

Seven Sketches in Compositionality: Real-World Applications of Category Theory

Topic 1. Cascade effects — *Posets and adjunctions*

- I. Posets, and transitivity
- II. Monotone functions, variance, and structure preserving maps
- III. Galois connections and adjunctions
- IV. Closures, kernels, monads, comonads
- V. Contagion

Topic 2. Data transformations — *Categories, functors, and universal constructions*

- I. Databases: schemas, instances, transformations
- II. Definition of category and functor, the category **Set**
- III. Graphs and autonomous dynamical systems

Topic 3. Resource theory — *Monoidal posets and categorification*

- I. Resources requirements and process plans
- II. Definition of monoidal posets
- III. Categorification: “from whether to which”

Topic 4. Collaborative design — *Enriched categories and profunctors*

- I. Problem statement: doing one part in a larger project
- II. Monotone design problems
- III. Boolean-enriched categories, functors, and profunctors
- IV. The compact-closed monoidal category
- V. Framed bicategory structure

Topic 5. Signal flow graphs — *Props and graphical proof systems*

- I. Props
- II. Generators and relations

- III. The prop of linear relations
- IV. Theorem: Sound and complete
- V. Signal flow graphs and stream semantics
- VI. Direction of signal flow as a derived concept

Topic 6. Electric circuits — *Wiring diagrams and functorial semantics*

- I. String diagrams for categories and monoidal categories
- II. Wiring diagrams (operads)
- III. The hypergraph category of labelled graphs
- IV. Functors that preserve extra structure
- V. The hypergraph functor from circuits of resistors to linear relations

Topic 7. Logic of behavior — *Sheaves, toposes, and internal languages*

- I. Problem statement: logic for composing machines
- II. The topos of sheaves on the interval domain.
- III. Type theories and semantics
- IV. How a safety proof might look