Cluster pictures for hyperelliptic curves

Raymond van Bommel
Massachusetts Institute of Technology
Simons Collaboration on Arithmetic Geometry, Number Theory, and Computation

These slides can be downloaded at
`raymondvanbommel.nl/talks/vantage.pdf`

Massachusetts Institute of Technology

Cluster pictures for hyperelliptic curves

Raymond van Bommel

Massachusetts Institute of Technology
Simons Collaboration on Arithmetic Geometry, Number Theory, and Computation

joint with:

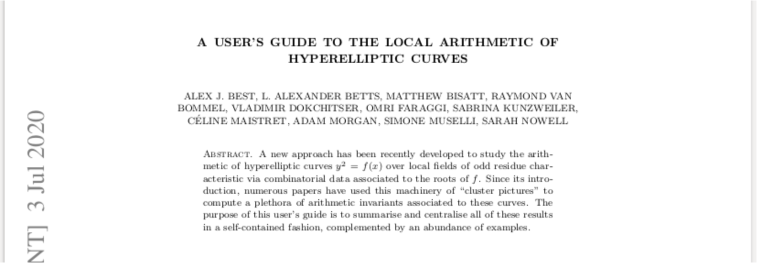
Alex J. Best	(Boston University)
L. Alexander Betts	(Max-Planck-Institut für Mathematik)
Matthew Bisatt	(University of Bristol)
Vladimir Dokchitser	(University College London)
Omri Faraggi	(University College London)
Sabrina Kunzweiler	(Universität Ulm)
Céline Maistret	(Boston University)
Adam Morgan	(Max-Planck-Institut für Mathematik)
Simone Muselli	(University of Bristol)
Sarah Nowell	(University College London)

Massachusetts Institute of Technology

Acknowledgements

We would like to thank:

- Edgar Costa for technical assistance.
- ICERM for hosting the virtual workshop *'Arithmetic Geometry, Number Theory, and Computation'* where this work has been done.
- The numerous organisations that supported the individual members of the group. A full list is in in our preprint, [ArXiv:2007.01749].



NT] 3 Jul 2020

A USER'S GUIDE TO THE LOCAL ARITHMETIC OF HYPERELLIPTIC CURVES

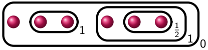
ALEX J. BEST, L. ALEXANDER BETTS, MATTHEW BISATT, RAYMOND VAN BOMMEL, VLADIMIR DOKCHITSER, OMRI FARAGGI, SABRINA KUNZWEILER, CÉLINE MAISTRET, ADAM MORGAN, SIMONE MUSELLI, SARAH NOWELL

ABSTRACT. A new approach has been recently developed to study the arithmetic of hyperelliptic curves $y^2 = f(x)$ over local fields of odd residue characteristic via combinatorial data associated to the roots of f. Since its introduction, numerous papers have used this machinery of "cluster pictures" to compute a plethora of arithmetic invariants associated to these curves. The purpose of this user's guide is to summarise and centralise all of these results in a self-contained fashion, complemented by an abundance of examples.

Outline

This talk will consist of three parts:

- What is a cluster picture?
- What can you do with a cluster picture?
- Cluster pictures in SageMath and the LMFDB.



Definitions

In this talk, K is a local field of odd residue characteristic p, e.g. $K = \mathbb{Q}_p$. Let C/K be a hyperelliptic curve of genus g given by the equation

$$y^2 = c \prod_{r \in \mathcal{R}} (x - r), \qquad \text{where } \mathcal{R} \subset K^{\mathrm{sep}} \text{ is finite.}$$

Definition (cluster, depth)

A non-empty subset of $\mathfrak{s} \subset \mathcal{R}$ is called a *cluster*, if it is of the form $D \cap \mathcal{R}$, where D is some p-adic disc inside K^{sep}. If $|\mathfrak{s}| \geq 2$, then the valuation of the radius of the smallest such disc D, is called the (absolute) *depth* $d_{\mathfrak{s}}$ of $\mathfrak{s}$.

Definition (parent, child)

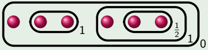
If $\mathfrak{c} \subsetneq \mathfrak{p}$ are two clusters, such that there is no cluster $\mathfrak{s}$ with $\mathfrak{c} \subsetneq \mathfrak{s} \subsetneq \mathfrak{p}$, then $\mathfrak{p}$ is the *parent* of $\mathfrak{c}$, and $\mathfrak{c}$ is a *child* of $\mathfrak{p}$.

