To verify that (5.51) is indeed a solution of the 2D Stokes equations, we first note that generally

$$\partial_j|\boldsymbol{x}| = \partial_j(x_i x_i)^{1/2} = x_j(x_i x_i)^{-1/2} = \frac{x_j}{|\boldsymbol{x}|} \tag{5.52a}$$

$$\partial_j|\boldsymbol{x}|^{-n} = \partial_j(x_i x_i)^{-n/2} = -n x_j(x_i x_i)^{-(n+2)/2} = -n\frac{x_j}{|\boldsymbol{x}|^{n+2}}. \tag{5.52b}$$