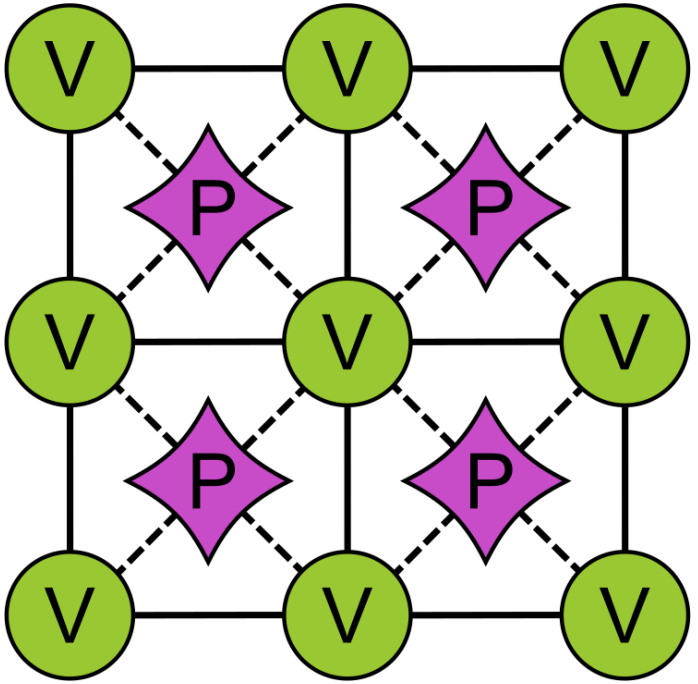






anti-ferro *vs.* ferro





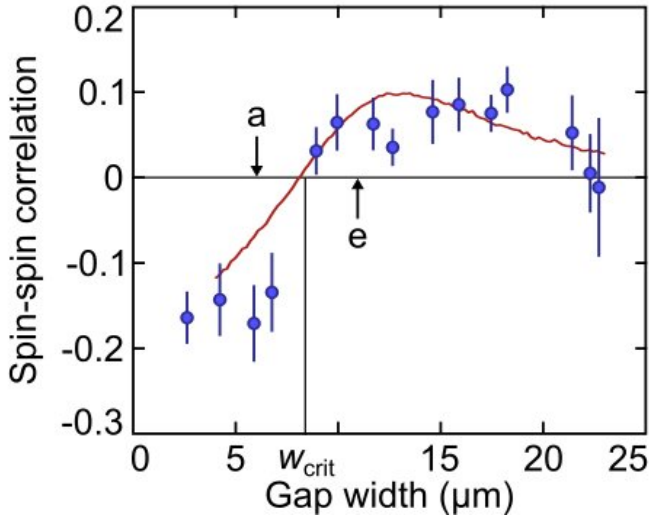
$$H(\mathbf{V},\mathbf{P}) = -J_v \sum_{V_i \sim V_j} V_i V_j - J_p \sum_{V_i \sim P_j} V_i P_j + \sum_{V_i} \left(\tfrac{1}{2} a_v V_i^2 + \tfrac{1}{4} b_v V_i^4\right) + \sum_{P_j} \tfrac{1}{2} a_p P_j^2$$