Definition (relative depth)

For a cluster $\mathfrak{s} \subsetneq \mathcal{R}$ with parent $\mathfrak{p}$, the *relative depth* of $\mathfrak{s}$ is $\delta_{\mathfrak{s}} := d_{\mathfrak{s}} - d_{\mathfrak{p}}$. By convention, we define $\delta_{\mathcal{R}} = d_{\mathcal{R}}$.

Example

Consider $C_1\colon y^2 = (x+2)(x-1)(x-2)(x-3)(x^2-27)$ over $\mathbb{Q}_3$. It has the following cluster picture:

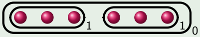


Here, the red dots depict the individual roots inside $\mathcal{R}$ and the number written next to each cluster, except for $\mathcal{R}$, is its relative depth. The number next to the big cluster $\mathcal{R}$ is the (absolute) depth of the cluster.

From the picture, it is immediately obvious that the Galois group has to fix each cluster of size > 1.

Example

For $C_2\colon y^2 = 3(x^2-2)((x-3)^2-2)((x+3)^2-2)$ over $\mathbb{Q}_3$, we get:



Here, Frobenius swaps the two clusters of size 3, but the inertia group acts trivially on the clusters.

Reduction types

Theorem (potential good reduction, [Theorem 5.5])

The curve C has potential good reduction (i.e. it obtains good reduction after an extension of the base field) if and only if there is no cluster $\mathfrak{s}$ such that $1 < |\mathfrak{s}| < 2g + 1$. The Jacobian of C has potential good reduction if and only if every cluster $\mathfrak{s} \neq \mathcal{R}$ has odd cardinality.

Example

Both C_1 and C_2, and the Jacobian of C_1 do not have potential good reduction, but the Jacobian of C_2 does have potential good reduction, as all the clusters except for $\mathcal{R}$ have size 1 or 3.

We can also determine from the cluster picture:

- whether the curve C has good or semistable reduction,
- whether the Jacobian of C has good or semistable reduction,
- the potential toric rank of the Jacobian J of C (i.e. the dimension of the toric part of J, after a base extension which makes J semi-abelian).

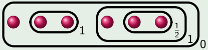
Minimal regular model

Example

Consider $C_1\colon y^2 = f(x) = (x+2)(x-1)(x-2)(x-3)(x^2-27)$ over $\mathbb{Q}_3$, and consider the cluster $\mathfrak{s} = \{-2, 1\}$ of depth $d_{\mathfrak{s}} = 1$. We then define

$$f_{\mathfrak{s}}(x) = \frac{1}{9} f(3^{d_{\mathfrak{s}}} \cdot x + 1) = \frac{1}{9} \cdot 3x(3x+3)(3x-1)(3x-2)((3x-1)^2 - 27),$$

where this factor $\frac{1}{9}$ is exactly chosen in such a way to make each of the factors primitive (i.e. the coefficients lie in $\mathcal{O}_K$, but not all in $\mathfrak{m}_K$).



We consider the subscheme of $\mathbb{A}^2_{\mathcal{O}_K}$ given by $y^2 = f_{\mathfrak{s}}(x)$. We let $\mathcal{U}_{\mathfrak{s}}$ be the open subscheme obtained by removing the points in the special fibre corresponding to double roots of $f_{\mathfrak{s}}$.

We can define such a scheme $\mathcal{U}_{\mathfrak{s}}$ for any cluster $\mathfrak{s}$. We can glue them to some other schemes $\mathcal{W}_{\mathfrak{s}}$ and $\mathcal{U}_{P,\mathfrak{s}}$ to obtain a regular model of C over $\mathcal{O}_K$, when C is semistable. It is exactly understood which components to blow down to obtain a minimal regular model. The dual graph, and the reduction map to the special fibre can all be expressed in terms of the cluster picture.

Conductor

Definition (even/übereven cluster)

A cluster $\mathfrak{s}$ is called *even* if $|\mathfrak{s}|$ is even. Moreover, $\mathfrak{s}$ is called *übereven* if all children of $\mathfrak{s}$ are even.

