March 12, 2003: David Vogan (MIT) "A blip in the Shimura correspondence for F_4." (Joint with Dan Barbasch.)

Suppose $G = KAN$ is an Iwasawa decomposition of the split real form of a simply connected simple group. There is a unique connected double cover $\widetilde{G}$ of G. There is a "pseudospherical" principal series for $\widetilde{G}$ that behaves formally much like the spherical principal series for G; for example, the representations in the series are parametrized essentially by Weyl group orbits in $\widehat{A}$, the space of (possibly non-unitary) characters of A. A little more precisely, there is a second simply connected simple group G' sharing a common split component A, and the same Weyl group; and a natural "Shimura correspondence"

$$\{\text{spherical representations of } G'\} \leftrightarrow \left\{\text{pseudospherical representations of } \widetilde{G}\right\}.$$

In case $G = SL(n)$, Jing-Song Huang proved that this correspondence respects unitarity. Similar results were proved for other groups (including G_2), by calculating the two sides separately and comparing. Last year joint work of Jeff Adams, Dan Barbasch, Annegret Paul, Peter Trapa, and I led to a general approach to this question. (The terminology comes from Shimura's work on automorphic forms for $SL(2)$, and there ought to be an automorphic version of the results; but what we have done is entirely local over $\mathbb{R}$.)

Recently Dan Barbasch and I were filling in some details in this approach for the exceptional groups. To our surprise, we found evidence that the correspondence may work in only one direction for unitary representations: that unitary representations of G' may correspond to unitary representations of $\widetilde{G}$, but not conversely. In this talk I will set up an example in F_4, and explain where to look for the mysterious extra unitary representations of $\widetilde{F}_4$.