and from this we find

$$\begin{aligned}\partial_k G_{ij} &= -\frac{\hat{x}_k}{|\boldsymbol{x}|} G_{ij} + \frac{\kappa}{|\boldsymbol{x}|^2}\left(\Pi_{ik}\hat{x}_j + \Pi_{jk}\hat{x}_i\right) \\ &= \frac{\kappa}{|\boldsymbol{x}|^2}\big(-\hat{x}_k\delta_{ij} + \hat{x}_j\delta_{ik} + \hat{x}_i\delta_{jk} - 3\hat{x}_k\hat{x}_i\hat{x}_j\big). \end{aligned} \tag{5.64}$$

Inserting this expression into (5.59), we obtain the far-field dipole flow in 3D

$$\boldsymbol{u}(\boldsymbol{x}) = \frac{F\ell}{4\pi\mu|\boldsymbol{x}|^2}\left[3(\boldsymbol{n}\cdot\hat{\boldsymbol{x}})^2 - 1\right]\hat{\boldsymbol{x}}. \tag{5.65}$$

As shown in Ref. [DDC$^+$11], Eq. (5.65) agrees well with the mean flow-field of a bacterium.