Theorem (conductor, [Theorem 12.1])

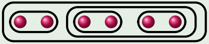
Suppose C/K is semistable. The valuation of the conductor of $\mathrm{Jac}(C)$ equals

$$n_C = \begin{cases} |A| - 1 & \textit{if } \mathcal{R} \textit{ is übereven}, \\ |A| & \textit{otherwise}, \end{cases}$$

where $A = \{\textit{even clusters } \mathfrak{s} \neq \mathcal{R} \mid \mathfrak{s} \textit{ is not übereven}\}$.

Example

Consider a curve with the following cluster picture:



Then A consists of the three clusters of size 2. The cluster of size 4 and $\mathcal{R}$ are both übereven. Hence, the conductor exponent equals $|A| - 1 = 2$.

Differentials

Let $\mathcal{C}/\mathcal{O}_K$ be a regular model. The standard differentials $\omega_i = x^i \frac{dx}{y}$ typically do not give rise to generators for $\omega_{\mathcal{C}/\mathcal{O}_K}(\mathcal{C})$ (i.e. the differentials used in the definition of the period in the BSD formula). However, there does exist a scalar $\frac{\omega^\circ}{\omega} \in K^*$ such that

$$\omega_0^\circ \wedge \cdots \wedge \omega_{g-1}^\circ = \frac{\omega^\circ}{\omega} \cdot \omega_0 \wedge \cdots \wedge \omega_{g-1} \quad \text{in } \bigwedge^g \Omega^1_{C/K}(C),$$

where $\omega_0^\circ, \ldots, \omega_{g-1}^\circ$ is a basis of $\omega_{\mathcal{C}/\mathcal{O}_K}(\mathcal{C})$.

Theorem (differentials, [Theorem 14.6])

Suppose C is semistable. Then

$$8 \cdot \mathrm{val}_K\left(\frac{\omega^\circ}{\omega}\right) = 4g \cdot \mathrm{val}_K(c) + \sum_{\substack{\mathfrak{s} \\ |\mathfrak{s}| \text{ is even}}} \delta_{\mathfrak{s}}(|\mathfrak{s}| - 2)|\mathfrak{s}| + \sum_{\substack{\mathfrak{s} \\ |\mathfrak{s}| \text{ is odd}}} \delta_{\mathfrak{s}}(|\mathfrak{s}| - 1)^2.$$

In fact, we can explicitly describe generators for $\omega_{\mathcal{C}/\mathcal{O}_K}(\mathcal{C})$ in terms of $\omega_0, \ldots, \omega_{g-1}$, and data extracted from the cluster picture of C.

Disriminant and minimal discriminant

Theorem (discriminant and minimal discriminant, [Theorem 15.1/15.2])

The valuation of the discriminant of the defining model for C is

$$\mathrm{val}_K(\Delta_C) = \mathrm{val}_K(c)(4g+2) + \sum_{\mathfrak{s}} \delta_s |\mathfrak{s}|(|\mathfrak{s}|-1).$$

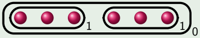
If C/K is semistable and the residue field of K has more than $2g+1$ elements, then

$$\frac{\mathrm{val}_K(\Delta_C) - \mathrm{val}_K(\Delta_C^{\min})}{4g+2} = \mathrm{val}_K(c) - E + \sum_{\substack{\mathfrak{s} \\ |\mathfrak{s}|>g+1}} \delta_{\mathfrak{s}}(|\mathfrak{s}|-g-1),$$

where $E=0$ unless there are two clusters of size $g+1$ that are swapped by Frobenius and $\mathrm{val}_K(c)$ is odd, in which case $E=1$.

Example

For $C_2\colon\, y^2 = 3(x^2-2)((x-3)^2-2)((x+3)^2-2)$ of genus 2 over $\mathbb{Q}_3$,



we get $\mathrm{val}_K(\Delta_C) = 10+0+6+6 = 22$ and $\mathrm{val}_K(\Delta_C^{\min}) = 22$ as well.

Other things

Some other things that can be computed using cluster pictures:

- minimal regular strict normal crossings model (tame reduction case),
- Tamagawa numbers (semistable reduction case),
- decomposition of the ℓ-adic Galois representation $H^1_{\text{ét}}(C/\overline{K}, \mathbb{Q}_\ell)$ (when ℓ is invertible in the residue field),
- tame and wild part of the conductor exponent in the non-semistable case,
- root numbers, i.e. the sign occurring in the conjectured functional equation for the L-function of $\mathrm{Jac}(C)$ (tame reduction case),
- different criteria to determine if a model is a minimal Weierstraß model.

