5.1 Navier-Stokes equations

Consider a fluid of conserved mass density $\varrho(t, \boldsymbol{x})$, governed by continuity equation

$$\partial_t \varrho + \nabla \cdot (\varrho \boldsymbol{u}) = 0, \tag{5.1}$$

where $\boldsymbol{u}(t, \boldsymbol{x})$ is local flow velocity. According to standard hydrodynamic theory, the dynamics of $\boldsymbol{u}$ is described by the Navier-Stokes equations (NSEs)

$$\varrho \left[\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}\right] = \boldsymbol{f} - \nabla p + \nabla \cdot \hat{T}, \tag{5.2}$$

where $p(t, \boldsymbol{x})$ the pressure in the fluid, $\hat{T}(t, \boldsymbol{x})$ the deviatoric[2]) stress-energy tensor of the fluid, and $\boldsymbol{f}(t, \boldsymbol{x})$ an external force-density field. A typical example of an external force $\boldsymbol{f}$, that is also relevant in the biological context, is the gravitational force

$$\boldsymbol{f} = \varrho \boldsymbol{g}, \tag{5.3}$$

where $\boldsymbol{g}(t, \boldsymbol{x})$ is the gravitational acceleration field.