Moreover, we find for the Laplacian

$$
\begin{aligned}
\partial_k\partial_k J_{ij} &= \frac{\partial_k}{4\pi\mu}\left[-\delta_{ij}\frac{x_k}{|\boldsymbol{x}|^2}+\delta_{ik}\frac{x_j}{|\boldsymbol{x}|^2}+\delta_{jk}\frac{x_i}{|\boldsymbol{x}|^2}-2\frac{x_i x_j x_k}{|\boldsymbol{x}|^4}\right] \\
&= \frac{1}{4\pi\mu}\left[-\delta_{ij}\partial_k\left(\frac{x_k}{|\boldsymbol{x}|^2}\right)+\delta_{ik}\partial_k\left(\frac{x_j}{|\boldsymbol{x}|^2}\right)+\delta_{jk}\partial_k\left(\frac{x_i}{|\boldsymbol{x}|^2}\right)-2\partial_k\left(\frac{x_i x_j x_k}{|\boldsymbol{x}|^4}\right)\right] \\
&= \frac{1}{4\pi\mu}\Bigg[-\delta_{ij}\left(\frac{\delta_{kk}}{|\boldsymbol{x}|^2}-2\frac{x_k x_k}{|\boldsymbol{x}|^4}\right)+\delta_{ik}\left(\frac{\delta_{jk}}{|\boldsymbol{x}|^2}-2\frac{x_j x_k}{|\boldsymbol{x}|^4}\right)+\delta_{jk}\left(\frac{\delta_{ik}}{|\boldsymbol{x}|^2}-2\frac{x_i x_k}{|\boldsymbol{x}|^4}\right)- \\
&\qquad 2\left(\frac{\delta_{ik}x_j x_k}{|\boldsymbol{x}|^4}+\frac{x_i\delta_{jk}x_k}{|\boldsymbol{x}|^4}+\frac{x_i x_j\delta_{kk}}{|\boldsymbol{x}|^4}-4\frac{x_i x_j x_k x_k}{|\boldsymbol{x}|^6}\right)\Bigg] \\
&= \frac{1}{4\pi\mu}\Bigg[-\delta_{ij}\left(\frac{2}{|\boldsymbol{x}|^2}-2\frac{1}{|\boldsymbol{x}|^2}\right)+\left(\frac{\delta_{ij}}{|\boldsymbol{x}|^2}-2\frac{x_j x_i}{|\boldsymbol{x}|^4}\right)+\left(\frac{\delta_{ij}}{|\boldsymbol{x}|^2}-2\frac{x_i x_j}{|\boldsymbol{x}|^4}\right)- \\
&\qquad 2\left(\frac{x_j x_i}{|\boldsymbol{x}|^4}+\frac{x_i x_j}{|\boldsymbol{x}|^4}+2\frac{x_i x_j}{|\boldsymbol{x}|^4}-4\frac{x_i x_j}{|\boldsymbol{x}|^4}\right)\Bigg] \\
&= \frac{1}{2\pi\mu}\left(\frac{\delta_{ij}}{|\boldsymbol{x}|^2}-2\frac{x_j x_i}{|\boldsymbol{x}|^4}\right) \qquad (5.56)
\end{aligned}
$$