Implementation in SageMath

Together with Alex J. Best, I implemented a package for SageMath.

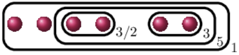
The code can be found at `github.com/alexjbest/cluster-pictures`.

In [3]:

Slide Type -

```
from sage_cluster_pictures.cluster_pictures import *
K = Qp(5)
x = polygen(K)
H = HyperellipticCurve((x^2 + 5^2)*(x^2 - 5^15)*(x - 5^6)*(x - 5^6 - 5^9))
R = Cluster.from_curve(H)
view(R)
```

Out[3]:



Slide Type Slide

Creating clusters

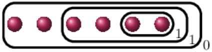
From roots:

In [3]:

Slide Type

```
view(Cluster.from_roots([K(1), K(2), K(5), K(10), K(25), K(50)]))
```

Out[3]:



Slide Type

From valuations:

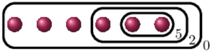
$$y^2 = (x-1)(x-2)(x-3)(x-p^2)(x-p^7)(x+p^7)$$

In [4]:

Slide Type -

```
view(Cluster([[oo, 0,  0,  0,  0,  0],
              [0,  oo, 0,  0,  0,  0],
              [0,  0,  oo, 0,  0,  0],
              [0,  0,  0 , oo, 2,  2],
              [0,  0,  0,  2,  oo, 7],
              [0,  0,  0,  2,  7,  oo]]))
```

Out[4]:



Slide Type Slide

Basic properties

In [5]: Slide Type -

```
print(R.children())
```

```
[Cluster with 4 roots and 2 children, Cluster with 1 roots and 0 children, Cluster with 1 roots and 0 children]
```

In [6]: Slide Type -

```
[unicode_art(D) for D in R.all_descendents()]
```

Out[6]:

```
['(((● ●)_3/2 (● ●)_3)_5 ● ●)_1',
 '((● ●)_3/2 (● ●)_3)_5',
 '(● ●)_3/2',
 '●',
 '●',
 '(● ●)_3',
 '●',
 '●',
 '●',
 '●']
```

In [7]: Slide Type

```
R.is_semistable(K)
```

Out[7]:

```
True
```

In [4]: Slide Type

```
(R.jacobian_has_potentially_good_reduction(), R.potential_toric_rank())
```

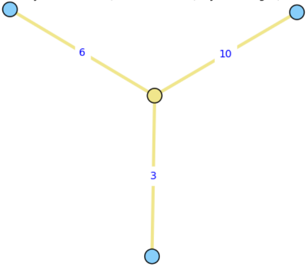
Out[4]:

```
(False, 2)
```

In [11]: Slide Type Slide

```
T = R.BY_tree(); T
```

Out[11]: BY-tree with 1 yellow vertices, 3 blue vertices, 3 yellow edges, 0 blue edges



In [12]: Slide Type

```
T.vertices()
```

Out[12]:

```
[Cluster with 6 roots and 3 children,
 Cluster with 4 roots and 2 children,
 Cluster with 2 roots and 2 children,
 Cluster with 2 roots and 2 children]
```

Slide Type: Slide

Via the cluster picture and the homology of the dual graph of the special fibre:

In [13]: (Slide Type: -)

```
R.root_number()
```

Out[13]: 1

Slide Type

Via the associated BY-tree:

In [14]: (Slide Type)

```
R.tamagawa_number()
```

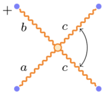
Out[14]: 108

In [15]: (Slide Type)

```
T, F = R.BY_tree(with_frob=True)
T.tamagawa_number(F)
```

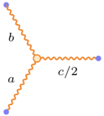
Out[15]: 108

FIGURE 1. The BY tree associated to X_f



In this diagram, the whole graph represents the tree T, while the blue/solid vertices represent the vertices of S (which has no edges in this example) – by contrast, the vertices of T not in S are represented by yellow/open circles and the edges of T not in S are represented by yellow/squiggly lines. The lengths of the edges are indicated by the parameters a, b and c, while the signed automorphism is indicated both with double-headed arrows for the underlying unsigned automorphism of (T, S) (which here has order 2) and with $\pm$ signs next to each connected component[10] of $T \setminus S$ (so here the sign is $+$).