LFT - Langevin dynamics

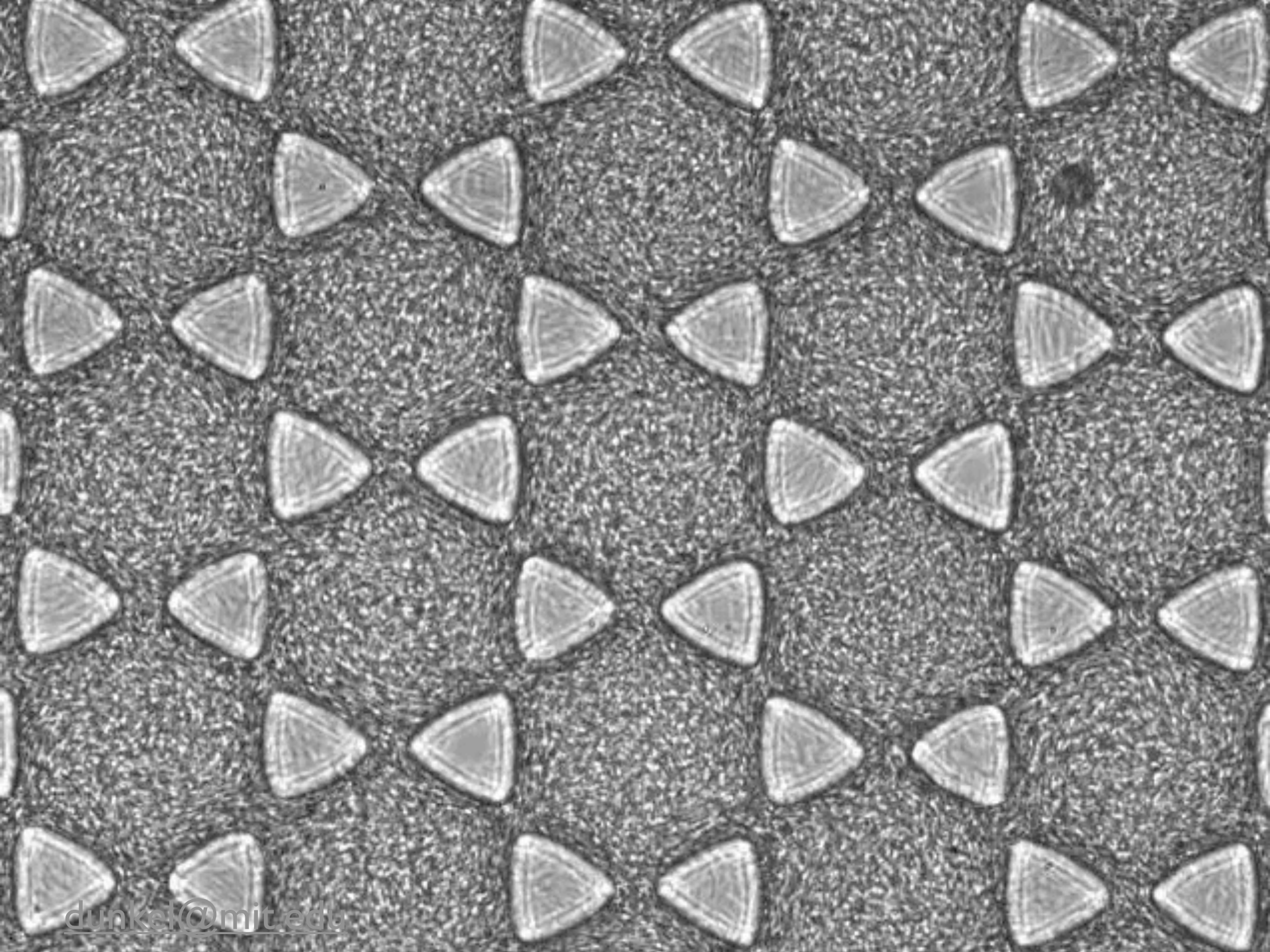
$$H(\mathbf{V},\mathbf{P}) = -J_v \sum_{V_i \sim V_j} V_i V_j - J_p \sum_{V_i \sim P_j} V_i P_j + \sum_{V_i} \left(\tfrac{1}{2} a_v V_i^2 + \tfrac{1}{4} b_v V_i^4\right) + \sum_{P_j} \tfrac{1}{2} a_p P_j^2$$

$$d\mathbf{V} = -(\partial H/\partial \mathbf{V})dt + \sqrt{2T_v}d\mathbf{W}_v$$

$$d\mathbf{P} = -(\partial H/\partial \mathbf{P})dt + \sqrt{2T_p}d\mathbf{W}_p$$



Wioland et al
Nature Physics 2016


dunkel@mit.edu

1. Bacteria interactions: steric + HD dipole

2. Collective vortex scale selection

3. Bacterial dynamics in idealized porous media can be described as spin systems

dunkel@mit.edu