18.100B Problem Set 1

Due in class Monday, February 9. You may discuss the problems with other students, but you should write solutions entirely on your own.

1. Text, page 22, number 9.

2. Text, page 22, number 10.

3. A *funny number* is defined to be an ordered pair $[a, b]$ of rational numbers. Define addition and multiplication of funny numbers by the rules

$$[a, b] + [c, d] = [a + c, b + d], \qquad [a, b][c, d] = [ac + 3bd, ad + bc].$$

Prove that axioms M4 and M5 on page 5 (existence of identity and inverses for multiplication) are satisfied by funny numbers.

(In fact the funny numbers satisfy all of the axioms for a field.)

4. What happens if you replace 3 in the definition of "funny number" by 4?