Since $T \setminus S$ is connected, there is only one term in the formula from theorem 3.0.1, and since $\epsilon(C_i) = +1$ for the unique component C_i, we have $(T'_i, S'_i) = (T, S)$ and $\tilde{c}_{1,i} = 1$. By inspection, $Q_i = 2$ and the quotient tree T''_i, along with its subgraph S''_i, are given by the following diagram



where again the blue/solid vertices indicate the subset $S' \subseteq T'$ and the labels indicate edge-lengths. The removal of any two of the three edges of this graph disconnects the three points of S' from one another, and hence the formula in theorem 3.0.1 provides that the Tamagawa number of X_f is

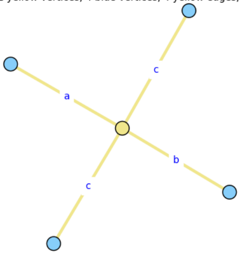
$$c_{X_f} = 2 \cdot \left(ab + b\frac{c}{2} + \frac{c}{2}a\right) = 2ab + bc + ca.$$

Example from a paper by L. Alexander Betts

In [40]: Slide Type Slide

```
a, b, c = var('a b c', domain="positive")
R = Cluster([[oo, a/2, 0,  0,  0,  0,   0,  0],
             [a/2, oo, 0,  0,  0,  0,   0,  0],
             [0,  0,  oo, b/2, 0,  0,   0,  0],
             [0,  0,  b/2, oo, 0,  0,   0,  0],
             [0,  0,  0,  0,  oo, c/2,  0,  0],
             [0,  0,  0,  0,  c/2, oo,  0,  0],
             [0,  0,  0,  0,  0,  0,  oo, c/2],
             [0,  0,  0,  0,  0,  0,  c/2, oo], ])
A,B = [x for x in R.children() if x.depth() in {a/2, b/2}]
C1,C2 = [x for x in R.children() if x.depth() == c/2]
T = R.BY_tree(); T
```

Out[40]: BY tree with 1 yellow vertices, 4 blue vertices, 4 yellow edges, 0 blue edges



In [44]: Slide Type -

```
F = BYTreeIsomorphism(T, T,
                      lambda x: {A:A, B:B, C1:C2, C2:C1, R:R}[x],
                      lambda Y: 1)
T.tamagawa_number(F)
```

Out[44]: $2\,ab + ac + bc$

Cluster pictures in the LMFDB

Cluster pictures have been computed for the following curves in the LMFDB:

- elliptic curves over $\mathbb{Q}$,
- elliptic curves over number fields,
- curves of genus 2 over $\mathbb{Q}$.

For the curves of genus 2, they are already visible on the webpages.

Genus 2 curve 39690.a.277830.1

Introduction
Overview Random
Universe Knowledge
L-functions
Degree 1 Degree 2
Degree 3 Degree 4
ζ zeros
Modular forms
Classical Maass
Hilbert Bianchi
Varieties
Elliptic curves over $\mathbb{Q}$
Elliptic curves over $\mathbb{Q}(\alpha)$
Genus 2 curves over $\mathbb{Q}$
Higher genus families
Abelian varieties over $\mathbb{F}_q$
Fields
Number fields
p-adic fields
Representations
Dirichlet characters
Artin representations
Groups
Galois groups
Sato-Tate groups

Show commands for: SageMath / Magma

Minimal equation

$y^2 + (x^3 + x)y = x^5 - 5x^3 - 7x^2 + 3x + 15$ (homogenize, simplify)

Invariants

Conductor:	N	=	39690	=	$2 \cdot 3^4 \cdot 5 \cdot 7^2$
Discriminant:	Δ	=	277830	=	$2 \cdot 3^4 \cdot 5 \cdot 7^3$

Igusa-Clebsch invariants

I_2	=	2304	=	$2^8 \cdot 3^2$
I_4	=	428472	=	$2^3 \cdot 3^2 \cdot 11 \cdot 541$
I_6	=	241891497	=	$3^2 \cdot 26876833$
I_{10}	=	-1111320	=	$-2^3 \cdot 3^4 \cdot 5 \cdot 7^3$

(Igusa invariants, G2 invariants)

Automorphism group

$\mathrm{Aut}(X) \simeq C_2$
$\mathrm{Aut}(X_{\overline{\mathbb{Q}}}) \simeq C_2$

Rational points

All points: $(1:0:0), (1:-1:0), (5:20:4), (5:-225:4)$

Number of rational Weierstrass points: 0

This curve is locally solvable everywhere.

