September 15: Pavel Etingof, "Hochschild cohomology of preprojective algebras."

Preprojective algebras of quivers were introduced by Gelfand and Ponomarev as a technical tool of studying quiver representations. More recently it became clear that these algebras are very interesting objects by themselves. Namely, their varieties of representations, called quiver varieties, turned out to play a central role in geometric representation theory, and consequently were studied in detail by many people, notably W. Crawley-Boevey, G. Lusztig, and H. Nakajima.

I will talk about Hochschild cohomology of preprojective algebras Π, which was computed in a work by Crawley-Boevey, Ginzburg, and myself (excluding the finite and affine Dynkin cases). First of all, it turns out that the cohomology is concentrated in degrees 0,1,2, (in particular, deformations of Π are unobstructed). Next, we show that $H^0(\Pi) = \mathbf{C}$, $H^1(\Pi) = \mathbf{C}E \ltimes (L/R)$, and $H^2(\Pi) = L$, where $L = \Pi/[\Pi, \Pi]$, R is the degree zero part of Π and L (spanned by the vertex idempotents), and E is the derivation of L induced by the grading of Π. The space L has a Lie algebra structure ("necklace bracket"), which determines the bracket on $H^1(\Pi)$ and more generally the structure of a Gerstenhaber algebra on the total cohomology $H^*(\Pi)$. Finally, we calculate the Hilbert series of L. The proofs are based on the theory of quiver varieties and the theory of random unitary matrices of large size.