3D case To compute the dipole flow field in 3D, we need to compute the partial derivatives of the Oseen tensor

$$G_{ij}(\boldsymbol{x}) = \frac{1}{8\pi\mu|\boldsymbol{x}|}\left(1 + \hat{x}_i\hat{x}_j\right) \ , \qquad \hat{x}_k = \frac{x_k}{|\boldsymbol{x}|}. \tag{5.61}$$

Defining the orthogonal projector (Π_{ik}) for $\hat{x}_k$ by

$$\Pi_{ik} := \delta_{ik} - \hat{x}_i\hat{x}_k, \tag{5.62}$$

we have

$$\partial_k|\boldsymbol{x}| = \frac{x_k}{|\boldsymbol{x}|} = \hat{x}_k, \tag{5.63a}$$

$$\partial_k\hat{x}_i = \frac{\delta_{ik}}{|\boldsymbol{x}|} - \frac{x_k x_i}{|\boldsymbol{x}|^3} = \frac{\Pi_{ik}}{|\boldsymbol{x}|}, \tag{5.63b}$$

$$\partial_n\Pi_{ik} = -\frac{1}{|\boldsymbol{x}|}\left(\hat{x}_i\Pi_{nk} + \hat{x}_k\Pi_{ni}\right), \tag{5.63c}$$