Mordell-Weil group of the Jacobian

Group structure: $\mathbb{Z}$

Generator	D_0	Height	Order
$D_0 - (1:-1:0) - (1:0:0)$	$2x^2 + xz - 7z^2 = 0,\ 2y = -7xz^2 + 5z^3$	0.042240	∞

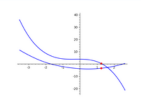
2-torsion field: 6.2.576108288000.65

BSD invariants

Hasse-Weil conjecture: unverified

Properties

Label	39690.a.277830.1



Conductor	39690
Discriminant	277830
Mordell-Weil group	$\mathbb{Z}$
Sato-Tate group	$\mathrm{USp}(4)$
$\mathrm{End}(J_{\overline{\mathbb{Q}}}) \otimes \mathbb{R}$	$\mathbb{R}$
$\mathrm{End}(J_{\overline{\mathbb{Q}}}) \otimes \mathbb{Q}$	$\mathbb{Q}$
$\mathrm{End}(J) \otimes \mathbb{Q}$	$\mathbb{Q}$
$\overline{\mathbb{Q}}$**-simple**	yes
GL_2**-type**	no

Related objects

Isogeny class 39690.a
Twists
L-function

Learn more about

Completeness of the data
Source of the data
Reliability of the data
Genus 2 curve labels

LMFDB

⌂ → Genus 2 curves → $\mathbb{Q}$ → 39690 → a → 277830 → 1

Feedback · Hide Menu

Genus 2 curve 39690.a.277830.1

Introduction
Overview Random
Universe Knowledge

L-functions
Degree 1 Degree 2
Degree 3 Degree 4
ζ zeros

Modular forms
Classical Maass
Hilbert Bianchi

Varieties
Elliptic curves over $\mathbb{Q}$
Elliptic curves over $\mathbb{Q}(\alpha)$
Genus 2 curves over $\mathbb{Q}$
Higher genus families
Abelian varieties over $\mathbb{F}_q$

Fields
Number fields
p-adic fields

Representations
Dirichlet characters
Artin representations

Groups
Galois groups
Sato-Tate groups

Show commands for: SageMath / Magma

Simplified equation

$y^2 = x^6 + 4x^5 + 2x^4 - 20x^3 - 27x^2 + 12x + 60$ (minimize, homogenize)

Invariants

Conductor:	N	=	39690	=	$2 \cdot 3^4 \cdot 5 \cdot 7^2$
Discriminant:	Δ	=	277830	=	$2 \cdot 3^4 \cdot 5 \cdot 7^3$

Igusa-Clebsch invariants

I_2	=	2304	=	$2^8 \cdot 3^2$
I_4	=	428472	=	$2^3 \cdot 3^2 \cdot 11 \cdot 541$
I_6	=	241891497	=	$3^2 \cdot 26876833$
I_{10}	=	-1111320	=	$-2^3 \cdot 3^4 \cdot 5 \cdot 7^3$

(Igusa invariants, G2 invariants)

Automorphism group

$\mathrm{Aut}(X) \simeq C_2$
$\mathrm{Aut}(X_{\overline{\mathbb{Q}}}) \simeq C_2$

Rational points

All points: $(1 : 0 : 0), (1 : -1 : 0), (5 : 20 : 4), (5 : -225 : 4)$

Number of rational Weierstrass points: 0

This curve is locally solvable everywhere.

Mordell-Weil group of the Jacobian

Group structure: $\mathbb{Z}$

Generator	D_0	Height	Order
$D_0 - (1 : -1 : 0) - (1 : 0 : 0)$	$2x^2 + xz - 7z^2 = 0,\ 2y = -7xz^2 + 5z^3$	0.042240	∞

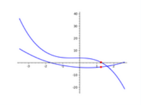
2-torsion field: 6.2.576108288000.65

BSD invariants

Hasse-Weil conjecture: unverified

Properties

Label	39690.a.277830.1



Conductor	39690
Discriminant	277830
Mordell-Weil group	$\mathbb{Z}$
Sato-Tate group	$\mathrm{USp}(4)$
$\mathrm{End}(J_{\overline{\mathbb{Q}}}) \otimes \mathbb{R}$	$\mathbb{R}$
$\mathrm{End}(J_{\overline{\mathbb{Q}}}) \otimes \mathbb{Q}$	$\mathbb{Q}$
$\mathrm{End}(J) \otimes \mathbb{Q}$	$\mathbb{Q}$
$\overline{\mathbb{Q}}$**-simple**	yes
GL_2**-type**	no

Related objects
Isogeny class 39690.a
Twists
L-function

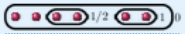
Learn more about
Completeness of the data
Source of the data
Reliability of the data
Genus 2 curve labels

2-torsion field: 6.2.576108288000.65

BSD invariants

Hasse-Weil conjecture:	unverified
Analytic rank:	1
Mordell-Weil rank:	1
2-Selmer rank:	1
Regulator:	0.042240
Real period:	11.33844
Tamagawa product:	2
Torsion order:	1
Leading coefficient:	0.957883
Analytic order of Ш:	1 (rounded)
Order of Ш:	square

Local invariants

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
2	1	1	1	$(1+T)(1+2T^2)$	
3	4	4	1	$1+T+3T^2$	
5	1	1	1	$(1-T)(1+2T+5T^2)$	
7	2	3	2	$(1+T)^2$	

Sato-Tate group

$\mathrm{ST} \simeq \mathrm{USp}(4)$
$\mathrm{ST}^0 \simeq \mathrm{USp}(4)$

Decomposition of the Jacobian

Simple over $\overline{\mathbb{Q}}$

Endomorphisms of the Jacobian

Not of GL_2-type over $\mathbb{Q}$

Endomorphism ring over $\mathbb{Q}$:

$\mathrm{End}(J) \simeq \mathbb{Z}$
$\mathrm{End}(J) \otimes \mathbb{Q} \simeq \mathbb{Q}$
$\mathrm{End}(J) \otimes \mathbb{R} \simeq \mathbb{R}$

All $\overline{\mathbb{Q}}$-endomorphisms of the Jacobian are defined over $\mathbb{Q}$.

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
2	1	1	1	$(1+T)(1+2T^2)$	
3	4	4	1	$1+T+3T^2$	

Data for a cluster picture

Cluster picture c3c3_2~3_0

Depth: 0
Size: 6
Potential toric rank of reduction of curve: 0
Potential good reduction of curve: False
Potential good reduction of Jacobian: True

permalink · (awaiting review)

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
5	1	1	1	$(1-T)(1+2T+5T^2)$	

Data for a cluster picture

Cluster picture c4c2_1~2_0

Depth: 0
Size: 6
Potential toric rank of reduction of curve: 1
Potential good reduction of curve: False
Potential good reduction of Jacobian: False

permalink · (awaiting review)

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
7	2	3	2	$(1+T)^2$	

Data for a cluster picture

Cluster picture c2c2_1~2c2_1_0

Depth: 0
Size: 6
Potential toric rank of reduction of curve: 2
Potential good reduction of curve: False
Potential good reduction of Jacobian: False

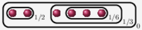
permalink · (awaiting review)

Cluster pictures for curve 39690.a.277830.1

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
3	4	6	5	$1-T$	

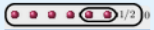
Data for a cluster picture

Cluster picture cc2_1~2c1c3_1~6_1~3_0



Depth: 0
Size: 6
Potential toric rank of reduction of curve: 1
Potential good reduction of curve: False
Potential good reduction of Jacobian: False

permalink · (awaiting review)

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
5	1	1	1	$(1-T)(1-T+5T^2)$	

Data for a cluster picture

Cluster picture c4c2_1~2_0

Depth: 0
Size: 6
Potential toric rank of reduction of curve: 1
Potential good reduction of curve: False
Potential good reduction of Jacobian: False

permalink · (awaiting review)

Prime	ord(N)	ord(Δ)	Tamagawa	L-factor	Cluster picture
7	2	2	1	$1+T^2$	

Data for a cluster picture

Cluster picture c2c2_1~2c2_1~2_0

Depth: 0
Size: 6
Potential toric rank of reduction of curve: 2
Potential good reduction of curve: False
Potential good reduction of Jacobian: False

permalink · (awaiting review)

Cluster pictures for curve 19845.b.178605.1